People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
Akli Mohand Oulhadj University of Bouira

جامعة البويرة

Civil engineering department
Third-year Bachelor's degree in Civil Engineering

COURSE HANDOUT

Structural analysis

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Introduction

Strength of Materials focuses on studying the static behavior of materials and structural components under the action of different types of force and thermal loadings. Its goal is to determine or verify their dimensions so that they can withstand the loads they are subjected to, ensuring satisfactory safety conditions and at the lowest possible cost.

The design of structures involves studying the internal stability of a structure, which means determining the internal forces and deformations. Structural analysis determines the relationship between the expected external load on a structural member and the resulting internal stresses and displacements that develop within the member during service. Structural members can be categorized as beams, columns, tension structures, frames, and trusses. The structural analysis course is designed to equip third-year civil engineering students with the foundational knowledge needed to predict the response of determinate and indeterminate structures to external loads, preparing them for further studies in structural mechanics.

The course is divided into four chapters. The first one focuses on the articulated truss systems which are structural frameworks composed of straight members connected at the joints. It describes the different calculation methods used for calculating the forces in members.

The second chapter addresses the determination of internal forces in determinate plane frames, which consist of assemblies of beams and columns, either with or without hinges. The cross section of all members has a centroidal principal axis lying in one plane; all applied loads and reactions are in the same plane. Unknown external reactions or internal members can be determined using only the conditions of equilibrium.

The methods for obtaining the influence lines for determinate beams, frames and trusses are discussed in the third chapter. The interest in influence lines stems from their ability to provide the behavior of structures under varying loads.

In the fourth chapter, the force mothed for analysis of indeterminate structures is introduced. This method involves the solution of linear simultaneous equations relating forces to displacements. A set of compatibility equations is formulated depending on the number of the redundant forces in the structure, and solving these equations simultaneously to determine the magnitude of the redundant forces. Once the redundant forces are known, the structure becomes determinate and can be analyzed completely using the conditions of equilibrium.

Please note that this course is specifically designed for Third-year Bachelor's degree in Civil Engineering.

CHAPTER I

TRUSS DETERMINATE SYSTEMS

1.1. Definitions

- Articulated Truss System

An articulated truss system (also known as a reticulated system or simply truss) is a collection of straight or curved elements, referred to as bars, interconnected at their ends by hinges. The points of connection for the bars are called nodes.

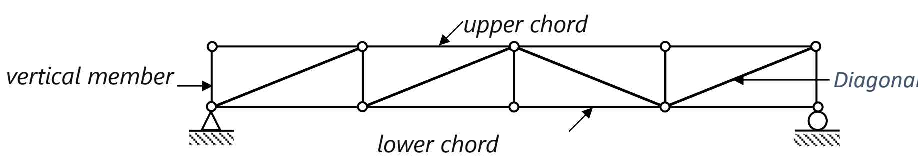


Figure 1.1 : Determinate beam

- Planar Articulated Truss System: when the axes of the bars and the applied loads are located in the same plane, it is referred to a planar system.
- Indirectly Loaded System: a truss system is said to be indirectly loaded if all external forces are applied exclusively to the nodes. If the loads are applied at arbitrary points, especially at locations on the bars other than the nodes, then it is referred to as a directly loaded system.
- Isostatic (determinate) System: If the static equilibrium equations alone are sufficient for the complete determination of the system, meaning they allow for the calculation of reactions and forces at

every point in the system, the system is considered isostatic. Otherwise, if not, the system is termed indeterminate.

1.2. Trusses indirectly loaded

Only statically determinate planar trusses loaded indirectly will be considered in this chapter.

a) Theorem:

When a planar articulated truss system, composed of straight bars, is indirectly loaded, each bar in the system is subjected only to a constant axial force.

Consider a bar in the truss. Since the system is in equilibrium, each constituent bar is also in equilibrium. Being articulated, the ends of the bar are not subject to any moment. The only solicitations it experiences are the systems of concentrated forces at the ends.

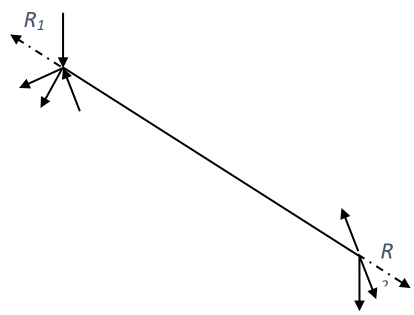


Figure 1.2. Bar in the truss loaded indirectly

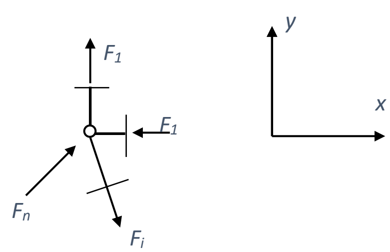


Figure 1.3. Node in the truss loaded indirectly

Each system of forces has a resultant. The resultants (R1 and R2) must necessarily be equal and opposite for equilibrium to be achieved. Ultimately, the bar is subjected only to a constant axial force, which can be either tension or compression.

b) Isostatic Condition: As the bars are subjected only to axial forces, at each node of the truss, there is a system of forces in equilibrium. The equilibrium of a system acting on a particle, a node for example, is confirmed if the resultant is zero or if the projections along two perpendicular directions (e.g., x and y) are zero ($\Sigma Fx = 0$, $\Sigma Fy = 0$).

If 'n' represents the number of nodes (supports are also nodes, n = 10 for the system in Figure 1.1), the number of equilibrium equations of statics that can be written is equal to 2n.

Let 'b' be the number of bars and 'l' be the number of connections at the supports. The isostatic condition is expressed as:

$$2n = b+l \tag{1.1}$$

However, it is important to note that the condition (1.1) may prove insufficient to prove the isostatic nature of a truss; the system must also be geometrically invariant.

A simple rule, known as the "triangular mesh rule," is used to verify if the system is determinate and stable. This rule is stated as follows: if, starting from a triangular mesh, it is possible to reconstruct the system by adding 2 bars at a time, then the system is determinate and stable.

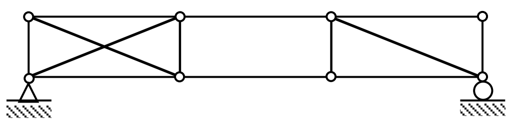


Figure 1.4. System satisfying condition 1.1 but unstable.

1.3. Calculation methods

The calculation methods for determinate articulated truss systems can be divided into two categories: analytical methods and graphical methods. The most common graphical method is Cremona's method (Cremona diagram). It involves constructing the force polygon at each node. The most common analytical methods are the method of joints and the method of sections. All three mentioned methods will be presented.

It is worth noting that, regardless of the method used, the calculations should always start with the determination of reactions.

1.3.1 Method of joints

Principle: The method involves isolating the considered joint by making cuts that release forces in the bars and projecting all forces, normal forces, and external forces acting on the node along two perpendicular axes.

Calculations must start with a joint where only two bars meet (2 unknowns, 2 equations). Then, proceed to a node with no more than two unknowns.

Example of Application:

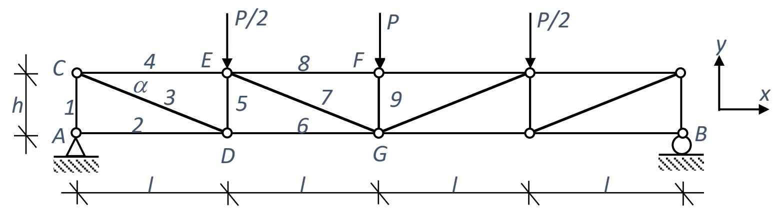


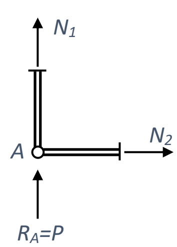
Figure 1.5. Determinate beam.

Node A

The choice of the direction of forces in the bars is arbitrary. The chosen direction corresponds to tension; the calculations will reveal the exact nature of the force borne by each bar.

$\Sigma F_x = 0 \Rightarrow N_2 = 0$

$\Sigma F_y = 0 \Rightarrow N_1 = -P$ (the "-" sign indicates that bar 1 is subjected to compression).



Node C

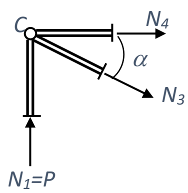
$\Sigma F_x = 0 \Leftrightarrow N_3 \cos\alpha + N_4 = 0$

$\Sigma F_y = 0 \Leftrightarrow P - N_3 \sin\alpha = 0$

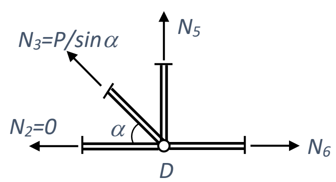
Where :

$N_3 = \dfrac{P}{\sin\alpha}$ (tension)

And $N_4 = -\dfrac{P}{tg\alpha}$ (*compression*)



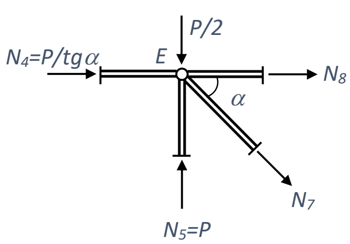
Node D



$$\Sigma F_x = 0 \Rightarrow N_6 = N_3 \cos\alpha = \\ = \frac{P}{tg\alpha} \text{ (traction)}$$

$$\Sigma F_y = 0 \Rightarrow N_5 = -N_3 \sin\alpha = \\ = -P \text{ (compression)}$$

Node E

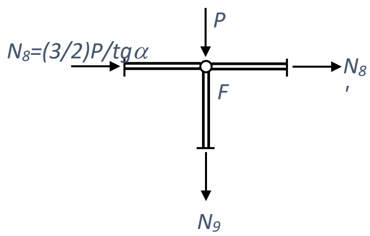


$$\Sigma F_x = 0 \Leftrightarrow \frac{P}{tg\alpha} + N_8 + N_7 \cos\alpha = 0$$

$$\Sigma F_y = 0 \Rightarrow N_7 = \frac{P}{2\sin\alpha} \text{ (tension)}$$

$$\Rightarrow N_8 = -\frac{3}{2}\frac{P}{tg\alpha} \text{ (compression)}$$

Node F



$$\Sigma F_x = 0 \Rightarrow N_8' = -\frac{3}{2}\frac{P}{tg\alpha} \text{ (compression)}$$

$$\Sigma F_y = 0 \Rightarrow N_9 = -P \text{ (compression)}$$

The figure 1.6 below illustrates the nature of the force in the studied bars:

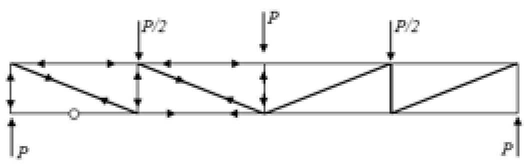


Figure 1.6. The nature of the forces.

