Research Article

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New inequalities of Gronwall type for the stochastic differential equations

Abstract: In this paper, we establish some new nonlinear integral inequalities of Gronwall type for Itô integrals. These inequalities generalize some inequalities which can be used in applications as handy tools to study the qualitative as well as quantitative properties of solutions of some stochastic differential equations. We will use this inequalities to show the existence and uniqueness of solutions for nonlinear EDS.

Keywords: Stochastic differential equation, Itô integral, Gronwall–Bellman inequality

MSC 2010: 60H10, 54C60

DOI: 10.1515/rose-2014-0035
Received May 3, 2014; accepted April 14, 2015

Communicated by: Stanislav Molchanov
Dedicated to Professor Hamid Osmanov

1 Introduction

In the theory of differential equations the Gronwall's lemma was widely used in various applications since its first appearance in the article by Bellman [4] in 1943. In this article [4] the author gave a fundamental lemma of Gronwall–Bellman in order to study the stability and asymptotic behavior of solutions of differential equations. In view of the important applications of the Gronwall–Bellman's inequality, see [3]. The Gronwall lemma has seen several generalizations to various forms.

In the stochastic field, this lemma has its fields of application for the study of the existence and uniqueness of solutions of stochastic differential equations (SDEs). Several forms of Gronwall inequality have been given for the integral of Itô [2, 6], in this paper we will give other forms of Gronwall inequality for the Itô and Startonovich integral finding their applications in various case to show the existence and uniqueness of solutions for nonlinear SDE.

Theorem 1.1 (The Gronwall–Bellman inequality). *Let $\alpha, \beta : [0, T] \to \mathbb{R}$ be integrable with*

$$0 \leq \alpha(t) \leq \beta(t) + L \int_0^t \alpha(s)\, ds$$

for $t \in [0, T]$ where $L > 0$. Then

$$\alpha(t) \leq \beta(t) + L \int_0^t e^{L(t-s)} \beta(s)\, ds$$

for $t \in [0, T]$.

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