Dynamic and thermal study of the influence of an axial magnetic field on flow between two horizontal cylinders in rotation

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Abstract :

This study is interested in the effect of an axial magnetic field imposed on incompressible flow of electrically conductive fluid between two horizontal coaxial cylinders. The imposed magnetic field is assumed uniform and constant, we also take into account the effect of heat generation due to viscous dissipation. The inner and outer cylinders are maintained at different and uniform temperatures. The movement of the fluid is due to the rotation of the cylinders with a constant speed. An exact solution of the equations governing the flow was obtained in the form of Bessel functions. A finite difference implicit scheme was used in the numerical solution. The velocity and temperature distributions were obtained with and without the magnetic field. The results show that for different values of the Hartmann number, the velocity between the two cylinders decreases as the Hartmann number increases. Also, it is found that by increasing the Hartmann number, the average Nusselt number decreases. On the other hand, the Hartmann number does not affect the temperature.

Key words: **Rotating cylinders, magnetic field, Bessel function, finite difference.**

1 Introduction

The study of flow of electrically conductive fluids, called magnetohydrodynamic (MHD) has attracted much attention due to its various applications. In astrophysics and geophysics, it is applied to the study of stellar structures, terrestrial cores and solar plasma. In industrial processes, it finds its application in MHD pumps, nuclear reactors, the extraction of geothermal energy, metallurgical and crystal growth in the field of semiconductors, the control of the behavior of fluid flow and heat and mass transfer and the stability of convective flows. Several studies have been conducted to evaluate the effect of magnetic field on the convective flows for different conditions. Tatsuo Sawada et al [1] carried out experimental investigations about the natural convection of a magnetic fluid between two concentric cylinders and horizontal isotherms. H. Ben Hadid, and D. Henry [2] investigated numerically the effect of a constant magnetic field on a three-dimensional buoyancy-induced flow in a cylindrical cavity, they put in light the structural changes of the flow induced by the magnetic field for each field orientation. Singh, SK Jha, BK and Singh, AK [3] presented exact solutions for fully developed natural convection in open-ended vertical concentric annuli under a radial magnetic field. El Amin, MF [4] studied the effects of both first- and second-order resistance due to the solid matrix on forced convective flow from a horizontal circular cylinder in the presence of a magnetic field and viscous dissipation, with a variable surface temperature boundary condition. The study of the effects of the azimuthal magnetic field of an electrically conducting fluid in a rotating annulus has also been presented by Kurt, E et al [5]. Sankar, M et al [6] studied numerically a natural convection of a low Prandtl number electrically conducting fluid under the influence of either axial or radial magnetic field in a vertical cylindrical annulus. They showed that the magnetic field can be suppress the flow and heat transfer. W. Wrobel et al [7] presented an experimental and numerical analysis of a thermo-magnetic convective flow of paramagnetic fluid in an annular enclosure with a round rod core and a cylindrical outer wall under gravitational and magnetic environments. OD Makinde and OO Onyejekwe [8] investigated a steady flow and heat transfer of an electrically conducting fluid with variable viscosity and electrical conductivity between two parallel plates in the presence of a transverse magnetic field. SC Kakarantzas et al [9] studied numerically the combined effect of a horizontal magnetic field and volumetric heating on the natural convection flow and heat transfer of a low Prandtl number fluid in a vertical annulus. Omid Mahian et al [10] presented an analysis of the first and second laws of thermodynamics to show the effects of MHD flow on the distributions of velocity, temperature and entropy generation between two concentric rotating cylinders. R.H Mozayyeni and A.B Rahimi [11] investigated numerically the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths, in the presence of a constant magnetic field with a radial MHD force direction, considering the effects of viscous heat dissipation in the fluid in both steady and unsteady states.

Although the exact solutions for the Hartmann flow and the MHD Couette flow have been achieved for more than seventy years, the solutions for a heat transfer in flow between concentric rotating cylinders, also known as Taylore Couette flows, under external magnetic field have been restricted to high Hartmann numbers.

The aim of the present study is to examine analytically and numerically the effects of an external axial magnetic field applied to the forced convection flow of an electrically conducting fluid between two horizontal concentric cylinders, considering the effects of viscous heat dissipation in the fluid. It should be noted that the natural convection is supposed négligeable in this work, which is not always the case of the vertical cylinder. The forced flow is induced by the rotating inner cylinder, in slow constant angular velocity and the other is fixed.

2 Formulation of the problem

Consider a laminar flow of a viscous incompressible electrically conductive fluid between two coaxial cylinders. The inner cylinder of radius r_1 is rotated at a constant speed Ω_1 and the outer cylinder of radius r_2 is fixed. The inner and outer walls are maintained at a constant and different temperatures T_1 and T_2 respectively, while the top and bottom walls are insulated.