Arrows towards the nodes = compression

Arrows towards the center = tension

0 = zero force

1.3.2 Section Method (or Ritter's Method)

Principle: The method involves making a cut in the system that does not intersect more than 3 non-concurrent bars (except in specific cases), effectively dividing the truss into two parts. To determine the force in one of the bars, the equilibrium equation for the rotation of one of the two parts about the point of intersection of the other bars is written (Figure 1.7).

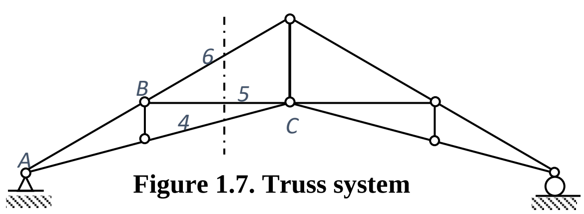


Figure 1.7. Truss system

$\Sigma M/A = 0 \Rightarrow N5 = \ldots$

$\Sigma M/B = 0 \Rightarrow N4 = \ldots$

$\Sigma M/C = 0 \Rightarrow N6 = \ldots$ (Right part)

Note: The point of intersection of the bars, with respect to which the moments are calculated, is not necessarily a node of the system (hence the advantage of working graphically).

Special cases:

1. Two intersected bars are parallel (intersection point rejected to infinity) (Figure 1.8).

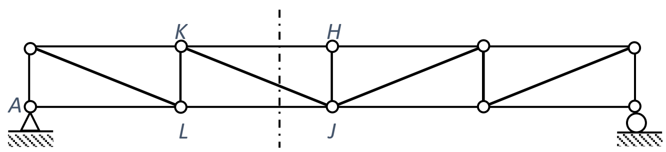


Figure 1.8: beam in N.

The force N_{KH} is obtained from the equation $\Sigma M/J = 0$, and the force N_{LJ} in bar LJ is obtained from: $\Sigma M/K = 0$. To calculate N_{KJ}, an equation of translational equilibrium, $\Sigma Fy = 0$, for example, is used, or an equation of rotational equilibrium about a support, $\Sigma M/A = 0$, for example.

2. More than three cut bars: Ritter's method can be applied provided that all cut bars converge except one.

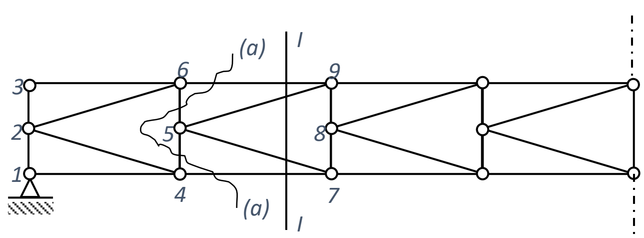


Figure 1.9 : Beam inn K.

CUT a-a (Figure 1.9) features three concurrent bars 4-5, 5-6, and 6-9 at 6, and the equation $\Sigma M/6=0$ yields the force N_{47}. With N_{47} known, we make the cut I-I, and there are only three unknown forces remaining.

Advantage of the section method: it allows for the direct calculation of the force in any bar and thus, serves as an excellent mean of verifying results obtained by other methods.

Example:

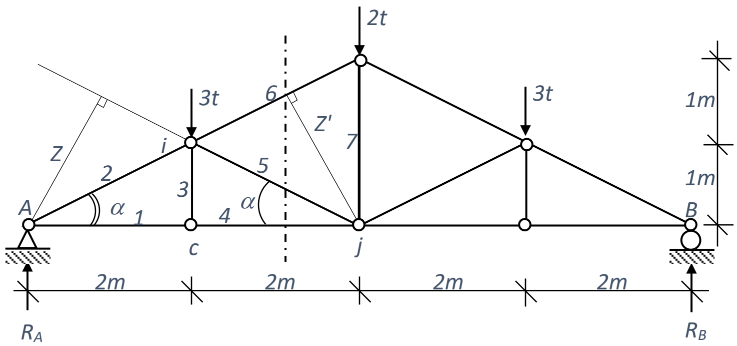


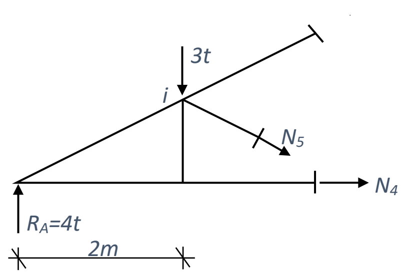
Figure 1.10

Reactions: $R_A = R_B = \frac{8}{2} t = 4\ t$

$\Sigma M/i = 0 \Leftrightarrow 2R_A - N_4 = 0 \Rightarrow N_4 = 8\ t$ (tension)

$\Sigma M/A = 0 \Leftrightarrow 2x3t + ZN_5 = 0,$

with : $Z = \frac{4}{\sqrt{5}} m$



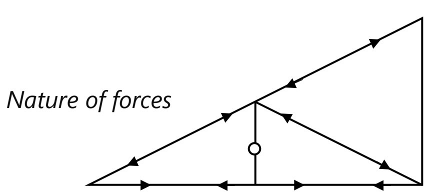
$\Rightarrow\ N_5 = -\frac{3}{2}\sqrt{5}\ t$

$\Sigma M/j = 0 \Leftrightarrow Z'N_6 - 2x3t + 4R_B = 0 \Rightarrow Z'N_6 = -10$ tm

$\sin\alpha = \frac{1}{\sqrt{5}} = \frac{Z'}{4}$, So : $Z' = 4\sqrt{5}\ m$

and : $N_6 = -5.59\ t$

To calculate the forces in bars 1, 2, and 3, we write the translational equilibrium equations at points A and C. Alternatively, the Ritter's method can also be applied.



Note: In practice, the lever arm distances can be measured graphically, which has the advantage of simplifying the work.

1.3.3 Cremona Method (Cremona Diagram)

Principle: The method involves plotting the equilibrium polygon of the forces applied at each node. Since all nodes are in equilibrium, the polygons are necessarily closed.

To apply the method, it is necessary for the system to have at least one node where only two bars meet.

The steps of the method are as follows:

1. Represent the system on a length scale.
2. Calculate the reactions, then number:

a) Intervals between external forces by rotating in one direction, such as clockwise.

b) Intervals of the truss network (internal domains delimited by the bars).

Thus, each bar is characterized by two numbers indicating adjacent intervals (domains).

3. Construct the polygon of external forces on a chosen force scale; this polygon is closed since the external forces are balanced by the reactions (global equilibrium). Indicate the direction of the forces with arrows.
4. Next, draw the polygon of forces acting on each node (external forces and forces in the bars), starting with a node where only two bars meet, then moving to a node with only two unknown forces.

Note: The directions of the forces are known (orientations of the bars), and their magnitude and direction are obtained by closing each polygon.

Example of Application: Let's calculate the forces in the bars of the beam represented in Figure 1.11, already calculated by the Ritter's method.

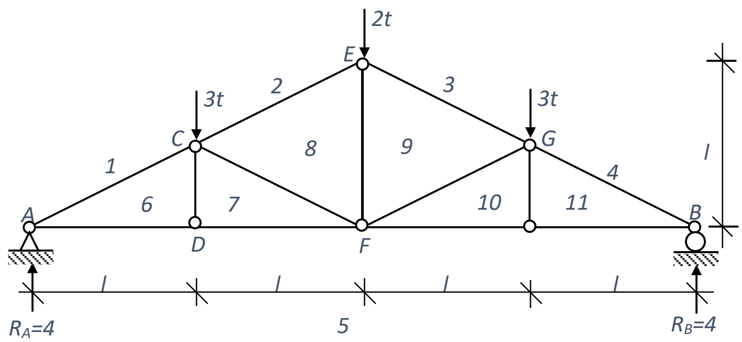


Figure 1.11: truss system

The resolution of the problem is carried out in the following steps.

1- Represent the structure on a length scale (Figure 1.11).

2- Numbering of external domains (bounded by applied forces and reactions): 1, 2, 3, 4, and 5 (clockwise, Figure 1.11).

3- Numbering of internal domains (meshes): 6, 7, 8, 9, 10, 11 (from left to right). Letters could have been chosen instead of numbers (Figure 1.11).

Now, each force (external or internal) can be numbered before moving on to the next step. Each force is characterized by the two numbers of the adjacent domains in its direction. Internal forces acting on the nodes are numbered in a clockwise direction (Figure 1.12).

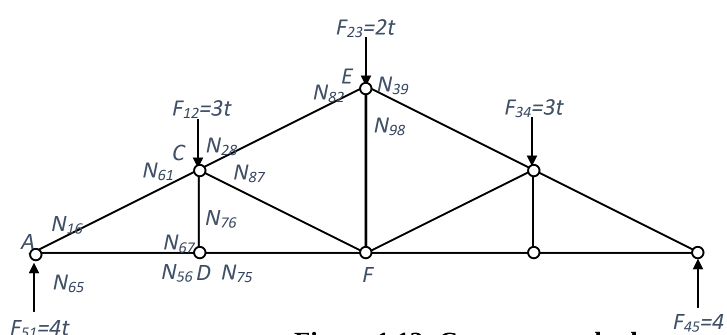


Figure 1.12: Cremona method

4- Draw the polygon of external forces (applied forces and reactions). This polygon is represented by the vertical segment: 1-2-3-4-5-1 (Figure 1.13).

5- Construction of the polygons of forces acting on each node.

a) Node A: The intervening forces are N_{16}, N_{65}, and F_{51}. This last force is known and represented on the polygon of external forces. Note that only point 6 is undetermined.

