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Effects of non-Darcy natural convection over a vertical plate
with different convective boundary conditions incorporating
gyrotactic microorganisms on dispersion

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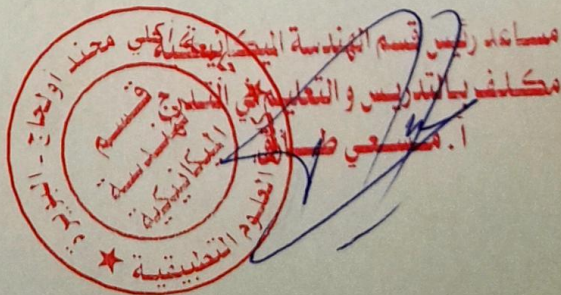
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Dedication

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Abstract

This thesis investigates natural convection over a vertical plate embedded in a non-Darcy porous medium saturated with a nanofluid containing gyrotactic microorganisms. A mathematical model incorporating Brownian motion, thermophoresis, bioconvection, and cross-diffusion effects was developed. The governing partial differential equations were transformed into a system of nonlinear ordinary differential equations using similarity variables and solved numerically with MATLAB's bvp4c solver.

Parametric analysis revealed that increasing the convective parameter enhances heat and mass transfer, while higher Darcy resistance reduces flow. Bioconvection improves velocity profiles, and nanoparticle effects significantly influence temperature and concentration distributions. These results offer valuable insight into complex transport phenomena in porous nanofluid systems with biological activity.

Keywords : Natural convection, vertical plate, nanofluids, Non-Darcy porous media, Bioconvection, Gyrotactic microorganisms.

Résumé

Ce mémoire étudie la convection naturelle le long d'une plaque verticale immergée dans un milieu poreux non Darcy saturé par un nanofluid contenant des micro-organismes gyrotactiques. Un modèle mathématique intégrant le mouvement brownien, la thermophorèse, la bioconvection et les effets de diffusion croisée a été développé. Les équations différentielles partielles régissant le phénomène ont été transformées en un système non linéaire d'équations différentielles ordinaires à l'aide de variables de similarité, puis résolues numériquement à l'aide du solveur *bvp4c* de MATLAB.

L'analyse paramétrique a montré qu'une augmentation du paramètre convectif améliore les transferts de chaleur et de masse, tandis qu'une résistance Darcy plus élevée réduit l'écoulement. La bioconvection améliore les profils de vitesse, et les effets des nanoparticules influencent fortement les distributions de température et de concentration. Ces résultats offrent une meilleure compréhension des phénomènes de transport complexes dans les systèmes de nanofluides poreux contenant une activité biologique.

Mots-clés : Convection naturelle, plaque verticale, nanofluides, milieu poreux non Darcy, bioconvection, micro-organismes gyrotactiques.

الملخص

تتناول هذه الأطروحة دراسة الحمل الطبيعي على صفيحة عمودية مغمورة في وسط مسامي غير دارسي ومشبع بسائل نانوي يحتوي على كائنات دقيقة جيروتاكتية. تم تطوير نموذج رياضي يدمج تأثيرات الحركة البراونية، والانجذاب الحراري، والحمل الحيوي، وظواهر الانتشار المتقاطع. وقد تم تحويل المعادلات التفاضلية الجزئية الحاكمة إلى نظام من المعادلات التفاضلية العادية غير الخطية باستخدام متغيرات التشابه، وتم حلها عددياً باستخدام البرنامج الحسابي (bvp4c) في برنامج (Matlab).

أظهرت التحاليل البارامترية أن زيادة معامل الحمل الحراري تعزز من انتقال الحرارة والكتلة، في حين أن زيادة مقاومة دارسي تقلل من تدفق المائع. كما أن الحمل الحيوي يعمل على تحسين منحنيات السرعة، بينما تؤثر الجسيمات النانوية بشكل كبير على توزيعات الحرارة والتركيز. وتقدم هذه النتائج فهماً أعمق لظواهر النقل المعقدة في أنظمة السوائل النانوية المسامية ذات النشاط البيولوجي.

الكلمات المفتاحية: الحمل الطبيعي، صفيحة عمودية، السوائل النانوية، وسط مسامي غير دارسي، الحمل الحيوي، الكائنات الدقيقة الجيروتاكتية.

Table des matières

Abstract	7
Résumé	8
الملخص	9
General introduction:.....	15
Chapter I: Theoretical research	18
I. Introduction:	19
I.1. Heat transfer:	19
I.1.1. Conduction:	20
I.1.2. Radiation:	20
I.1.3. Convection:.....	20
I.2. Porous media:.....	21
I.2.1. Definition of porous media:.....	21
I.2.2. Characteristics of porous media:	22
I.2.3 Traditional Darcy's law and its limitations:	23
I.2.4. Darcy effects and their significance in high-permeability media:	23
I.3. Role of nanofluids in heat transfer enhancement:	26
I.3.1. Properties of nanofluids:	27
I.3.2. Applications and benefits in thermal system:.....	27
I.3.3. Mathematical models describing nanofluids behaviors:	27
I.4. Gyrotactic microorganisms and bioconvection:.....	28
I.4.1. Definition of gyrotactic microorganisms:	28
I.4.2. Mechanisms of microorganism notion and dispersion:	29
I.4.3. role and applications in mechanical engineering:	30
Literature review	31
I.5. literature review of existing research:	32
Chapter II: Governing Equations and Model Formulation	36
II.1. Introduction:	37
II.2. Translation of Theoretical Framework into Mathematical Equations:.....	38
II.3. physical problem and mathematical analysis:	44
II.3.1. Mathematical equations:	46
Chapter III: Numerical Method and Algorithm	51
III.1. Analytical resolution:.....	52
III.2. program validation:	54
Chapter IV: Results and discussion	55
IV. Result and discussion:	56

IV.1. Dimensionless velocity profile $S'(\eta)$:	56
IV.2. Dimensionless temperature profile $\theta(\eta)$:	58
IV.3. Dimensionless nanoparticle concentration profile $\gamma(\eta)$:	60
IV.4. Dimensionless microorganism density distribution $\chi(\eta)$:	61
IV.5. Heat and mass transfer rates:	63
Conclusion:	66
V. General conclusion:	69
References:	72

List of figures:

Figure (1): Illustration about the three types of heat transfer (case of attic of a house). [2] ...	19
Figure (2): porosity and permeability illustration in a porous medium [8]	22
Figure (3): some types of microorganisms [22]	28
Figure (4): heat transfer from a vertical isothermal wall to a linearly stratified porous medium [27]	32
Figure (5): Heat transfer data for water-based nanofluids with oxide nanoparticles and metal nanoparticles. [17]	33
Figure (6): streamline and plots of temperature and stream function distribution at different position of hot and cold sources. [29]	34
Figure (7): Physical model and coordinate system	45
Figure (8): Effects of the convective parameter h and the modified Darcy parameter N_d on the dimensionless velocity profile.....	57
Figure (9): effect of convective parameter h and the bioconvection Rayleigh number R_b on the dimensionless velocity profile.....	58
Figure (10): Effects of the convective parameter h , the Brownian motion parameter N_b and the thermophoresis parameter N_t on the dimensionless temperature profile	59
Figure (11): Effects of the convective parameter h and the Dufour number Du on the dimensionless temperature profile	60
Figure (12): Effects of the Soret number S_r , nanoparticle and traditional Lewis numbers Le and Le_n on the dimensionless nanoparticle concentration profile	61
Figure (13): Effects of the bioconvection Lewis number Le_b and the Péclet number Pe on the dimensionless microorganism density profile.	62
Figure (14): Effects of the convective parameter h , the Brownian motion parameter N_b and the thermophoresis parameter N_t on the dimensionless microorganism density profile.	63