The two cylinders are electrically isolated. The flow is subjected to a magnetic field B of constant magnitude, uniform and axially oriented. We assume that the magnetic Reynolds number Rm is much smaller than unity. One can then consider the flow equations within the low-Rm approximation in which the coupling between the velocity and the magnetic field is weak. If furthermore the magnetic field is uniform and externally applied, its time variations can be neglected and the set of flow equations further simplified to involve only the Navier-Stokes equations and the conservation of the electric current including Ohm's law. [12]. Also we assume that no polarization voltage is applied (the electric field is zero). Further, in this study the viscous dissipation term in the energy equation is considered.

3 Analytical study

The flow is assumed to be steady, laminar and unidirectional, therefore the radial and axial components of the velocity and the derivatives of the velocity with respect to θ and z are zero. Under these assumptions and in cylindrical coordinates, the non-dimensional equations governing the flow together with the boundary conditions in the azimuthal direction can be written as follows (where the stars are dropped for convenience):

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\left(\frac{Ha^2}{(1-\eta)^2}+\frac{1}{r^2}\right)v=0 \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)=-Ec\,\mathrm{Pr}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)^2 \tag{2}$$

$$r=\eta:\ v(r)=1\ ,\ \theta=1 \tag{3}$$

$$r=1:\ v(r)=0\ ,\ \theta=0 \tag{4}$$

Where:

$r^*=\dfrac{r}{r_2}, v^*=\dfrac{v}{\Omega_1 r_1}, \eta=\dfrac{r_1}{r_2},\ Ha=B_0 d\sqrt{\dfrac{\sigma}{\rho\upsilon}},\ \theta=\dfrac{T-T_2}{T_1-T_2},\ \mathrm{Pr}=\dfrac{\upsilon}{a},\ Ec=\dfrac{(\Omega_1 r_1)^2}{Cp\Delta T}$: are the dimensionless variables and parameters.

We begin by solving the momentum equation by integrating Eq. 1 and then advance a solution for the energy equation.

The solution of the momentum equation is:

$$v(r) = C_1 I_1\left(\frac{Ha}{1-\eta}r\right) + C_2 K_1\left(\frac{Ha}{1-\eta}r\right) = 0 \tag{5}$$

Where: I_1 is the modified Bessel function of the first kind of order 1, and K_1 is the modified Bessel function of the second kind of order 1.
The velocity profile in the annular space is therefore:

$$v(r) = \left[\frac{K_1\left(\frac{Ha}{1-\eta}\right) - bK_1\left(\eta\frac{Ha}{1-\eta}\right)}{I_1\left(\eta\frac{Ha}{1-\eta}\right)K_1\left(\frac{Ha}{1-\eta}\right) - K_1\left(\eta\frac{Ha}{1-\eta}\right)I_1\left(\frac{Ha}{1-\eta}\right)}\right] I_1\left(\frac{Ha}{1-\eta}r\right) + \left[\frac{bI_1\left(\eta\frac{Ha}{1-\eta}\right) - I_1\left(\frac{Ha}{1-\eta}\right)}{I_1\left(\eta\frac{Ha}{1-\eta}\right)K_1\left(\frac{Ha}{1-\eta}\right) - K_1\left(\eta\frac{Ha}{1-\eta}\right)I_1\left(\frac{Ha}{1-\eta}\right)}\right] K_1\left(\frac{Ha}{1-\eta}r\right) \tag{6}$$

To obtain the temperature field from Eq. (2), we performed calculations by using the 1, 2 terms and 3 terms used by [10] of the expansions of the Bessel functions $I_1\left(\frac{Ha}{1-\eta}r\right)$ and $K_1\left(\frac{Ha}{1-\eta}r\right)$ for small values of Ha, we find an insignificant difference, so to simplify the calculations we have chosen in this work the expansions of 1 term, it can be used as following:

$$I_1\left(\frac{Ha}{1-\eta}r\right) \cong \frac{1}{2}\left(\frac{Ha}{1-\eta}\right)r, K_1\left(\frac{Ha}{1-\eta}r\right) \cong 1\Big/\left(\frac{Ha}{1-\eta}\right)r \tag{7}$$

By replacing the values of I_1 and K_1 from the above expansions in the velocity distribution equation, Eq. (5),then we introduce the new velocity distribution in equation (2) to find the temperature field.
The solution to the temperature profile is given by:

$$\theta = C_3 \ln(r) + C_4 - Ec\Pr C_2^{\,2}\Big/\left(\frac{Ha}{1-\eta}\right)^2 r^2 \tag{8}$$

Where C_3 and C_4 are the constants of integration. The expressions for C_3 and C_4 not presented here to conserve the space.