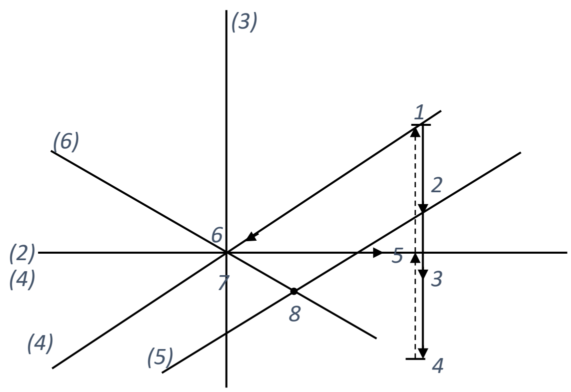
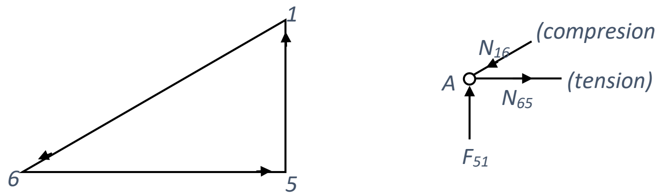


Figure 1.13 : Construction of polygon of forces.

starting from point 1, draw a line parallel to bar AC (N_{16}), and from point 5, draw a line parallel to AD (N_{65}). The intersection of these two parallel lines determines the sought-after point 6. To determine the direction of forces N_{16} and N_{65}, close the polygon starting from the known force, F_{51} (see diagrams below).

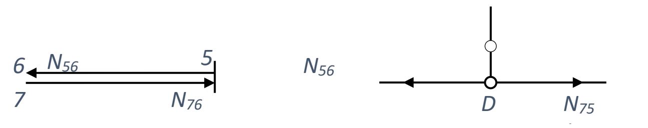


The arrows obtained by closing the polygon of forces acting on node A indicate the nature of each force.

b) Next, move to node D where only the forces in bars DF and DC are unknown.

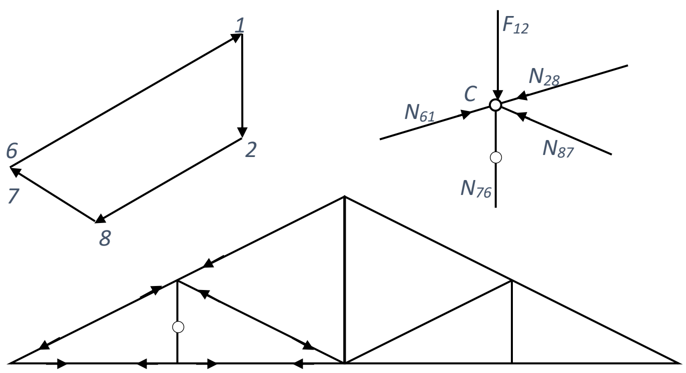
Intervening forces: N_{56} (known since N_{65} is known), N_{67}, and N_{75}. In this case as well, only point 7 is undetermined.

From node 6, draw a line parallel to DC (N_{67}), and from 5, draw a line parallel to DF (horizontal) (N_{75}). The intersection of these two parallel lines occurs at point 6, so point 7 coincides with 6. The polygon of forces at D (N_{56}, N_{67}, and N_{75}) is limited to the segment 5-7; therefore, the force $N_{67} = 0$ (see diagrams below).



c) Node C: Intervening forces: N_{61}, F_{12}, N_{28}, N_{87}, and N_{76} ($N_{67} = N_{76} = 0$). Only point 8 remains to be determined.

From point 2, draw a line parallel to CE (N_{28}); then from point 7, draw a line parallel to CF (N_{87}). The intersection of these two parallel lines determines the position of point 8. Close the polygon to determine the direction of the unknown forces (N_{87} and N_{28}) ($N_{61} \rightarrow F_{12} \rightarrow N_{28} \rightarrow N_{87}$ and N_{76}) (see diagrams below).



1.4. Remarks

1- Combined use of Cremona's diagram and Ritter's method.

During a Cremona diagram construction, nodes where more than two bars meet with unknown forces cannot be passed. Ritter's method allows for the traversal of these nodes. It is sufficient to make one or more cuts providing the values of the forces in the "redundant" bars. This case often occurs in structures known as "Polonceau" trusses (see Figure 1.14).

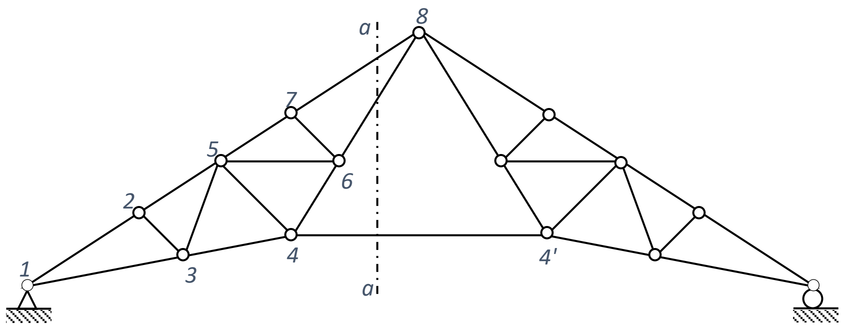


Figure 1.14: Polonceau truss.

Having started Cremona at 1, upon reaching 4, we encounter three unknown forces (N_{45}, N_{46}, and $N_{44'}$). The cut a-a' allows the direct calculation of the force $N_{44'}$ ($\Sigma M/8=0$); after which, the Cremona diagram can be continued as usual.

2- Bars not in action (N=0). In the example of Figure 1.15, five bars are not in action (N=0); nevertheless, they are necessary because they contribute to:

- ensuring the rigidity and isostatic nature of the system;
- reducing the lengths of buckling;
- facilitating construction arrangements.

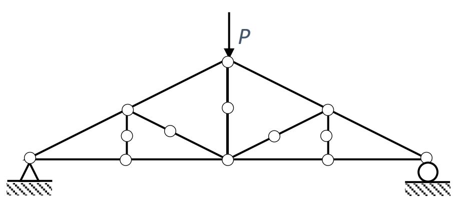


Figure 1.15 : Beam with several non-loaded bars

CHAPTER II
DETERMINATE PLANE FRAMES

2.1. INTRODUCTION

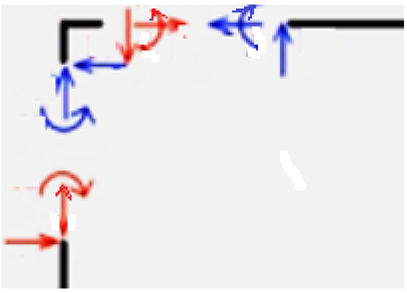
Plane frames allow the assembly of beams and columns. Plane frames can be made up of I-beams, H-beams, tubes, with variable or non-variable sections, as well as box sections and truss elements. All elements of these types of structure contribute to resistance to both vertical and horizontal forces. The greater inertia of the beam and column elements of the Plane frames is necessarily in the plane of the frames, ensuring greater resistance to bending in this plane.

Plane frames can have hinges or be completely rigid. A rigid joint can transmit two force components (N and T) and moment. At a hinge, the internal moment is zero. Differential settlements and thermal variations can be absorbed by articulated systems. When multiple panels are stiffened in the same row or on multiple stacked levels, "multiple Plane frames" are obtained. A Plane frames therefore admits three types of loading:

- Tensile and compressive loads (most often applied to columns)
- Bending loads (most often applied to beams)
- Bending moments

2.2. Sign convention

For the bending moment: the moment that stretches the left fibers of a column is positive. For beams, the moment that stretches the bottom fibers is positive. Regarding shear force and normal force, they are positive when their diagrams are plotted outside the Plane frames.



2.3. Structure equilibrium balance

The first question to consider when approaching the static calculation of a structure is the hyperstaticity of the structure. If the number of equilibrium equations is equal to the number of unknowns (the reactions), the system is statically determinate.

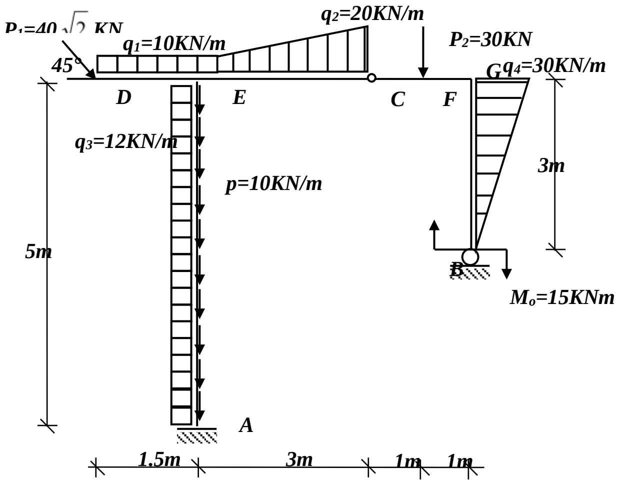
To establish the equilibrium balance of plane frames with hinges and determine if the frame is statically determinate or not, the system is subdivided into subsystems at each hinges. Pinned supports are added in place of the hinges. The total number of unknowns in all subsystems must be equal to the total number of equilibrium equations (three equations in each subsystem).

2.4. Calculation method

Like all statically determinate systems, the method of sections can be used.

2.5. Example

Calculate the reactions of the system shown below, then calculate and plot the internal force diagrams M, N, T (bending moment, axial force, shear force).