List of tables:

Table (1): Comparison of the maximum relative error ϵ^*	54
Table (2) : values of $NuxRax12$ and $ShxRax12$ for selected values of h , N_d and N_b	63
Table (3): values of $NuxRax12$ and $ShxRax12$ for selected values of h , N_d and N_t	64
Table (4): values of $NuxRax12$ and $ShxRax12$ for selected values of h , N_d and Du	65

Nomenclature:

Symbols	Signification	Units
x	Horizontal coordinate	m
y	Vertical coordinate	M
u	Velocity component in the x-direction	m/s
v	Velocity component in the y-direction	m/s
T	Temperature of the nanofluid	K
C	Nanoparticle concentration	Kg/m^3 or mol/m^3
n	Density of motile microorganisms	$cells/m^3$
μ	Dynamic viscosity of the fluid	Pa-s
ρ	Density of the fluid	Kg/m^3
β	Thermal expansion coefficient	$1/K$
g	Gravitational acceleration	m/s^2
D_B	Brownian diffusion coefficient	m^2/s
D_T	Thermophoretic diffusion coefficient	m^2/s
D_n	Diffusivity of microorganisms	m^2/s
D_m	Chemical molecular diffusivity	m^2/s
α	Thermal diffusivity	m^2/s
k	Thermal conductivity	$W/m.K$
η	Similarity variable	-
$\theta(\eta)$	Dimensionless temperature profile	-
$\gamma(\eta)$	Dimensionless nanoparticle concentration profile	-
$\chi(\eta)$	Dimensionless microorganism density profile	-
Nb	Brownian motion parameter	-
Nt	Thermophoresis parameter	-
Nd	Modified non-Darcy number	-
Nr	Buoyancy ratio parameter	-
Du	Dufour number	-
Sr	Soret number	-
Ln	Nanoparticle Lewis number	-
Le	Traditional Lewis number	-
Lb	Bioconvection Lewis number	-
Rb	Bioconvection Rayleigh number	-
Pe	Bioconvection Péclet number	-
h	Convective heat transfer parameter	-
bi	Bioconvection constant	-

General introduction

General introduction:

The study of natural convection has gained significant attention in recent years due to its broad range of applications in engineering, geophysics and biomedical sciences. Natural convection arises from buoyancy forces that develop due to temperature and concentration variations within a fluid. When these flow occurs in porous media, additional complexities arise, requiring a deeper understanding of fluid dynamics, heat transfer and mass transport.

In many practical applications, such as geothermal energy extraction, chemical reactors, filtration processes and biological systems, the governing transport phenomena involve the interaction of heat, mass and momentum transfer through a porous medium. A porous medium is a material containing interconnected void spaces, allowing fluid to pass through. The traditional modelling of fluid flow in porous media is based on Darcy's law, which assumes a linear relationship between velocity and pressure gradient. However, for high permeability and high velocity flows, non-Darcy effects must be considered. These include inertia and boundary layer effects, which significantly alter the flow structure and heat transfer characteristics. Non-Darcy natural convection provides a more realistic representation of flow behavior in such cases, making it essential for accurate modelling.

A porous medium can be found in various natural and industrial settings, including soil, rocks, biological tissues and engineered materials like metal foams. The presence of a porous medium affects both heat and mass transfer due to increased surface area for interaction and enhanced mixing properties. In heat transfer, porous materials often act as thermal conductors or insulators depending on their properties. The effective thermal conductivity of a porous medium is determined by the solid matrix and the fluid filling the pores, leading to different convection and conduction behaviors.

Similarly, in mass transfer, porous structures influence the dispersion and diffusion of species, which is particularly relevant in fields like chemical engineering, biomedical applications and environmental science. The combination of heat and mass transfer in a porous medium plays a crucial role in processes such as oil recovery, groundwater flow and industrial drying operations.

In recent years, the use of nanofluids has emerged as an effective way to enhance heat transfer properties. A nanofluid is a suspension of nanoparticles (such as metallic, metal oxide, or carbon based nanoparticles) in a base fluid (such as water, ethylene glycol, or oil). Due to

their high thermal conductivity, nanoparticles improve the overall heat transfer rate and modify the convective heat transfer characteristics.

Nanofluids are particularly useful in energy systems, cooling devices, biomedical applications and Microfluids, where efficient thermal management is critical. Their ability to enhance thermal conductivity, viscosity and surface tension effects makes them a promising choice for optimizing convective heat transfer processes. However, their behavior in porous media under non-Darcy convection conditions is still a developing area of research.

Another significant aspect of this study is the role of gyrotactic microorganisms in modifying natural convection and dispersion processes. Gyrotactic microorganisms, such as certain species of algae and bacteria, exhibit a unique swimming behavior that is influenced by fluid shear and gravitational forces. Their ability to self-propel and orient themselves in response to external stimuli introduces a new mechanism of bioconvection¹.

In applications such as biofuel production, wastewater treatment and biomedical fluid flow, the presence of microorganisms affects mass transfer and nutrient mixing. By incorporating gyrotactic microorganisms into the analysis, this study extends traditional convective heat and mass transfer models to include biological interactions. Understanding how these microorganisms interact with nanofluids and porous structures can lead to advancements in biotechnological applications and engineered bio-inspired systems.

The boundary conditions imposed on the system significantly influence the overall heat and mass transfer characteristics. In this study, different convective boundary conditions are considered, simulating realistic thermal and concentration variations at the vertical plate. These boundary conditions include:

- constant heat flux;
- variable surface temperature and concentration;
- mixed convective conditions where both conduction and convection occur simultaneously;

Each of these conditions represents a specific physical scenario that can be applied in industries such as thermal insulation, solar energy collector and chemical processing. The choice of boundary conditions affects the stability, flow structure and overall efficiency of the transport process, making their analysis essential.

¹ A process where microorganisms collectively move in a fluid, creating complex flow patterns.

Given the complexity of the combined effects of non-Darcy convection, nanofluids, gyrotactic microorganisms and porous medium, this research aims to provide comprehensive analysis of their interplay. The main objectives of this study are:

- 1- To investigate the influence of non-Darcy natural convection on heat and mass transfer over a vertical plate in a porous medium.
- 2- To examine the role of nanofluids in enhancing heat transfer properties under different boundary conditions.
- 3- To explore the effects of gyrotactic microorganisms on dispersion and bioconvection phenomena.
- 4- To analyze the impact of different convective boundary conditions on the overall fluid flow and heat transfer behavior.
- 5- To develop mathematical models and numerical simulations that can accurately predict the combined effects of these factor in real world applications.

To address these objectives, the thesis is organized into two main chapters;

-The first chapter provides a comprehensive overview of the fundamental concepts relevant to this study, including natural convection, heat and mass transfer in porous media, nanofluids, gyrotactic microorganisms, and convective boundary conditions. Its main purpose is to establish a solid theoretical foundation, highlight key contributions in the field, identify research gaps, and discuss recent developments with practical relevance to engineering and scientific applications.

-The second chapter is devoted to the numerical resolution of this system using, followed by a detailed parametric study to analyse the influence of key dimensionless parameters on the velocity, temperature, concentration, and motile microorganism profiles.