4 Numerical study

In this numerical study, we consider a two-dimensional and axisymmetric unsteady flow. We opted for the numerical formulation speed - pressure due to its rapidity of prediction, its lower cost, and its ability to simulate real conditions. The finite difference scheme adopted for the resolution is very similar to that used by R.Peyret [13], A.Ghezal et al [14] and[15], this is a semi implicit scheme of Crank-Nicholson type. We used the Marker And Cell (MAC) for the spatial discretization. The iterative procedure is assumed converged when the following test is verified max $(|L_u|,|L_v|,|L_w|,|L_\theta||D|) \prec \varepsilon$, where L_u, L_v, L_w, L_θ and D represents operators differences relating to system equations corresponding to the problem variables u, v, w,θ and Π respectively, ε is of the order of 10^{-3} depending on the considered case. We then proceeded to a study of the mesh sensitivity of the field of study. This study led us to retain a mesh of 336 nodes along the direction r and 48 nodes in the z direction.

4.1 Mathematical equations

Based on these dimensionless variables, the conservation equations of mass, momentum and energy are written as follows (where the stars are dropped for convenience):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{\delta \Pi}{\delta r} + \frac{1-\eta}{Ta}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) - \frac{Ha^2 u}{(1-\eta)Ta} \tag{10}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{vu}{r} + w\frac{\partial v}{\partial z} = \frac{1-\eta}{Ta}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right) - \frac{Ha^2 v}{(1-\eta)Ta} \tag{11}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z} = -\frac{\partial \Pi}{\partial z}+\frac{1-\eta}{Ta}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{12}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial r}+w\frac{\partial \theta}{\partial z}=\frac{1-\eta}{Pr\,Ta}\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{(1-\eta)Ec\Phi}{Ta} \tag{13}$$

Where: $Ha = Bd\sqrt{\frac{\sigma}{\rho\upsilon}}$ is the Hartmann number, $Ta = \frac{\Omega_1 r_1 d}{\nu}$ is the Taylor number, $d = R_1 - R_2$: is the width of the annular space.

$$\Phi = 2\left[\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{u}{r}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2$$ is the viscous dissipation function.

The rate of heat transfer in non – dimensional for the inner and outer cylinder is given by:

$$Nu_i(z)=-\gamma\left.\frac{\partial\theta}{\partial r}\right|_{r=\eta}\ ,\quad Nu_e(z)=-\gamma\left.\frac{\partial\theta}{\partial r}\right|_{r=1}\qquad \text{With } \gamma=1-\eta \tag{14}$$

The average Nusselt number on the inner and outer cylinder is given by:

$$\overline{Nu_i}=\frac{1}{L}\int_0^z Nu_i(z)dz\,,\quad \overline{Nu_e}=\frac{1}{L}\int_0^z Nu_e(z)dz \tag{15}$$

4.1.1 Initial and boundary conditions

At the time t=0: $u(r,z,0)=v(r,z,0)=w(r,z,0)=\Pi(r,z,0)=\theta(r,z,0)=0$ (16)

The boundary conditions are as follows:

$$\begin{aligned}
&r=\eta \quad z\ge 0: && u(r_1,z)=v(r_1,z)=w(r_1,z)=0,\ \theta(r_1,z)=1\\
&r=1 \quad z\ge 0: && u(r_2,z)=v(r_2,z)=w(r_2,z)=\theta(r_2,z)=0\\
&\eta<r<1 \quad z=0: && u=v=w=0\,,\ \frac{\partial\theta}{\partial z}=0\\
&z=L && w=0,\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=\frac{\partial\theta}{\partial z}=0
\end{aligned} \tag{17}$$

5 Results and discussion

In order to understand the physical situation of the problem and the effects of the hartmann and Eckert numbers entering the problem, we have computed the numerical and analytical values of the velocity, temperature, and the Nusselt number. The results are presented in (figures 1, 2) and table 1.

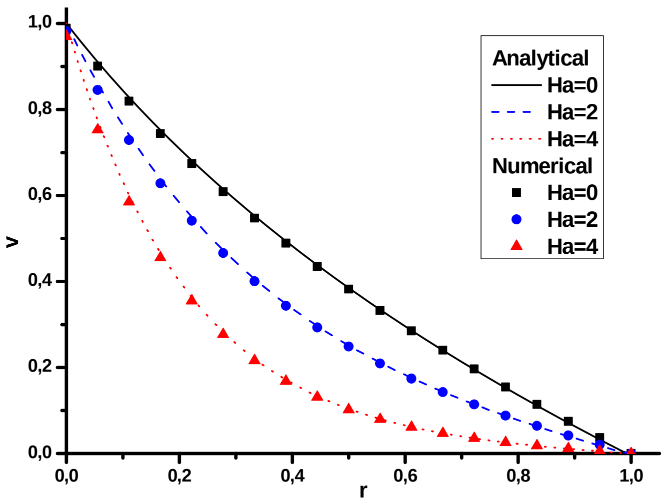

FIG. 1 – velocity profile as a function of Hartmann number, for η = 0.5, Ta=20,t=120.