- Calculation of reactions:

$$\Sigma F_x = 0 \Rightarrow H_A = -55KN$$

$$\Sigma F_y = 0 \Rightarrow V_A + V_B = 180KN$$

$$M_C^{Right} = 0 \Rightarrow M_C^{Right} = -90 + 2V_B = 0 \Rightarrow V_B = 45KN$$

$$V_A = 135KN$$

$$M_C^{Left} = 0 \Rightarrow M_A = 83.75KN.m$$

- Calculation of the internal Forces:

Column AE: $\mathbf{0 \leq x \geq 5m}$

$$\mathrm{N(x) = -135 + 10x \Rightarrow N(0) = -135KN, N(5) = -85KN}$$

$$\mathrm{T(x) = 55 - 12x \Rightarrow T(0) = 55KN, T(5) = -5KN}$$

$$\mathrm{T(x) = 0 \Rightarrow x = 4.58m}$$

$$\mathrm{M(x) = 83.75 - 55x} + 6x^2 \Rightarrow \mathrm{M(0) = 83.75KN.m, M(5) = -41.25KN.m,}$$

$$\mathrm{M(4.58) = -42.3KN.m}$$

Beam DE: $\mathbf{0 \leq x \geq 1.5m}$

$$\mathrm{N(x) = 40KN}$$

$$\mathrm{T(x) = -40 - 10x \Rightarrow T(0) = -40KN, T(1.5) = -55KN}$$

$$M(x) = -40x - 5x^2 \Rightarrow M(0) = 0KN.m, M(1.5) = -71.25KN.m$$

Beam EC: $\mathbf{1.5 \leq x \geq 4.5m}$

$$N(x) = -45KN$$

Knowing that: $\frac{10}{3} = q/(x - 1.5)$

$$T(x) = 30 - 10(x - 1.5) - 5/3(x - 1.5)^2 \Rightarrow T(1.5) = 30KN, T(4.5) = -15KN$$

$$T(x) = 0 \Rightarrow x = 3.7m$$

$$M(x) = -\frac{5}{9}(x - 1.5)^3 - 5(x - 1.5)^2 + 30x + 532.5 \Rightarrow M(1.5) = -30KN.m, M(4.5) = 0KN.m$$

$$M(3.7) = 113.88KN.m$$

Beam CG: $\mathbf{4.5 \leq x \geq 6.5m}$

$\mathbf{4.5 \leq x \geq 5.5m}$ (To the right)

$$N(x) = -45KN$$

$$T(x) = -15KN$$

$$M(x) = -30\,(5.5 - x) - 45 - 15 + 45(6.5 - x) \Rightarrow M(4.5) = 0KN.m, M(5.5) = -15KN.m$$

$\mathbf{5.5 \leq x \geq 6.5m}$ (To the right)

$$N(x) = -45KN$$

$$T(x) = -45KN$$

$$M(x) = -45 - 15 + 45(6.5 - x) \Rightarrow M(5.5) = -15KN.m, M(6.5) = -60KN.m$$

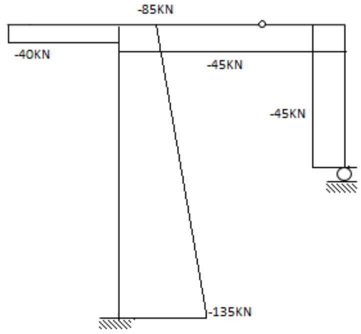
Column BG: $\mathbf{0 \leq x \geq 3m}$

$$N(x) = -45KN$$

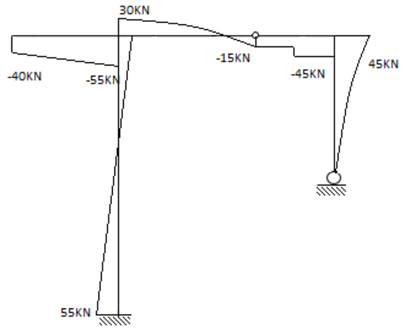
Knowing that: $\frac{30}{3} = q/x$

$$T(x) = 5x^2 \Rightarrow T(0) = 0KN, T(3) = 45KN$$

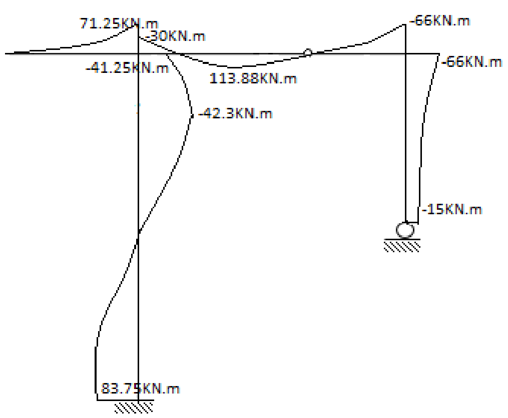
$$M(x) = -15 - -\frac{5}{3}x^3 \Rightarrow M(0) = -15KN.m, M(3) = -60KN.m$$



Normal forces diagram



Shear forces diagram



Bending moment diagram

CHAPTER III

INFLUENCE LINES IN STRUCTURAL ANALYSIS

3.1. Introduction

Structures such as bridges must be designed to resist moving loads as well as their own weight. When loads change position, it can lead to shifts in internal forces within the structure, including moments, shears, and axial forces. Since structures are designed for the critical loads that may occur in them, influence lines are used to obtain the position on a structure where a moving load will cause the largest stress.

Influence lines can be defined as a graph whose ordinates show the variation of the magnitude of a certain response function of a structure as a unit load traverses across the structure. Influence lines offer a clear means of understanding the interplay between loads and structural responses.

3.2. The interest of influence lines

The interest in influence lines stems from their ability to provide the behavior of structures under varying loads:

a. Graphical representation: influence lines offer a graphical representation of how loads are distributed and how they affect internal forces within a structure. This visualization helps to understand which parts of the structure are subjected to the highest forces under different loading conditions.

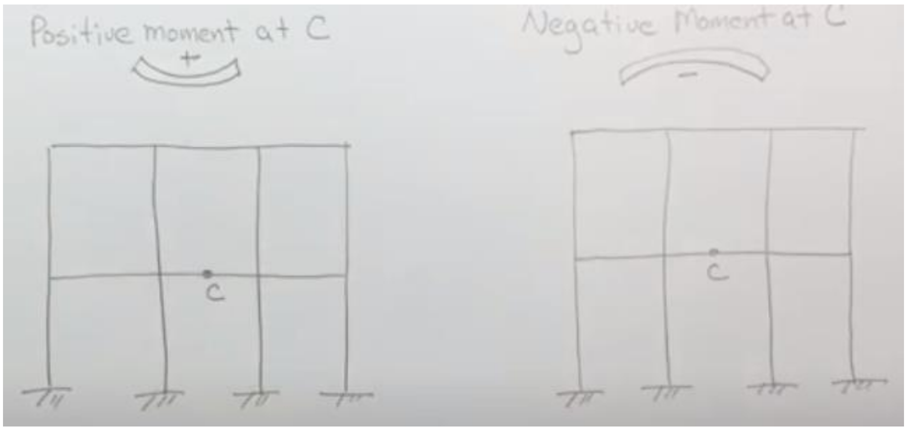
b. Response to unit load: influence lines depict the variation in a particular internal force or reaction in response to a unit load applied at different positions along the structure. By considering a unit load, engineers can easily scale the influence line to determine the response to actual loads of varying magnitudes.

c. Critical locations: influence lines highlight critical points along the structure where the internal force or reaction is maximized or minimized. These critical points correspond to locations where the structure is most sensitive to changes in load position, and where design considerations are often focused.

d. Optimization of structural design: by studying influence lines, we can optimize the design of structural elements to efficiently distribute loads and minimize the potential for stress concentrations. This optimization process leads to more cost-effective and reliable structural designs.

e. Dynamic analysis: influence lines can also be used for dynamic analysis, allowing engineers to predict the response of structures to moving loads, such as vehicles on bridges or trains on railway tracks.

Example:



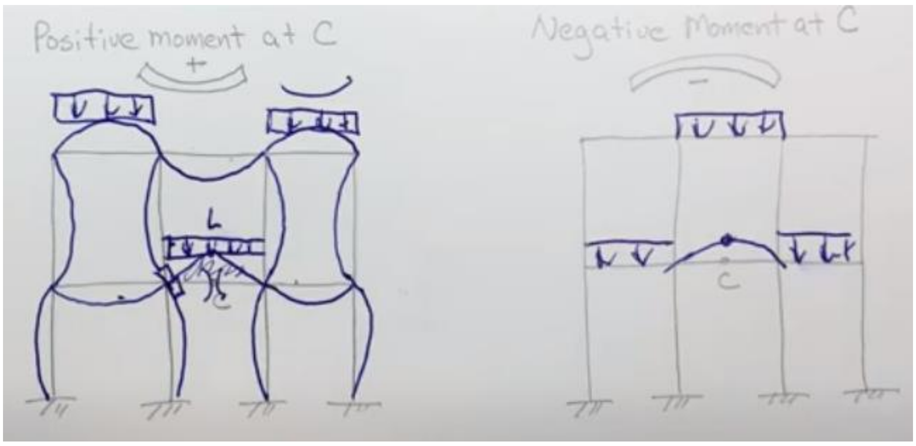


Figure 3.1 : Distributed loading pattern that will cause the greatest positive and negative moment at C.

3.3. Calculation methods

A shearing force or bending moment diagram shows the magnitude of the shearing force or bending moments at different points of the structure due to the static or stationary loads that are acting on the structure, while the influence lines for certain functions of a structure at a specified point of the structure show the magnitude of that function at the specified point when a unit moving load traverses across the structure. The influence lines of determinate structures can be obtained by the static equilibrium method or by the kinematic or Muller-Breslau method.

3.3.1 The static equilibrium method

To grasp the basic concept of influence lines, consider the simple beam shown in Figure 2. Statics help to determine the magnitude of the reactions at supports *A* and *B*, and the shearing force and bending moment at a section *n*, as a unit load of arbitrary unit, moves from right to left.