Chapter I: Theoretical research

I. Introduction:

This chapter provides an in-depth review of the existing literature on key topics related to this study, including natural convection, heat and mass transfer in porous media, the role of nanofluids, gyrotactic microorganisms and connective boundary conditions

The primary goal is to develop a strong theoretical framework that underpins this research, analyses significant contributions in the field and identify existing research gaps that necessitate further study. Additionally, the chapter highlights the latest advancements in these domains, discussing their practical implication for heat and mass transfer applications in engineering and scientific research.

I.1. Heat transfer:

Heat transfer is the discipline of thermal engineering that studies the movement of thermal energy from one physical system to another. This energy transfer occurs due to a temperature difference and can take place through three primary mechanisms: conduction, convection and radiation. [1]

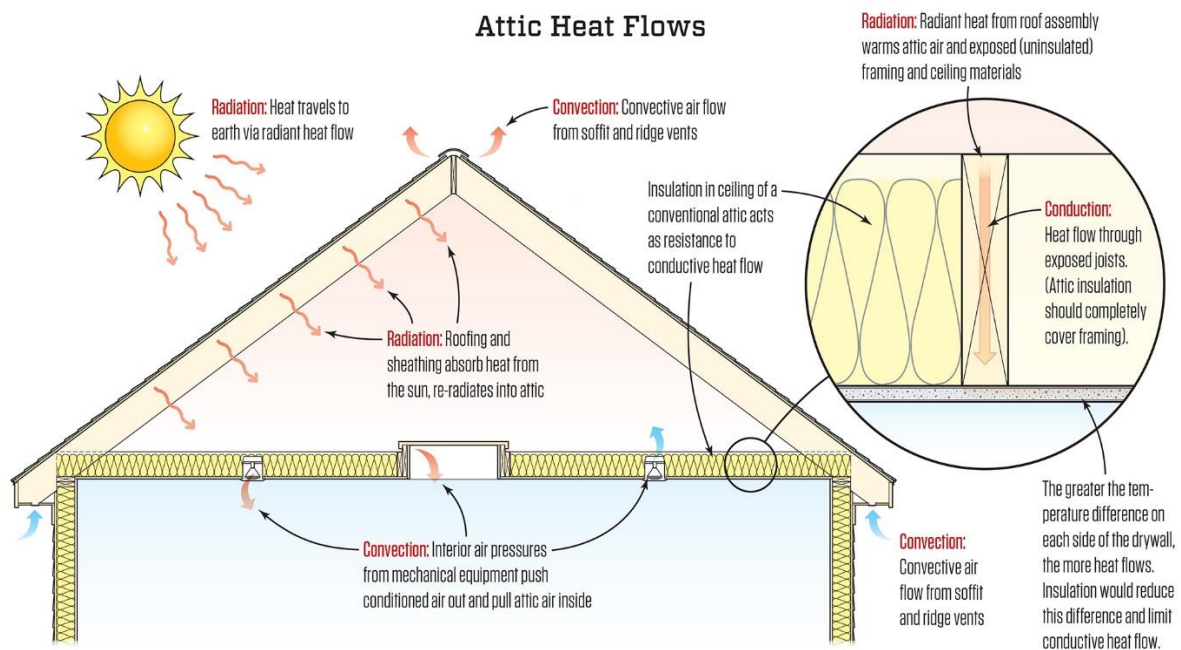


Figure (1): Illustration about the three types of heat transfer (case of attic of a house). [2]

I.1.1. Conduction:

Thermal conduction is a key mode of heat transfer involving the transfer of energy through direct contact without bulk motion of the material. It occurs via microscopic interactions, like lattice vibrations or free electron movement, and is crucial in systems where solids interface with fluid flows—such as porous media. Its effectiveness depends on the material's thermal conductivity and plays a significant role in heat transfer within nanofluids and porous structures

I.1.2. Radiation:

Radiation is a heat transfer mode that operates via electromagnetic waves and can occur in a vacuum. It depends on a surface's temperature and emissivity—its ability to emit thermal energy compared to a perfect blackbody. Radiation becomes particularly important in high-temperature systems, such as combustion chambers or solar collectors, and can also influence heat transfer at the boundaries of porous or nanofluid systems.

I.1.3. Convection:

Convection is the process by which thermal energy is transferred within fluids (liquids and gases) as a result of the combined effects of conduction and fluid motion, it is essential to the larger study of heat transfer. Convection enhances heat transmission by involving the bulk movement of the fluid, in contrast to pure conduction, which depends only on molecular interactions. Depending on the kind of fluid motion, convection may be divided into three primary categories: natural, mixed and forced convection. In forced convection, which is frequently found in HVAC systems and industrial heat exchangers, the fluid motion is extremely driven by mechanical devices like fans or pumps. In the intermediate scenario of mixed convection, the flow and heat transmission properties are influenced by both natural and driven caused. In this present study, we focus on natural convection. [3]

I.1.3.1 Natural convection:

Natural convection is a heat transfer process driven by buoyancy forces caused by temperature induced density differences in a fluid sink, creating a continuous circulation. This phenomena, governed by the energy conservation equation, plays a vital role in natural and engineering systems, influencing weather patterns, industrial cooling and thermal management.

The process of natural convection depends on three essential factors:

- A temperature difference that causes variation in fluid density.
- Gravity, which allows buoyant forces to drive the moment
- A fluid medium capable of flowing freely in response to these forces

Together, these elements create the continuous circulation that defines natural convection.

The intensity of natural convection is measured by the Rayleigh number, which indicates whether heat transfer is dominated by conduction when this number goes beyond a certain limit (>1700), the fluid start moving and heats transfer shifts from conduction to convection. [4]

Boussinesq approximation:

The Boussinesq approximation is a mathematical simplification used in the study of buoyancy-driven flows, where fluid density variations are negligible except in the buoyancy (gravitational body force) term of the momentum equation. This assumption allows for the coupling of temperature-induced density changes to the fluid motion, enabling accurate modelling of natural convection without solving the full compressible Navier–Stokes equations. In the context of the present study—dealing with nanofluid flow containing gyrotactic microorganisms in a porous medium—the approximation facilitates a tractable formulation while retaining the essential physics governing thermally induced motion. It ensures that thermal expansion effects contribute to flow development, crucial for capturing natural convection mechanisms in porous structures under moderate temperature gradients. [5]

I.2. Porous media:

I.2.1. Definition of porous media:

Many natural and synthetic materials are porous. A porous material is composed of a solid framework that contains numerous pores, which are irregularly and randomly distributed throughout its structure. These empty spaces can either be interconnected or isolated.

A fluid can only flow through a porous medium if most of these pores are connected to one another. When all the pores are filled with the same fluid phase (whether liquid or gas) the material is said to be saturated with that fluid.

Near the surface of the solid grains, molecular attraction forces create this layers of water around the particles. This water is known as bound or absorbed water. Beyond these zones of attraction, the fluid can move freely, meaning that flow paths vary depending on the distance between the solid grains. [6]

I.2.2. Characteristics of porous media:

To study flow and heat transfer in porous media, properties like permeability and porosity must be identified and measured at a large (macroscopic) scale, where the medium can be treated as continuous, since these properties vary with the scale of observation.

