Research Article

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Existence of almost periodic solutions of stochastic differential equations with periodic coefficients

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Abstract: In this work, under some conditions, we will prove that a scalar stochastic differential equation with periodic coefficients admits almost periodic solutions.

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1 Introduction

For an ordinary differential equation

$$dx = f(t, x)\, dt,$$

whose second member is periodic in t, Massera [15] showed that the existence of a bounded solution implies also the existence of a periodic solution with the same period with f. This result is obviously incorrect in the case where the function $f(t, x)$ is almost periodic [21, p. 181].

The existence of almost periodic solutions for stochastic differential equations is one of the important problems discussed by mathematicians. Several studies were conducted on almost-periodicity for this type of equations, see, for example, [6, 7, 20].

The notion of almost-periodicity for random processes and in particular for stochastic differential equations has developed considerably because of the large number of works in this area since the work of Tudor and his collaborators, see [2, 17–19]. In these papers they proved almost-periodicity in the distribution of the solutions for some stochastic differential equations with almost periodic coefficients.

Recently, Bezandry and Diagana [3, 4] proved that some stochastic differential equations with almost periodic coefficients admit solutions which satisfy the strong property of almost-periodicity in the mean square sense.

We deal with the following SDE:

$$dx = a(t, x)\, dt + b(t, x)\, dB_t, \tag{1.1}$$

where the coefficients $a(t, x)$, $b(t, x)$ are periodic in t, however, x_t does not represent a periodic path because the Brownian motion B is not periodic in t. Nevertheless, we will show that the SDE (1.1) can have, under certain assumptions, an almost-periodic solution.

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