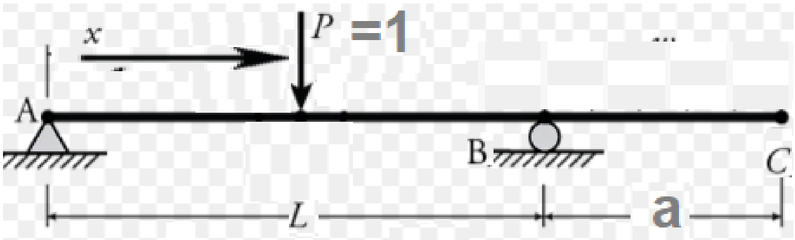


Figure 3.2 : Determinate beam

Beam Reactions

$$\Sigma F_x = 0 \Rightarrow H_A = 0$$

$$\Sigma F_y = 0 \Rightarrow V_A + V_B = 1$$

$$\Sigma M/_A = 0 \Rightarrow L * V_B - 1 * x = 0 \Rightarrow V_B = x/L$$

$$\Rightarrow V_A = 1 - x/L$$

$$\mathrm{x} = 0 \Rightarrow V_A = 1, V_B = 0$$

$$\mathrm{x} = \mathrm{L} \Rightarrow V_A = 0, V_B = 1$$

$$\mathrm{x} = \mathrm{a} + \mathrm{L} \Rightarrow V_A = -\frac{\mathrm{a}}{\mathrm{L}}, V_B = 1 + \mathrm{a}/\mathrm{L}$$

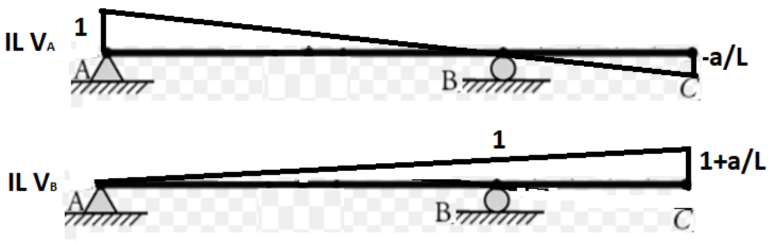


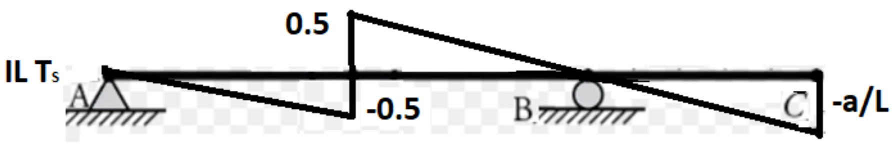
Figure 3.2 : influence lines of reactions

Shearing Force at Section S (x=L/2)

$$0 \leq x \geq L/2$$

$$T_S = V_A - 1$$

$$\frac{L}{2} \leq x \geq L + a$$



$$T_S = V_A$$

Figure 3.3 : influence lines of Shearing Force at Section S

Bending Moment at a Section S

$$0 \leq x \geq L/2$$

$$M_S = \frac{\mathrm{L}}{2} * V_A - 1(\frac{L}{2} - x)$$

$$\frac{L}{2} \leq x \geq L + a$$

$$M_S = \frac{\mathrm{L}}{2} * V_A$$

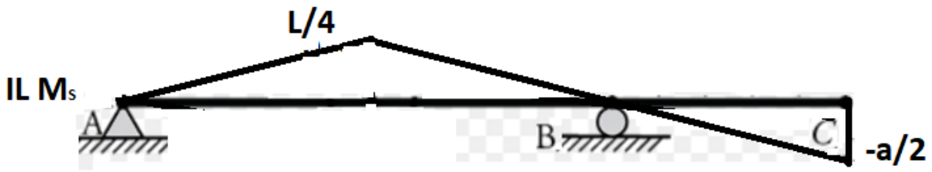


Figure 3.4 : influence lines of bending moment at Section S

3.3.2 The kinematic or Muller-Breslau method (kinematic method)

This method allows to draw influence lines faster than if we use the equilibrium calculations described in the previous section. In fact, it is likely that if we wanted to construct a shear or moment influence line using this method, then we would not have to find the reaction influence lines for the structure first.

The basis of the Müller-Breslau Principle is that we can find the influence line for a determinate beam by:

1. Removing the restraint caused by the parameter that we want to find the influence line for; then,
2. Displace or rotate the resulting structure by one unit.

If the restraint that is removed is an internal shear or a vertical or horizontal support, then the second step is to displace the structure at that same location. If the restraint that is removed is an internal moment or a rotational restraint, then the second step is to rotate the structure at that same location.

If the structure is determinate, then removing the support will cause the structure to become unstable. Therefore, it will be able to move freely, without any stiffness. The trick to the Müller-Breslau Principle is that you must figure out how the rest of the structure will move after applying the unit displacement or rotation. Since the determinate structure has lost its stiffness it also means that none of the member will bend when the unit displacement or rotation is applied. All the pieces will remain straight.

So, for a vertical or horizontal reaction component, the influence line for that reaction is constructed by removing the reaction component and then displacing the structure by 1.0 in the location of the reaction component and in the same direction as that component.

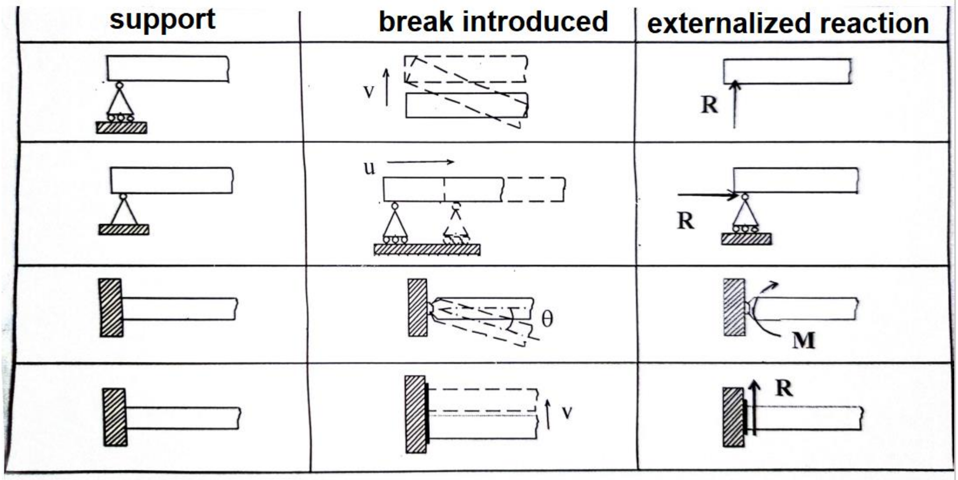


Figure 3.5 : Removing the restraint caused by the reaction that we want to find the influence line for

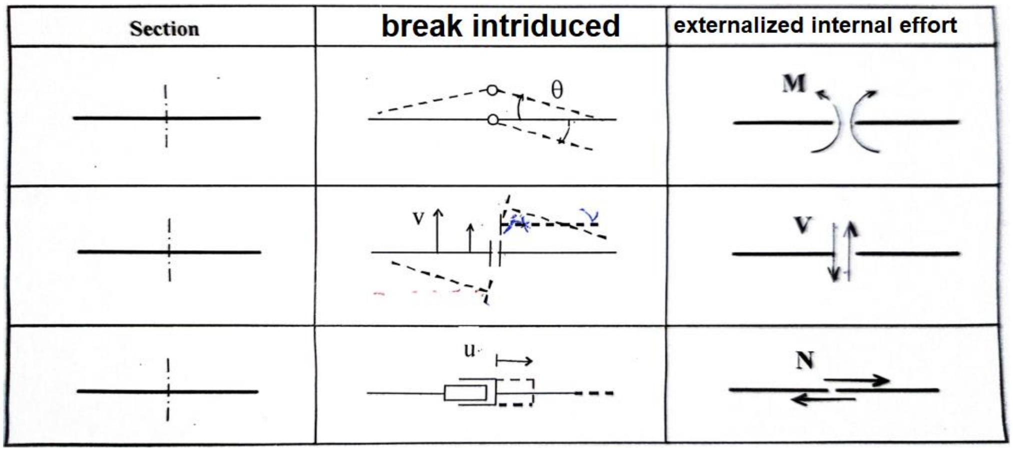


Figure 3.6 : Removing the restraint caused by the internal forces that we want to find the influence line for

Beam Reactions

Consider the simple beam shown in Figure 3.7.

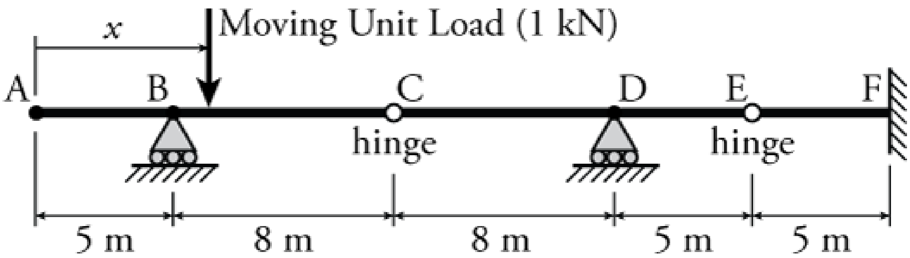


Figure 3.7 : determinate beam

To find the influence line for the vertical reaction at point B, the process is shown in Figure 3.8. The roller only provides a vertical reaction, so we remove the roller support (making the structure unstable in the process), and move point B upwards by 1.0 as shown in the figure. Since the beam is now unstable, it is free to move in this way, rotating at the hinge at point C. The resulting shape of the structure now represents the influence line for the vertical reaction at point B. The value of the influence line at B is 1.0 (the amount that we moved point B upwards).

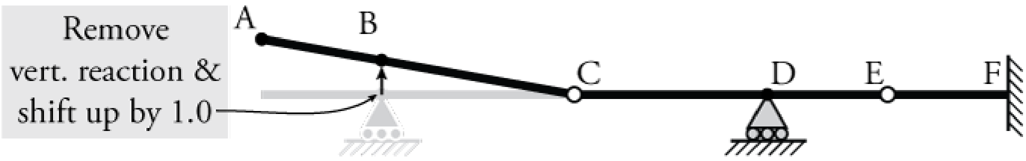


Figure 3.8 : the process for the determination the influence line for the vertical reaction at point B

Shearing Force at Section S

To find the influence line for internal shear at a point, you must break the beam in shear at that point and displace the broken ends relative to each other (so one will go up and the other end will go down); When the beam is displaced, the slope of the beam on either side of the break must be the same because we have only removed the beam's ability to take shear at the break, not internal moment. This restriction on the slope allows the determination of the relative movement of either side of the beam break using similar triangles.

To find the influence line for the shear in the beam at a point 2m to the right of point D (shown as point D') we must break the beam in shear at point D' and displace the broken ends. So, for the section on the left of the break (DD'), since the break is on the right side of this section beam, it will move downwards. Likewise, the other side of the break (D'E) will move upwards. The total relative displacement of the two ends should be 1.0. To find the value of the displacement at each displaced end, we can use similar triangles.

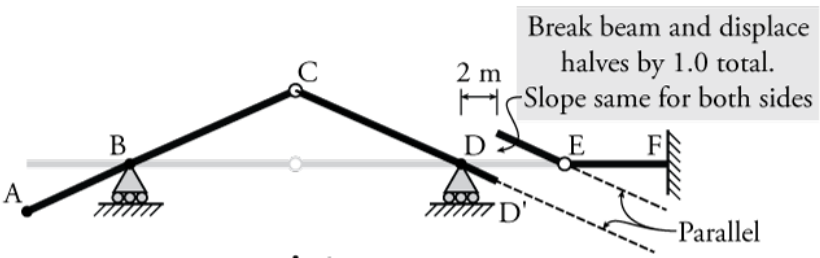


Figure 3.9 : the process for the determination the influence line for the shear in the beam at point D'

Bending Moment at a Section S

An internal moment influence line for a certain point is constructed by breaking the beam for moment at that point (while retaining the beam's ability to take shear). This is easier to conceptualize than the shear case because adding an internal hinge to the beam at that point easily releases the internal moment restraint. Therefore, to find the influence line for an internal moment at a point, add a hinge to the beam at that point and then rotate the beam at that point by a total of 1.0 rad.