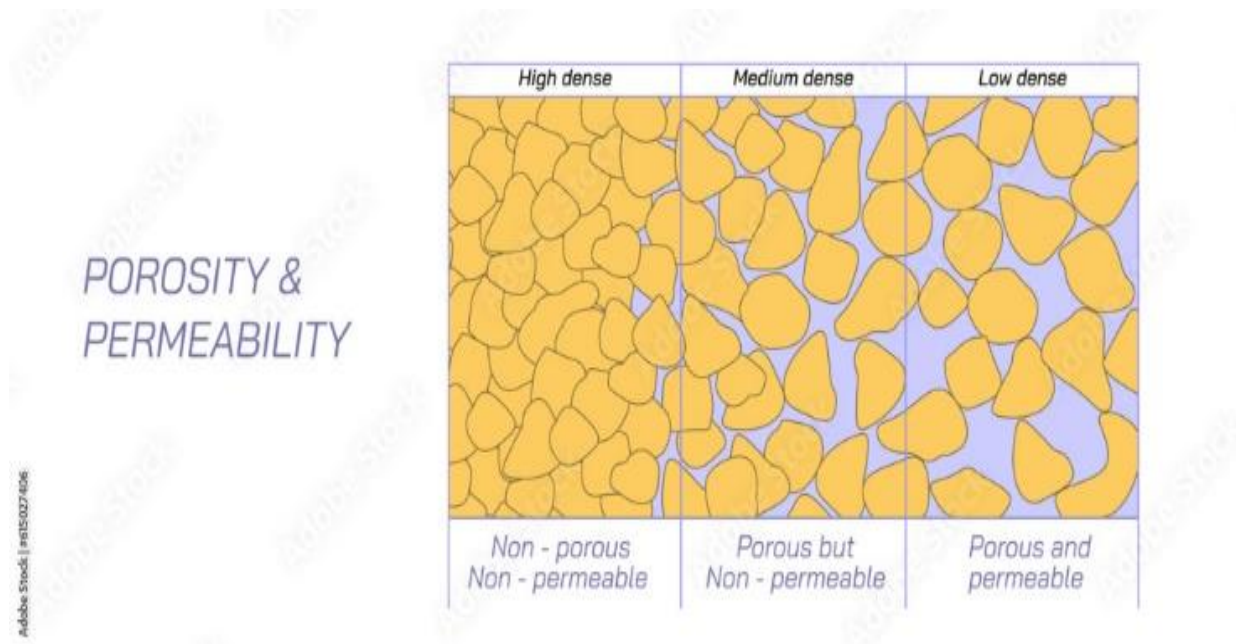


Figure (2): porosity and permeability illustration in a porous medium [7]

a)- Porosity:

Porosity simply measures how much empty spaces exists within a material. It tells us what portion of the total volume is made up of pores or voids and it's usually given as a percentage. This property is especially important in areas like soil science, water flow studies, and material design, where understanding how fluids move through a material matters.

b) permeability:

permeability is a measure of how easily a fluid (like water or air) can pass through a porous material. While porosity describes the volume of voids, permeability reflects how well these voids are connected, which determines the material's ability to transmit fluids.

I.2.3 Traditional Darcy's law and its limitations:

Darcy's law is a fundamental equation used to describe the flow of fluid through a porous medium. It was first formulated by Henry Darcy in 1856 and is widely applied in fields like hydrogeology, petroleum engineering, and soil science. [8]

Limitation of Darcy's law in the contest of porous media:

While Darcy's law is widely used for understanding fluid flow through porous materials, it has several limitations:

- 1- Laminar flow: assumes smooth, steady flow. Fails under high velocities or coarse media where turbulence occurs.
- 2- Uniform material: based on homogenous and isotropic media-accuracy drops with heterogeneous or directionally variable materials.
- 3- Constant permeability: assumes fixed permeability, but real conditions often cause it to vary with saturation, pressure or clogging.
- 4- Fluid behavior: applies to Newtonian fluids with constant viscosity many practical fluids are non-Newtonian with variable flow properties.
- 5- Multi-phase flow: designed for single-phase flow, real porous media often contains multiple interacting fluid phase.
- 6- Scale effects: valid at macroscopic scale. At microscopic (pore) scale, assumptions break down due to structural complexities.
- 7- Steady-state: assumes unchanging conditions. Real systems often involve time-dependent behaviors requiring dynamic models. [9]

I.2.4. Darcy effects and their significance in high-permeability media:

Darcy's law provides a foundational framework for modelling fluid flow through porous media under laminar and low-velocity regimes, where viscous forces predominate and the pressure gradient is linearly related to the superficial velocity. However, in high-permeability structures such as gravel packs, metal foams, and fractured porous systems, the flow transitions toward higher velocities, making inertial effects non-negligible. Under such conditions, the assumptions underpinning Darcy's linear formulation break down, necessitating the incorporation of non-linear corrections to accurately describe the momentum transport. [10]

Forchheimer's extension of Darcy's law:

Darcy's law becomes inadequate for flow in porous media when inertial effects arise at moderate to high Reynolds numbers, particularly in high-permeability structures (e.g., metal foams, gravel, fractured formations). To capture nonlinear behavior, Forchheimer (1901) introduced a correction by adding a quadratic velocity term to the momentum equation. This nonlinear model better represents pressure losses due to inertial resistance in porous channels and is critical for accurately simulating non-Darcian flow regimes encountered in advanced thermal, energy, and environmental systems. [11]

Importance of the Forchheimer coefficient β :

The Forchheimer coefficient (β) characterizes the inertial resistance encountered in porous media flows at moderate to high velocities, where nonlinear pressure drop becomes significant. Unlike permeability K , which governs viscous effects, β accounts for inertial deviations due to flow acceleration, particularly in structures with low porosity or fine pore structures.

Its magnitude depends on the geometrical characteristics of the medium, such as pore size, particle diameter, and porosity. As ϵ decreases, inertial drag intensifies due to increased tortuosity. Since β is not a universal constant, it must be determined via experiment or empirical models like the Ergun equation. In volume-averaged CFD, incorporating both Darcy and Forchheimer terms ensures accurate modelling of total hydrodynamic resistance in porous systems.

Without including the Forchheimer correction, prediction of pressure drop, flow rate and thermal performance can be significantly inaccurate in high permeability systems. Engineers rely on this model to design efficient systems where momentum losses can't solely due to viscosity. [12]

Significance in high-permeability media:

- Underestimation of pressure losses can occur if non-Darcy effects are neglected.
- Important in hydrocarbon extraction, ground water hydrology, geothermal systems, and industrial filtration, where flow rates are often high.
- The Forchheimer coefficient (β) is material. Specific and often determined through

experiments or pore-scale simulations. [13]

Most studies do a good job of explaining how fluids move through porous materials using Darcy's law, which is a well-known and straightforward model. It assumes that the fluid flows slowly, and that the resistance comes mainly from the porous medium's structure. This works fine when the motion is gentle—like in low-speed groundwater flow. But real-world systems, especially in engineering and biology, often behave differently.

For example, when nanofluids are used (liquids mixed with very small solid particles), or when the fluid contains motile microorganisms that swim or create patterns (bioconvection), the flow becomes more intense and dynamic. In these cases, the inertia of the fluid (its tendency to resist changes in motion) starts to play an important role. Darcy's law doesn't include these effects, so it may oversimplify the problem.

That's why researchers turn to non-Darcy models, like the Forchheimer equation, which adds extra terms to account for inertial resistance. These models are better suited for flows where the speed is higher or where complex interactions—such as heat and mass transfer enhanced by nanoparticles or microorganisms—are present. In such cases, using a simple Darcy model could lead to serious errors in predicting the velocity of the fluid or how efficiently heat and solutes are transferred.

Furthermore, in bioconvective systems, microorganisms not only move but also affect the fluid around them. This introduces nonlinear effects that amplify the deviations from Darcy's assumptions. So, for flows involving high Rayleigh numbers, active biological agents, or thermal gradients, the non-Darcy approach gives a more realistic and reliable description of the system behavior.