The velocity is evaluated analytically and numerically for different values of Hartmann number in (figure 1).

The distribution of the velocity and the temperature for numerical results reaching a steady-state at t=120.There is not much difference in velocity and temperature at t = 60 compared to t = 120.
Comparing the numerical results with those obtained analytically, we find that the analytical and numerical results are in good agreement. We can notice that the velocity profile without magnetic field is quasi-linear, and an increase in Hartman number, which causes a reduction of the velocity in the annular space because the Coriolis force is counter-productive and the Lorentz electromagnetic force acts as a flow damper
In (Figure 2), we present the behaviour of the temperature radial profile, for various values of Hartmann number, it can be seen that the effect of magnetic field on the radial profile of temperature is insignificant. It is valid in the case of low and high values of Hartmann.
In (Table 1), the distribution of average Nusselt number on the outer and inner surfaces is presented for different Hartman numbers. It is observed that the effect of increasing Hartmann number is to decrease the magnitude of average Nusselt number on both surfaces of the cylinder. So, an considerably increasing in Hartmann number, which leads to a reduction of the centrifugal force, which results in a progressive decrease in the Nusslet number.
From this table, it can be noticed that the average Nusselt number on the outer cylinder is lower than on the inner cylinder, because the velocity and temperature gradient are higher for the cold inner cylinder than for the outer cylinder. Also the results show the effects of viscous dissipation terms on the rate of heat transfer, the average Nusselt number increases with an increase in the Eckert number on the outer cylinder, but it decreases on the inner cylinder. In fact, as the Eckert number is large the heat generated in the annulus increases due to viscous dissipation, and thus the temperature of the fluid increases. This causes a decrease in the temperature gradient close to the inner cylinder, and an increase in the gradient in the vicinity of the outer cylinder.

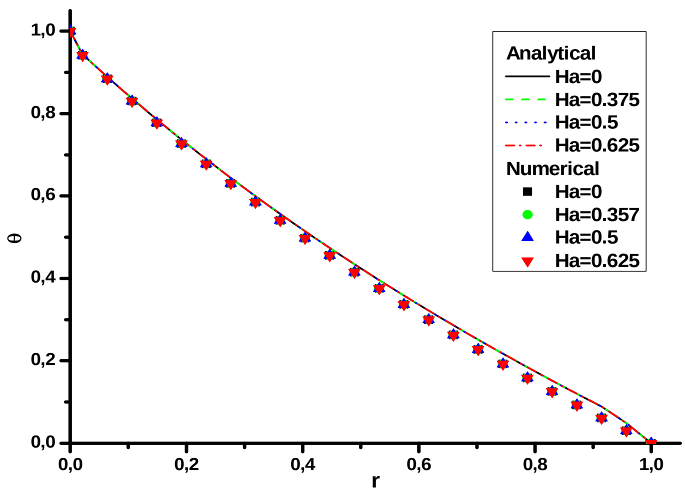

FIG. 2 – Temperature profile as a function of Hartmann number, for η = 0.5, Ta=20, Pr = 0.02, Ec=0.0001, *t=120..*

Ha	$\overline{Nu}_i$			$\overline{Nu}_e$		
	Ec=0.0001	Ec=0.5	Ec=5	Ec=0.0001	Ec=0.5	Ec=5
0	1,44284	1,42495	1,28173	0,72132	0,72612	0,76932
1	1,44276	1,4241	1,27291	0,72129	0,72555	0,7639
2	1,44272	1,42182	1,24949	0,72124	0,72354	0,75468
4	1,44254	1,41591	1,18979	0,72119	0,72354	0,74476
6	1,442	1,4108	1,13853	0,72117	0,72315	0,74093
8	1,44137	1,40683	1,09871	0,72117	0,72292	0,73873
10	1,44118	1,40378	1,06821	0,72117	0,72277	0,73721
20	1,44112	1,39695	0,99983	0,72116	0,72239	0,73342
50	1,44109	1,39776	1,00795	0,72116	0,7221	0,73052

Table.1 – Effect of Hartman number on average Nusselt number for different Eckert numbers on inner and outer cylinders, for $\eta = 0.5$, Ta=20,Pr = 0.02, $t=120$ (numerical results).

6 Conclusion

In conclusion therefore, the forced convection flow of an electrically conducting fluid between two horizontal concentric cylinders in the presence of an axial magnetic field and a viscous dissipation was studied. The exact solution is more general with variable boundary conditions. Therefore, we can easily obtain another exact solution with different conditions. Our results show that the application of a magnetic field may have interesting effects in the fluid motion and heat transfer because it causes a damping of movement in the system.

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