To find the influence line for the internal moment in the beam at a point 2m to the right of point D (shown as point D'), you have to add a hinge at D'. Then both sides of the hinge are rotated in the positive sense for internal moment, clockwise on the left end of a beam (right side of the hinge), and counter-clockwise on the right end of a beam (left side of the hinge). The total amount of rotation at the hinge should be equal to 1.0 rad as shown in the figure.

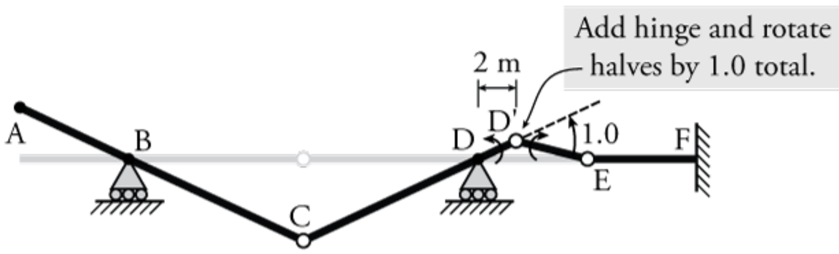


Figure 3.10 : the process for the determination the influence line for the internal moment in the beam at point D'

3.4. Uses of influence lines to determine response functions of structures subjected to concentrated point load

the influence line may be used to determine the effect of any magnitude point load on a beam. We have to do is multiply the magnitude of the applied point load by the value of the influence line at the location where the point load is applied. **F=IL(x1) P**

For example, a sample influence line for a vertical support reaction. If we apply a point load of 13 kN at point E, as shown, then the reaction force Cy will be equal to:

$C_y = 13\ \text{kN}\ (-0.4) = -5.2\text{kN}$

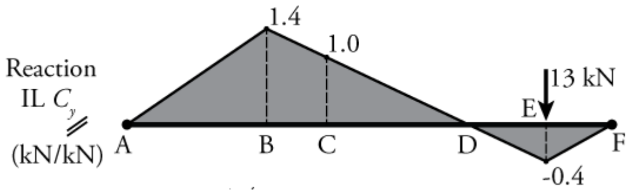


Figure 3.11 : uses of influence lines to determine the vertical support reaction at C in the beam subjected to concentrated point load

The influence line also gives us a visual indication of where a load should be placed to create the maximum positive or negative effect.

3.5. Uses of influence lines to determine response functions of structures subjected to a series of moving loads

This is a common design problem in bridge engineering, since one of the design loads is a series of loads caused by a "standard" truck, which can move anywhere along the length of the bridge. Since the truck has multiple wheels/axles, the total load of the truck is not spread evenly over its length, but is concentrated at the locations of the wheel axles.

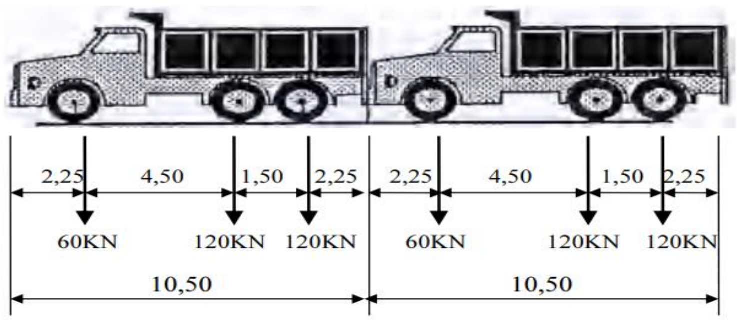


Figure 3.12 : a series of moving loads

The magnitude of a response function of a structure due to concentrated loads can be determined as the summation of the product of the respective loads and the corresponding ordinates of the influence line for that response function.

$$F = IL(x_1)\,P1 + IL(x_2)\,P2 + IL(x_3)\,P3 + \ldots + IL(x_n)\,P_n$$

A sample influence diagram for the shear in a beam at point B. we must find the maximum positive shear that will be caused by the set of moving loads.

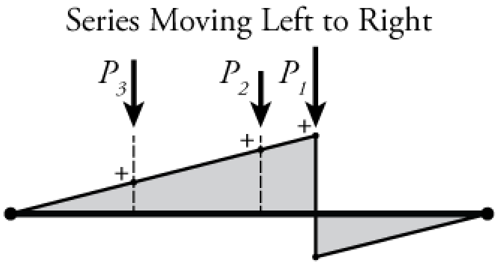


Figure 3.13 : influence diagram for the shear in a beam at point subjected to a series of moving loads

Example: A simple beam is subjected to three concentrated loads. Determine the magnitudes of the reactions and the shear force and bending moment at the midpoint of the beam using influence lines.

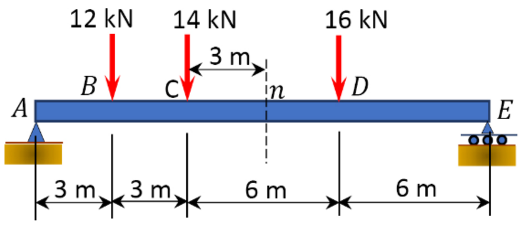


Figure 3.14 : beam subjected to three concentrated loads

3.6. Uses of influence lines to determine absolute maximum moment at any point along the structure

The absolute maximum shear force for a cantilever beam will occur at a point next to the fixed end, while that for a simply supported beam will occur close to one of its reactions. **The absolute maximum moment** for a cantilever beam will also occur close to the fixed end, while that **for simply supported beam is not readily known** and, thus, will require some analysis. To locate the position where the absolute maximum moment occurs in a simply supported beam, consider a beam subjected to three moving concentrated loads P_1, P_2, and P_3.

Although it is certain from statics that the absolute maximum moment will occur under one of the concentrated loads, the specific load under which it will occur must be identified, and its location along the beam must be known. Assume that the concentrated load under which the absolute maximum moment will occur is P3, and the distance of P3 from the centerline of the beam is x. To obtain an expression for x, first determine the resultant P_R of the concentrated loads, acting at a distance x′ from the load P3.

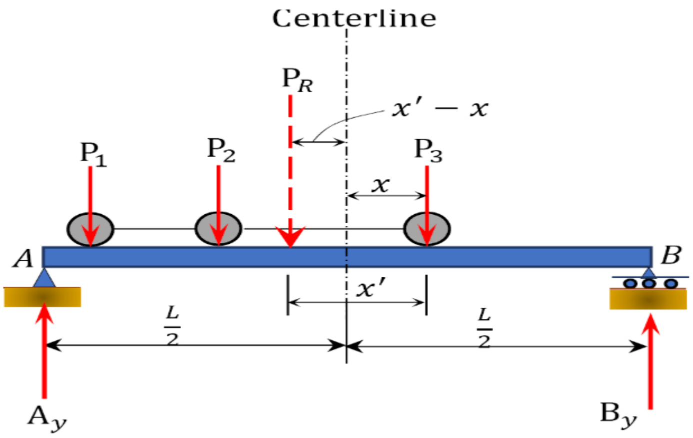


Figure 3.15 : The absolute maximum moment for simply supported beam

To determine the right reaction of the beam, take the moment about support A, as follows:

$$\sum M_A = 0$$
$$B_y L = P_R\left[\frac{L}{2} - (x' - x)\right] \Rightarrow B_y = \frac{P_R}{L}\left[\frac{L}{2} - (x' - x)\right]$$

$$M_3 = B_y\left(\frac{L}{2} - x\right) = \frac{P_R}{L}\left[\frac{L}{2} - (x' - x)\right]\left(\frac{L}{2} - x\right)$$
$$= P_R\left(\frac{L}{4} - + \frac{x'}{2} + \frac{xx'}{L} - \frac{x^2}{L}\right)$$

Thus, the bending moment under M3 is as follows:

The distance x for which M3 is maximum can be determined by differentiating this equation with respect to x and equating it to zero, as follows:

Therefore,

$$\frac{dM_3}{dx} = P_R\left(\frac{x'}{L} - \frac{2x}{L}\right) = 0$$
$$\frac{2x}{L} = \frac{x'}{L}$$
$$x = \frac{x'}{2}$$

Theorem: The absolute maximum moment in a simply supported beam occurs under one of the concentrated loads when the load under which the moment occurs and the resultant of the system of loads are equidistant from the center of the beam.

3.7. Uses of influence lines to determine response functions of structures subjected to distributed load

To do this, we simply find the area underneath the influence diagram for the parts of the diagram where a uniform distributed load is applied and then multiply that area by the magnitude of the uniform load.

Example: an influence diagram is shown for the vertical reaction at a point C (IL Cy). If a 16kN/m distributed load is applied between points B and C, we can find the effect of the distributed load on the reaction Cy by multiplying the area under the influence diagram in the applied load location (the trapezoidal area shaded in the figure) by the value of the distributed load (16kN/m):

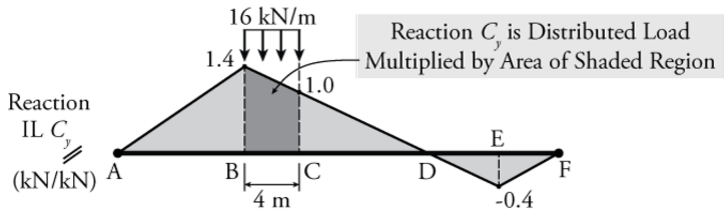


Figure 3.16 : influence lines for structures subjected to distributed load

Cy=(1.4+1) (4m) /02 (16kN/m)

Cy=76.8KN

If we would like to find the distributed loading pattern that will cause **the greatest positive moment at B** (M_B), then we should only load between points A and C and between points D and E as shown in the figure 3.17. If we were to also load the rest of the beam between C and D, then our moment would actually decrease. Patterned loading like this is a common design case in structural engineering, and this example makes it clear that sometimes <u>considering a distributed load to act along the entire length of a beam can actually be un-conservative</u>! We could actually get worse moment by leaving some of the load off.