In summary, while Darcy's law is suitable for slow, simple flows, non-Darcy models become essential when dealing with nanofluids, high-speed flow, or active suspensions like gyrotactic microorganisms. Choosing the right model is not just a technical detail—it significantly influences the accuracy of simulations and the reliability of design in energy systems, biomedical applications, or environmental engineering

I.3. Role of nanofluids in heat transfer enhancement:

Nanofluids are advanced heat transfer liquids made by dispersing tiny solid particles, known as nanoparticles, into regular fluids like water or oil. These particles are incredibly small (typically ranging from 1 to 100 nm), usually made of materials like metals (e.g., copper), metal oxide (e.g., Al_2O_3) or carbon-based substances (e.g., graphene).

The concept of nanofluids was first proposed by **Choi (1995)** at Argonne National Laboratory, who demonstrated that even a small addition of nanoparticles to a fluid could result in a significant enhancement in thermal conductivity. Since then, nanofluids have emerged as a promising solution for various heat transfer challenges in thermal engineering systems. [14]

What makes nanofluids especially useful is how these tiny particles enhance the way heat moves through the liquid. Their behavior isn't random it's governed by several interrelated physical mechanisms, including:

Brownian motion: the nanoparticles are in constant, stochastic motion, bumping into each other and the fluid molecules. This random movement causes extra microscopic mixing, which helps distribute heat more quickly and evenly. [15]

Thermophoresis: when there's a temperature difference in the fluid, the nanoparticles tend to drift from hotter regions to cooler ones. This flow helps balance out temperature zones and support faster heat removal. [16]

Nanoparticles clustering: sometimes nanoparticles gather into tiny groups. While this might sound like a problem, these clusters can create efficient paths for heat to move through, especially if they're well distributed. [17]

Interfacial layering: at the surface of each nanoparticle, a thin layer of liquid molecules forms. This layer behaves differently from the rest of the fluid and can enhance thermal conductivity at the interface, acting like a thermal bridge. [18]

Thanks to these effects, nanofluids have been shown to improve heat transfer rate by 10% to 40% or more even with small amounts of particles. This makes them ideal for use in cooling systems for electronics, automotive engines, solar panels and other thermal technologies.

I.3.1. Properties of nanofluids:

a)-Better heat conductivity: nanofluids conduct heat much better than regular fluids because the nanoparticles help to transfer heat more efficiently. This makes them perfect for systems that need good cooling, like car radiators or industrial machines.

b)- viscosity (thickness): nanofluids are slightly thicker than regular fluids, but the increase is often small enough that it doesn't cause problems in many systems. The viscosity increases as more particles are added, but it drops at higher temperatures.

c)- improved heat transfer: thanks to the nanoparticles, nanofluids can transfer heat much more effectively. This makes them useful in devices like heat exchangers, where efficient cooling is needed to keep machines from overheating.

d)- electrical conductivity: some nanofluids can conduct electricity better than regular fluids, especially if the nanoparticles are made of metals. This property is useful in sensors or energy storage systems. [14]

I.3.2. Applications and benefits in thermal system:

Nanofluids are widely used in systems that need efficient cooling in electronics like computers and data centers, they help prevent overheating by improving heat removal.

In car engines and radiators, they keep engines cooler and boost fuel efficiency, nanofluids are also effective in air conditioning and refrigerators, making these systems faster and more energy-saving.

In renewable energy, especially solar systems, nanofluids enhance the collection and transfer of solar heat, leading to better performance.

Advanced uses include nuclear reactors, where nanofluids help with emergency cooling and medical or laser devices that require precise temperature control. Their main advantage lies in better thermal conductivity, which allows faster and more efficient heat transfer. This makes systems smaller, smarter, and more energy-efficient across various industries. [18]

I.3.3. Mathematical models describing nanofluids behaviors:

Nanofluid behavior is often described using extended classical models of heat and fluid flow that incorporate the unique thermophysical properties of nanoparticles. The most widely

used models are based on the Navier-Stokes and energy equations, modified to include nanoparticle effects such as Brownian motion and thermophoresis.

Buonigorno's Model (2006) is a benchmark model for nanofluids. It introduces two slip mechanisms:

Brownian diffusion (random motion of nanoparticles) and thermophoresis (motion caused by temperature gradient), which significantly affect heat transfer characteristics in nanofluids. [16]

Tiwari-Das model (2007) simplifies nanofluid behavior by treating the nanofluid as a single-phase fluid with modified effective properties based on nanoparticle volume fraction. The energy equation includes nanoparticles concentration as a static parameter rather than solving for its distribution. [19]

I.4. Gyrotactic microorganisms and bioconvection:

I.4.1. Definition of gyrotactic microorganisms:

Gyrotactic microorganism are a special kind of tiny swimming organisms (like certain types of algae) that have an uneven internal structure (they are slightly heavier at the bottom).

This bottom-heavy design means they tend to align themselves upright and swim upward when in still water. However, in moving water, things become more complex. [20]

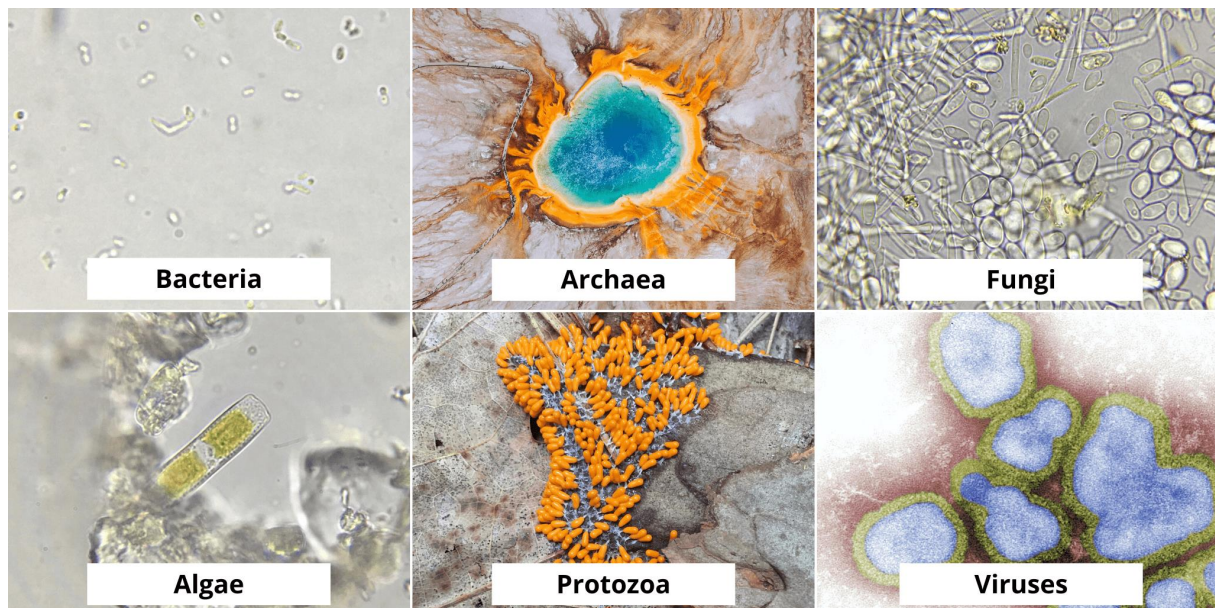


Figure (3): some types of microorganisms [21]

When the fluid around them is in motion (swimming, rotating or flowing) these microorganisms are influenced by two main forces. Gravity tries to keep them upright, while the moving water applies a spinning force. Gyrotactic is the term used to describe how they respond to this balance of forces. Instead of drifting aimlessly, they adjust their orientation and swim in specific directions depending on the flow.

