

Research Article

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Some new stochastic forms of Gronwall–Bellman inequalities and their applications

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Abstract: In this work, we establish some new forms of stochastic Gronwall–Bellman inequalities. We illustrate our work with examples of applications.

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MSC 2010: 60H10, 54C60, 60H05, 60H35

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1 Introduction

The theory of stochastic differential equations is a generalization of ordinary differential equations, it is developed in order to modulate physical, biological and economic phenomena, whose random aspect is a deciding factor.

In the stochastic field, the Gronwall lemma is used for the study of the existence and uniqueness of solutions of stochastic differential equations. Several forms of the Gronwall inequality have been given for the integral of Itô [1, 4]. In this paper we will give other forms of the Gronwall inequality for the Itô and Startonovich integral, which can be applied in various cases to show the existence and uniqueness of solutions for nonlinear SDE.

The Gronwall–Bellman lemma can be applied in the stochastic domain to demonstrate the existence and uniqueness of solutions of stochastic differential equations.

In [2], we found some new Gronwall–Bellman inequalities that are used in the stochastic domain. In this article we will use these results to establish other inequalities of the same type and give some examples of applications.

Let (Ω, F, P) be a complete probability space with natural filtration F_t . Let $W(t)$ be a Brownian motion adapted to $\{F_t, t \geq 0\}$, i.e., $W(0) = 0$ for all $t \geq 0$, and $W(t)$ is F_t -measurable. For a positive number L , $H_w^2[0, L]$ denotes the set of all separable nonanticipative functions $f(t)$ with respect to F_t defined on $[0, L]$ satisfying

$$E\left(\int_0^L f^2(t) dt\right) < \infty.$$

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