Likewise, if we want to find the distributed loading pattern that will cause the greatest negative moment at B, then we should only load the beam between points C and D.

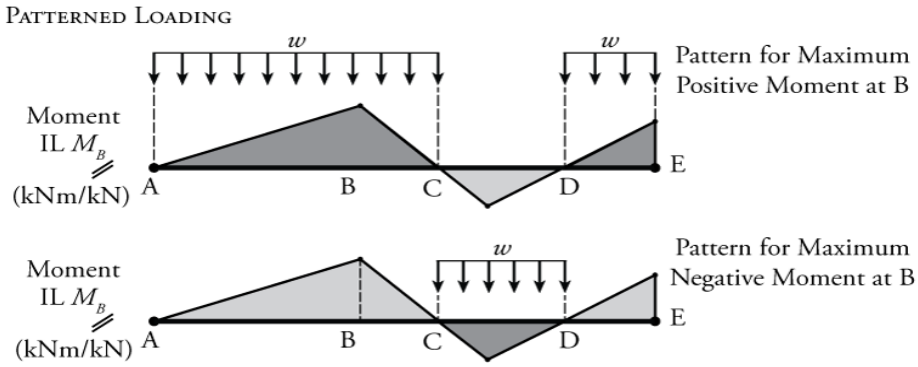


Figure 3.17 : distributed loading pattern that will cause the greatest moment at B

3.8. Influence Lines for Trusses

The construction of influence lines for trusses is similar to the construction of influence lines for beams; however, it is important to determine which path the moving load takes across the truss. On the truss shown in this figure, the road surface is level with the lower chord of the truss. Therefore, an influence line for this truss would be constructed along a direct line that joins the pin at the left with the roller on the right along the lower chord of the truss.

Another important difference for influence lines for trusses is that we must assume that the load can only be transferred to the truss at the intersection points between the members. This is necessary because the truss members themselves are often assumed in design to only take axial forces. Any loads between the intersection points on the truss elements would cause them to bend.

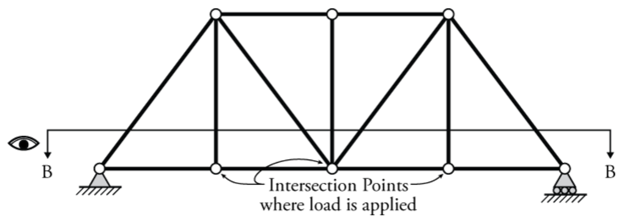


Figure 3.18 : influence lines for trusses

This means that any load that is located between intersection points will be split between the two closest intersection points (in proportion to how close the load is to each intersection point). This means that in between the intersection points, the values are just an interpolation between the two closest points. For truss influence lines, the consequence of this is that we can find the values for the influence line at the intersection points, and then just connect the points together using straight lines.

To find these influence lines, there is no easy Müller-Breslau principle. The method of nodes or method of sections must be used to find a truss member force as a function of the moving unit load position.

Example

An example truss is shown in this Figure 3.19. This truss supports a bridge and the deck of the bridge is concurrent with the bottom chord of the truss. So, the path for the moving unit point load is along a line joining points A and D. The distance x is the distance of the moving load from point A. Find the influence lines for the three members EF, BF and BC.

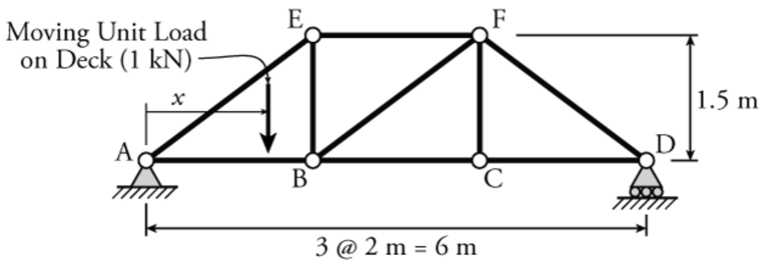
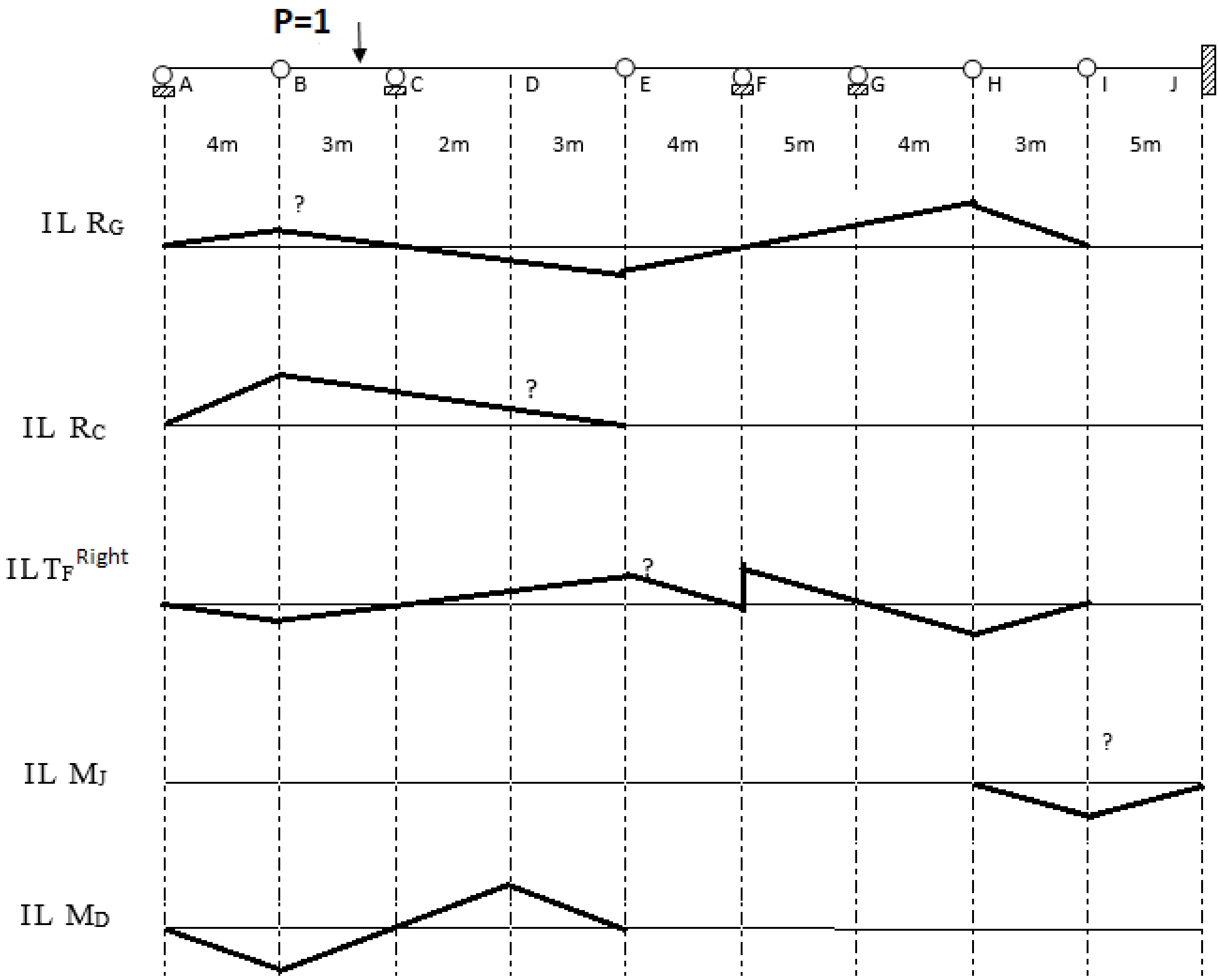


Figure 3.19 : Example of influence lines for trusses

3.9. Exercise

Given the beam shown below, plot the influence lines for the functions indicated below.

Specify the values at each question mark (?) point.


P=1
A
B
C
D
E
F
G
H
I
J
4m
3m
2m
3m
4m
5m
4m
3m
5m
IL RG
?
IL RC
?
IL TF Right
?
IL MJ
?
IL MD

CHAPTER IV

FORCE METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES

4.1. Compatible system - Lack of compatibility

An indeterminate system is said to be compatible with respect to its supports, or that the supports of an indeterminate system are compatible, when the reaction components are all zero in the absence of external loads (Figure 4.1a).

In the opposite case in the figure 4.1b, the supports are said to be incompatible (the reaction components are not all zero). The lack of compatibility of a support is represented by the linear or angular displacement it undergoes from its compatible position to its actual position.

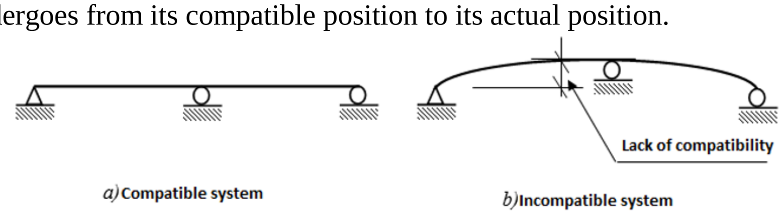


Figure 4.1 : compatible and incompatible system

4.2. Menabrea's Theorem

Theorem: The partial derivative of the internal potential energy (W) of a system with respect to an external redundant unknown or with respect to the common value of two internal redundant unknowns X_i revealed by a cut, is equal to the corresponding lack of compatibility c_i.

$$\frac{\partial W}{\partial X_i} = c_i \tag{4.1}$$

In the case of a compatible system, this derivative is always zero, i.e.:

$$\frac{\partial W}{\partial X_i} = 0 \tag{4.2}$$

The theorem follows from Castigliano’s Theorem (first form). This result is evident in the case of an external redundant unknown (i.e., a reaction), as the displacement in the direction of the reaction is zero. Menabrea's Theorem implies that redundant forces take values that minimize the internal potential energy expressed as a function of the loadings, including the redundant forces applied to the system under consideration.

Thus, for each redundant unknown X_i, Menabrea's Theorem provides a continuity equation. Solving the resulting system of equations allows finding the redundant unknowns and resolving the problem, reducing it to an isostatic one.