This behavior is not just biologically interesting (it has important consequences). For example, gyrotactic microorganisms tend to gather in certain areas of the fluid, especially where flow conditions help them stay aligned. [22]

This can lead to large scale patterns in the water, known as bioconvection, which affects how heat and nutrients move in the system.

I.4.2. Mechanisms of microorganism motion and dispersion:

The motion and dispersion of motile microorganisms in fluid environment are governed by a combination of biological propulsion mechanisms and physical interactions with the surrounding flow fluid, most commonly, microorganisms such as bacteria and algae swim using flagella or cilia, which generate propulsion through wave. [23]

In the absence of external flow, microorganisms perform random or biased movement patterns such as run-and-tumble (e.g., *E. coli*) helical swimming (e.g. Algae). However, when placed in a fluid flow, their trajectories are influenced by fluid shear, vorticity and gravitational torque. gyrotaxis, chemotaxis (response to chemical gradients), photaxis (response to light), and thermotaxis (response to temperature) are key taxis mechanisms that introduce directional bias into their motion. [24]

Dispersion, in this context, refers to how the collective movement of a population of microorganisms spread out over time in a fluid. Dispersion arises not only from individual random swimming but also from flow-induced alignment (e.g. Gyrotactic focusing), cell-cell interactions and boundary effects. These interactions can produce phenomena like bioconvection, where dense aggregations of microorganisms cause instability in the fluid and induce self-organized flow structures. [25]

Mathematically, microorganism motion is often modeled using advection-diffusion equations with swimming velocity terms and orientation dynamics incorporated. The distribution of microorganisms is influenced by both deterministic factors (e.g. taxis responses) and stochastic perturbations (e.g. rotational diffusion).

I.4.3. role and applications in mechanical engineering:

Motile microorganisms and nanofluids have emerged as innovative agents in mechanical engineering application, especially in areas dealing with thermal management, flow control and energy efficiency. Their unique physical behaviors at micro and nanoscale levels offer promising performance improvements across a range of mechanical systems.

1- Thermal management systems:

Motile microorganisms and nanofluids are widely used in heat exchangers, cooling loops and thermal storage systems. The addition of nanoparticles enhances thermal conductivity, while the motility of microorganisms can contribute to micro-convective flows-improving heat transfer performance. [18]

2- Microfluids and flow control:

In lab-on-chip devices and microscale mechanical systems, the presence of active microorganisms can modify flow profiles due to their self-propulsion and orientation under flow fields. This behavior is particularly relevant in bio-inspired designs for fluid manipulation and passive flow control. [23]

3- Porous media engineering and bioremediation:

Gyrotactic microorganism introduced into porous structures can enhance fluid mixing, contaminant degradation or targeted material deposition, which is useful in heat exchanges, catalytic reactors and environmental systems. [25]

4- Self-healing and smart materials:

Engineered microbes can be embedded in coating or composite materials to sense damage and respond by releasing healing agents. This is being explored for corrosion resistance and fatigue repair in mechanical components.

Literature review

I.5. literature review of existing research:

a) – Nanofluids and Non-Darcy flow through porous media:

Choi et al. [15]: presented the idea of nanofluid by putting nanoparticles in base fluids like water or oil, including carbon nanotubes, copper or alumina. Even at extremely low volume fraction ($<1\%$). Their investigation in ASME FED 231(1995) demonstrated an increase improvement curves, which are usually shown in plots comparing the conductivities of base fluid with nanofluid, and assessed thermal conductivity as a function of nanoparticle concentration. The original ASME proceeding and subsequent review papers contain these plots.

Nield and Bejan [26]: discussed how Darcy's law is inadequate for flow through porous material at moderate to high speeds. They developed a modified momentum equation that accounts for pressure drop nonlinearity at higher flow velocities by appending the Forchheimer inertial component to Darcy's law. They showed that at a certain velocity threshold, the pressure gradient no longer behaves linearly through theoretical derivation and concentration with experimental evidence. These results are visualized through velocity-vs-pressure gradient plots in editions of convection in porous media.

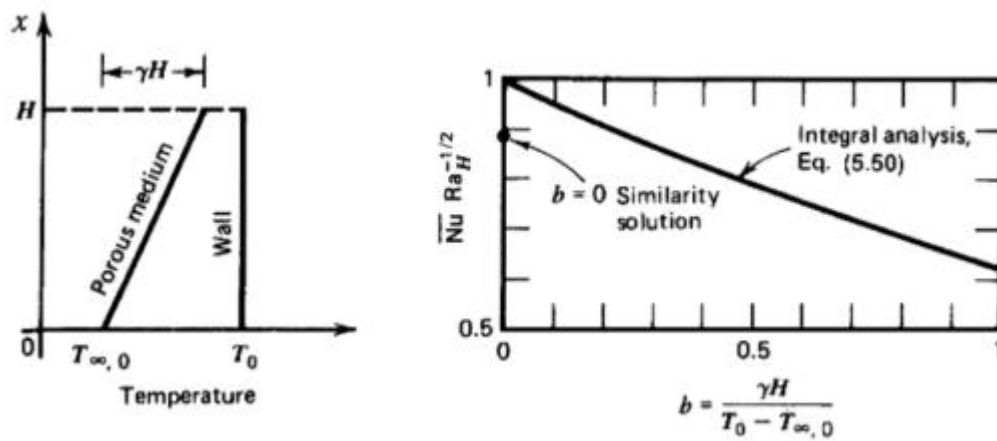


Figure (4): heat transfer from a vertical isothermal wall to a linearly stratified porous medium [26]

Khanafer et al. [27]: conducted numerical simulations to investigate the relationship between non-Darcy flow resistance and nanoparticle concentration porous materials. According to their findings, the addition of nanoparticles improves heat transfer by raising the average Nusselt number, but inertial effects also cause an increase in flow resistance. They displayed

graphs, streamlines and isothermal contour plots that displayed the inertia parameter and Nusselt number as a function of concentration.

b)- Natural convection with nanofluids and non-Darcy flow in porous media:

Buongiorno [16]: created a two-component model to distinguish between the phases of nanoparticles and base fluid. He determined that the two main slide processes influencing nanoparticle migration in temperature gradients are thermophoresis and Brownian motion. His model consists of linked equations for mass, momentum, energy and concentration of nanoparticle that show how slip effects alter spatial distribution are included in Buonigorno's work in the ASME journal of heat transfer.

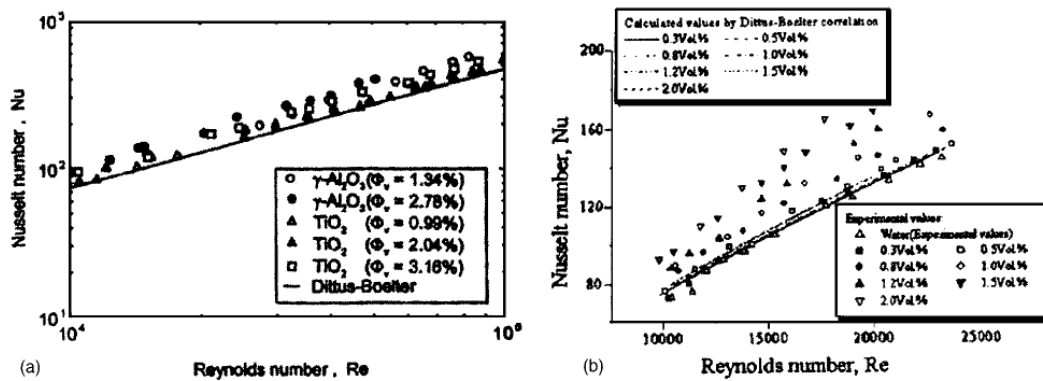


Figure (5): Heat transfer data for water-based nanofluids with oxide nanoparticles and metal nanoparticles. [16]

Niu et al. [28]: extended the Buongiorno two component nanofluid model to simulate natural convection under non-Darcy flow conditions (incorporating the Forchheimer inertial correction within porous cavities). Through comprehensive numerical simulations using combined effects of the Rayleigh number, nanoparticle volume fraction and inertial parameter on flow and temperature distribution. Their result revealed that increasing the Forchheimer inertia parameter notably suppressed convective circulation, transitioning the flow from strong multicellular patterns to a dominance of conductive heat transfer. This suppression also significantly reshaped thermal boundary layers, reducing the Nusselt number and weakening heat transfer efficiency. Their paper, typically published in International Journal of Thermal Sciences, includes detailed contour plots of temperature and streamlines for a spectrum of parameter combination, ideal for showcasing how non-Darcy effects alter nanofluid convection.

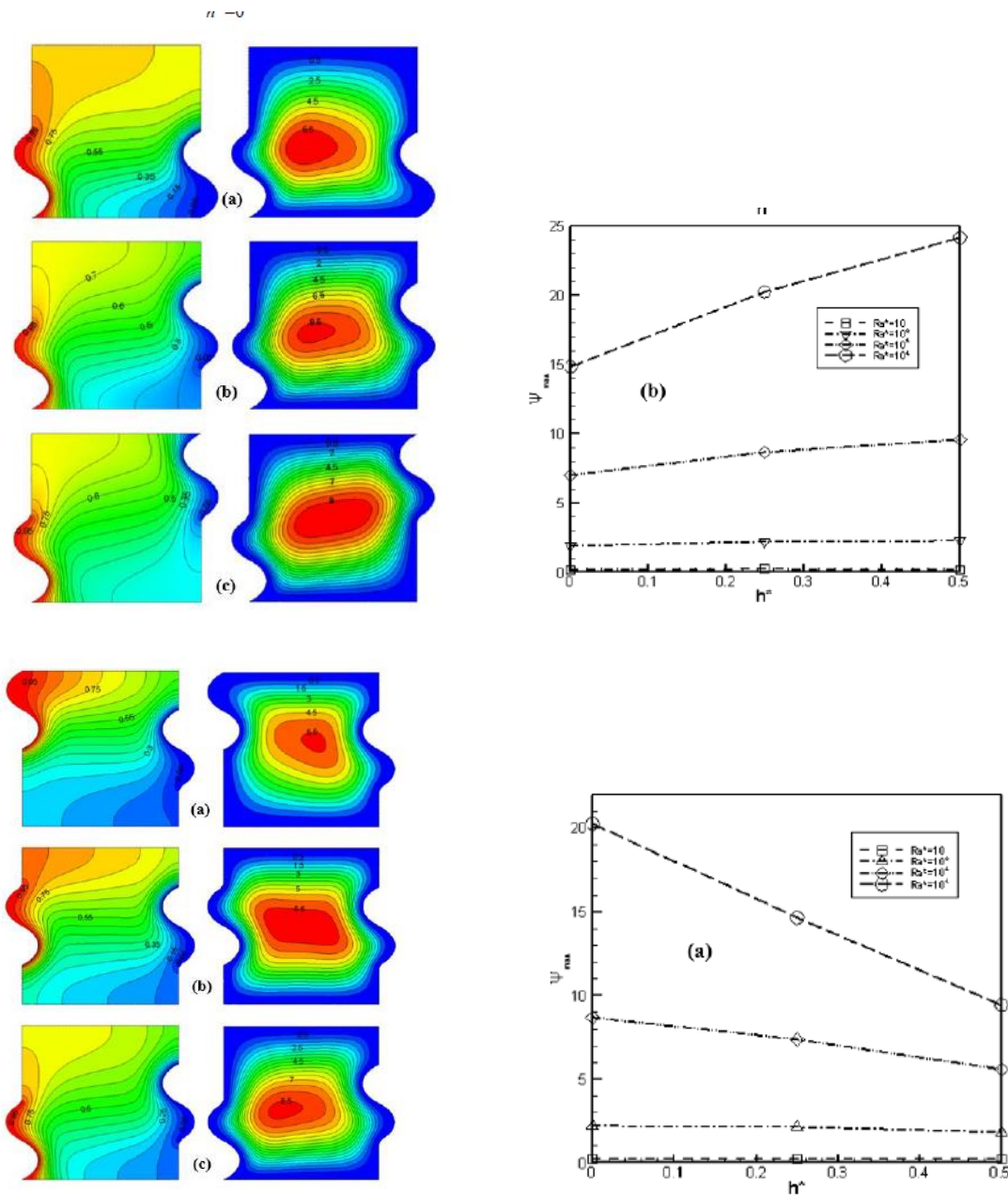


Figure (6): streamline and plots of temperature and stream function distribution at different position of hot and cold sources. [28]

Shermet and Pop [29]: further advanced the theoretical modelling of natural convection in porous media by including Brinkman viscosity correction alongside Darcy and Forchheimer terms in the flow equations. Adopting Buongiorno's slip model for nanoparticle dynamics, they examined buoyancy-driven convection in square enclosures filled with nanofluids. Their findings highlighted complex couplings: increased viscosity damping narrowed flow channels, thermophoresis and Brownian motion altered nanoparticle layering and non-Darcy inertia muddled these effects, leading to asymmetric thermal and concentration boundary layers. The

study is richly illustrated with isotherm, nanoparticle concentration maps and streamline, demonstrating how multiple physical mechanisms interact to shape overall convection processes.

c)- Gyrotactic microorganisms in nanofluids-based natural convection with non-Darcy flow in porous media:

Khanafer and Vafai [30]: introduced an innovative application of gyrotactic microorganisms- self motile microorganisms like algae- in porous nanofluid flow. By coupling thermally driven buoyancy, nanoparticle slip mechanisms and microorganism's behavior, their model captured bioconvection patterns emerging in porous media. Simulations showed that microorganisms induced plumes and vortices significantly enhanced mixing, leading to more uniform temperature fields and disrupted thermal boundary layers. Their paper, published in *International Journal of Heat and Mass Transfer*, contains contour maps of microorganism's density, flow plume structures and altered isotherms that vividly display the bioconvection enhancement.

Chamkha et al. [31]: formulated a comprehensive mathematical model that integrates Buongiorno's nanofluid framework, Darcy-Forchheimer flow dynamics, and gyrotactic microorganisms bioconvection within vertically aligned porous structures. Their numerical studies uncovered that microorganism concentration gradients not only influence but can stabilize convective patterns depending on pore permeability and nanoparticle loading. The study includes compelling 3D flow vector plots, combined concentration fields and streamline visualizations, clearly illustrating how biological and nanofluidic mechanisms interact to adjust heat transfer performance.