In the case of a linear planar system L, whose reduction elements are designated by M, N, and T, the general expression for W is as follows:

$$W = \frac{1}{2}\int_L \frac{M^2}{EI}\,ds + \frac{1}{2}\int_L \frac{N^2}{EA}ds + \frac{1}{2}\int_L \kappa\,\frac{T^2}{GA}\,ds$$

If the system is composed of several bars (beams) L_i, the integral is extended to each of them.

$$W = \frac{1}{2}\sum_i \int_{L_i} \frac{M^2}{EI}\,ds + \frac{1}{2}\sum_i \int_{L_i} \frac{N^2}{EA}\,ds + \frac{1}{2}\sum_i \int_{L_i} \kappa\,\frac{T^2}{GA}\,ds$$

If bending moment is predominant compared to other loadings, the expression of the energy is reduced to the first term.

$$W = \frac{1}{2}\sum_i \int_{L_i} \frac{M^2}{EI}\,ds$$

For planar systems composed of straight articulated bars (trusses) with loads applied at the nodes (indirectly loaded trusses), the energy is given by:

$$W = \frac{1}{2}\sum_i \int_{L_i} \frac{N^2}{EA}\,dx = \frac{1}{2}\sum_i N_i^2 \int_{L_i} \frac{dx}{EA}$$

And if the extensional stiffness EA is constant on each bar, it becomes:

$$W = \frac{1}{2}\sum_i \frac{N_i^2 L_i}{(EA)_i}$$

4.3. Principle of the Force Method

In this method, the unknowns are the redundant forces. A redundant force can be an external support reaction force or an internal member force, which if removed from the structure, will not cause any instability. To remove n redundant forces, n cuts must be made, one for each redundant unknown. For the determinate system to be equivalent to the initial system, each removed support must be replaced by the corresponding force (Figure 4.2).

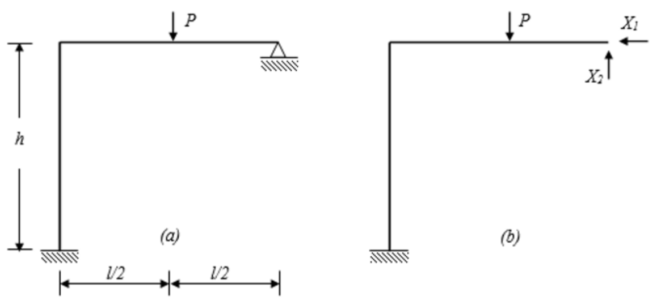


Figure 4.2 : primary structure

A set of compatibility equations is formulated using the equation 4.2, depending on the number of the redundant forces in the structure, and solving these equations simultaneously to determine the magnitude of the redundant forces. Once the redundant forces are known, the structure becomes determinate and can be analyzed completely using the conditions of equilibrium.

$$\begin{cases} \dfrac{\partial W}{\partial X_1} = 0 \\ \dfrac{\partial W}{\partial X_2} = 0 \end{cases} \tag{4.3}$$

The structure that remains after the removal of the redundant reaction is called the primary structure. A primary structure must always meet the equilibrium requirement. More than one primary structure is possible.

4.4. Continuity equations

For a compatible system of order n, there will be a set of n equations:

$$\frac{\partial W}{\partial X_1} = 0,\ \frac{\partial W}{\partial X_2} = 0,\ \ldots, \frac{\partial W}{\partial X_j} = 0,\ \ldots, \frac{\partial W}{\partial X_n} = 0 \tag{4.4}$$

These equations can be written in the following form, known as the Müller-Breslau formula:

$$\frac{\partial W}{\partial X_j} = 0,\ \Leftrightarrow\ \sum_{i=1}^{n} X_i \delta_{ji}^{u} + \delta_{jF} = 0 \qquad j = 1,\ 2,\ \ldots, n \tag{4.5}$$

The system of n continuity equations can be written in the following explicit form:

$$\begin{cases} \delta_{11}^{u} X_1 + \delta_{12}^{u} X_2 + \ldots + \delta_{1n}^{u} X_n + \delta_{1F} = 0 \\ \delta_{21}^{u} X_1 + \delta_{22}^{u} X_2 + \ldots + \delta_{2n}^{u} X_n + \delta_{2F} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \delta_{n1}^{u} X_1 + \delta_{n2}^{u} X_2 + \ldots + \delta_{nn}^{u} X_n + \delta_{nF} = 0 \end{cases} \tag{4.6}$$

or :

$$\left[\delta^{u}\right]\{X\} = \{-\delta_F\} \tag{4.7}$$

The equations of system (4.5) or (4.6) are called the canonical equations of the force method.

4.5. Meaning and calculation of the coefficients

The coefficients δ_{ij}^{u} are the influence coefficients. Their matrix δ_u is called the flexibility matrix. The general coefficient δ_{iF} represents the displacement of section i of the primary structure (in the i-direction, which is also the direction of the redundant unknown X_i) under the effect of the applied (given) loads (F).

4.6. Results Verification

Verifying the equilibrium of the nodes and entire parts of the studied structure, based on the force diagrams, provides a good means of checking the obtained results. Each node or part of the structure isolated by cuts must be in equilibrium under the action of the forces directly applied to it and the internal forces (M, N, T) acting at the edges of the cuts, which can be directly read from the diagrams to be checked.

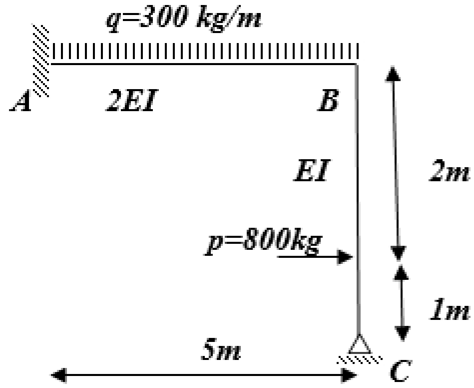
4.7. Meaning of the Continuity Equations.

each continuity equation expresses that the relative displacement of the edges of the cut releasing the considered redundant unknown is equal to the corresponding lack of compatibility.

In the case of a compatible system, this (relative) displacement is zero. When it concerns an external redundant unknown, the relative displacement is, in fact, the actual displacement.

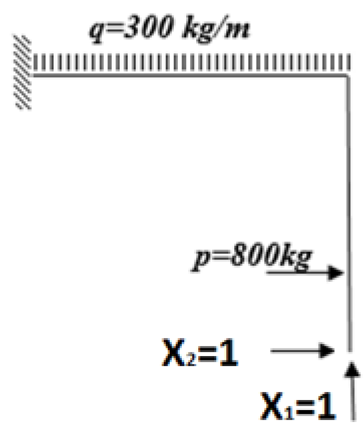
4.8. Application Example

Using the force method, calculate and plot the internal force diagrams M, N, T (bending moment, axial force, shear force) for the system below (EI= constant):

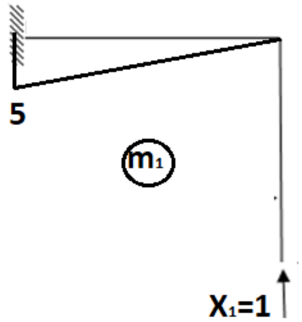


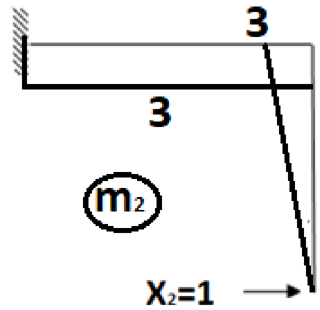
H=3c-a-2s=(3*1)-1-(2*0)=2

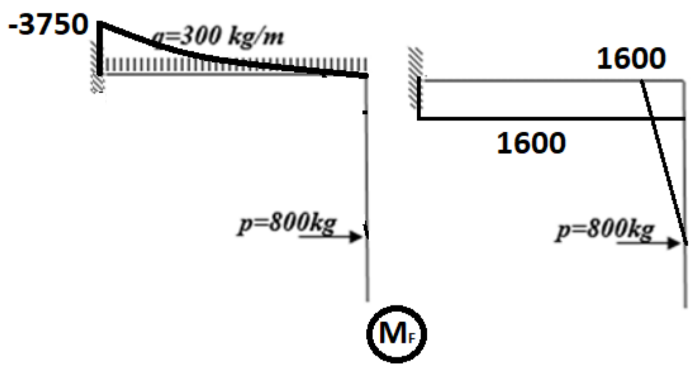
The primary structure adopted is:



Calculate of the influence coefficients:





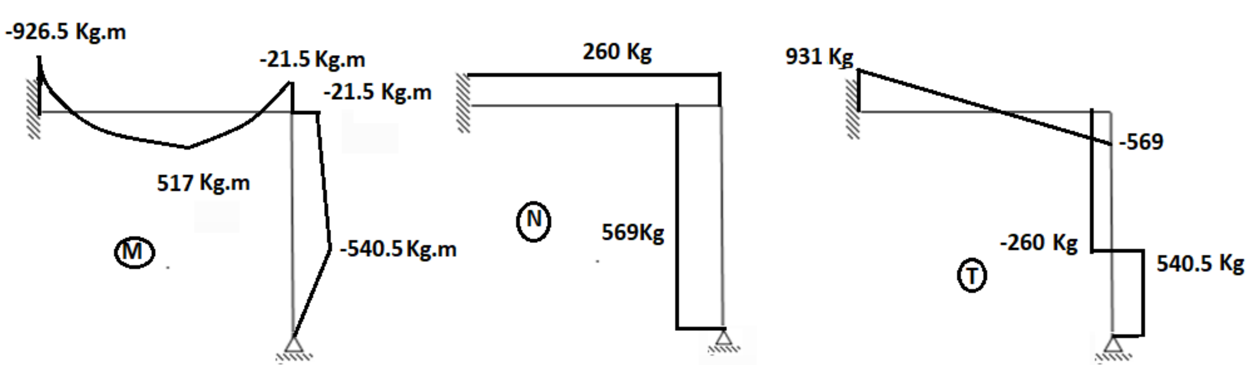


δ_{11}=125/3EI δ_{12}=75/4EI δ_{22}=63/2EI δ_{1F}=-6875/4EI… δ_{2F}=19075/3EI.

X1=568.97Kg… *…X2=-540.5Kg……………………..*

R_A=931Kg…… …….H_A=-259.5Kg……… …M_A=926.5Kg…….

The internal force diagrams M, N, T are:



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