Ghasemi et al. [32]: presented a refined mathematical treatment of bioconvection in nanofluid porous systems by combining transient and steady-state heat and mass transport equations along with microorganism momentum balances. Their simulations demonstrated that gyrotactic microorganisms significantly flattened thermal gradients, resulting in higher overall thermal uniformity and improved heat transport efficiency compared to nanofluid only scenarios. The paper includes time-series plots of Nusselt number, density profile graphs and spatiotemporal concentration maps, which vividly illustrate the enhanced thermal performance due to microorganism motility.

Chapter II: Governing Equations and Model Formulation

II.1. Introduction:

In this chapter, we develop the mathematical and numerical framework necessary to analyze natural convection flow over a vertical plate embedded within porous medium, considering the presence of nanofluids and gyrotactic microorganisms. Natural convection along a vertical surface is classical yet continuously evolving subject, owing to its importance in various engineering applications such as cooling technologies, geothermal systems and biological processes.

The study of vertical plate convection involves understanding how buoyancy forces, generated by temperature and concentration gradients, drive the fluid motion adjacent to the surface. In porous media, this phenomenon is further influenced by the resistance of the medium (captured by Darcy and non-Darcy effects) and by additional transport mechanisms introduced by suspended nanoparticles and motile microorganisms.

To capture these complex interactions, the governing equations for momentum, energy, nanoparticle concentration and microorganism distribution are systematically derived. These equations account for key effects such as Forchheimer drag (non-Darcy), Brownian motion thermophoresis and bioconvection. Appropriate boundary conditions are applied to realistically model the behavior at the surface and far from the plate.

Due to the highly coupled and nonlinear nature of the system and an analytical solution is not feasible. Therefore, we employ the Matlab BVP4C solver, a powerful tool for handling. Boundary value problems numerically the solver enables as to accurately predict velocity, temperature and concentration profiles, providing critical insights into the behavior of the system under various physical parameters.

By combining theoretical modeling with robust numerical technics, this chapter sets the foundation for a deeper understanding of enhanced heat and mass transfer mechanisms associated with natural convection over vertical plates in complex fluid systems.

II.2. Translation of Theoretical Framework into Mathematical Equations:

- **Fourier's Law of Heat Conduction:**

The rate of thermal conduction in a medium is described by Fourier's law, which states that the heat flux is proportional to the negative temperature gradient in the direction of heat transfer:

$$q = -k \frac{dT}{dx} \quad (\text{II.1})$$

where:

q is the heat flux (W/m^2)

k is the thermal conductivity of the material ($\text{W/m} \cdot \text{K}$)

$\frac{dT}{dx}$ is the temperature gradient.

This law implies that heat flows naturally from regions of higher temperature to regions of lower temperature, and the rate of this transfer increases with both the steepness of the temperature gradient and the material's conductivity.

- **Stefan–Boltzmann Law of Thermal Radiation:**

The radiative heat flux emitted by a surface is described by the Stefan–Boltzmann law. This law states that the energy radiated per unit area is proportional to the fourth power of the surface's absolute temperature:

$$q = \varepsilon \sigma T^4 \quad (\text{II.2})$$

where:

ε is the emissivity

σ is the Stefan Boltzmann constant;

T is the absolute temperature of the surface

This equation captures the strong dependence of radiation on temperature, making it increasingly important at high temperatures.

- **Governing equations of natural convection:**

The equations that describe the flow of a fluid assumed to be steady and incompressible except for the buoyancy effects as described by the Boussinesq approximation, are: [5]

A)- continuity equation:

$$\nabla \cdot \vec{U} = 0 \quad (\text{II.3})$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{II.4})$$

B)- momentum equation(Navier-strokes):

*a long the x-axis:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g \quad (\text{II.5})$$

$$\frac{\partial P}{\partial x} = -\rho_{\infty} g \quad (\text{II.6})$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} \right) = g(\rho_{\infty} - \rho) + \mu \frac{\partial^2 u}{\partial y^2} \quad (\text{II.7})$$

And β is the volumetric expansion coefficient of the fluid at constant pressure, is given by:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \quad (\text{II.8})$$

And in approximate from:

$$\beta = -\frac{1}{\rho} \left(\frac{\rho_{\infty} - \rho}{T_{\infty} - T} \right) \quad (\text{II.9})$$

$$\text{Or: } \rho_{\infty} - \rho = \rho \beta (T - T_{\infty}) \quad (\text{II.10})$$

-The difference $(T - T_{\infty})$ means that if the local temperature T is higher than T_{∞} , the buoyancy force acts upwards (hot fluids rise) and if T is lower, it acts down wards (cold fluids sink)

The momentum equation becomes:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = g \beta (T - T_{\infty}) + \nu \frac{\partial^2 u}{\partial y^2} \quad (\text{II.11})$$

For a perfect gas:

$$\rho = \frac{P}{RT} \rightarrow \beta = \frac{1}{\rho} \left(\frac{P}{RT^2} \right) = \frac{1}{T} \quad (\text{II.12})$$

C)- Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \quad (\text{II.13})$$

- **Buoyancy Term under Boussinesq Approximation:**

In accordance with the Boussinesq approximation, the fluid density ρ is assumed constant ($\rho = \rho_0$) in all equations except in the gravitational body force term of the momentum equation. In this term, the temperature-dependent density variation is retained as:

$$\rho g = \rho_0 g (1 - \beta (T - T_0)) \quad (\text{II.14})$$

Where:

ρ_0 is the reference density;

β is the thermal expansion coefficient;

T is the local temperature;

T_0 is the reference temperature;

g is the gravitational acceleration.

This formulation ensures that buoyancy forces, which drive natural convection, are appropriately linked to thermal variations in the system.

- **Quantification of Porosity :**

Porosity is defined as the ratio of the volume of voids (pores) to the total volume of the material:

$$\varepsilon = \frac{V_v}{V_t} \quad (\text{II.15})$$

Where:

ε : porosity (dimensionless or %)

V_v : volume of voids (pores)

V_t : total volume of the material (solid + void).

- Alternative from using solid volume if total and solid volumes are known:

$$\varepsilon = 1 - \frac{V_s}{V_t} \quad (\text{II.16})$$

Where

V_s : volume of solid material.

- In term of mass and density (bulk materials):

$$\varepsilon = 1 - \frac{\rho_b}{\rho_p} \quad (\text{II.17})$$

Where:

ρ_b : bulk density (total mass /total volume)

ρ_p : particle density (mass of solids /solid volume)

These expressions allow for flexible and accurate calculation of porosity depending on the physical data available.

❖ typical porosity ranges:

- sands: 25-50%
- Clays: 40-70%
- Rocks: 1-20 %
- foams, sponges or artificial Porous media: up to 90%+

- **Permeability and Fluid Transport in Porous Structures:**

In fluid mechanics, permeability often appears in Darcy's Law:

$$\phi = - \frac{KA}{\mu} \frac{dP}{dx} \quad (\text{II.18})$$

Where:

ϕ : flow rate

K: permeability (m^2)

A: cross-sectional area

μ : dynamic viscosity of the fluid

$\frac{dP}{dx}$: pressure gradient

For granular media, permeability can also be estimated using the Kozeny-Carmen equation:

$$K = \frac{d^2}{180} \frac{\varepsilon^3}{(1-\varepsilon)^2} \quad (\text{II.19})$$

Where:

d: average particle diameter(m)

ε : porosity (dimensionless)

This equation is useful for predicting permeability based on the size of the particles and porosity.

- **Darcy's Law and Forchheimer's Extension:**

Darcy's Law

Under low-velocity, laminar flow conditions, Darcy's law relates the pressure gradient to the fluid velocity as:

$$-\nabla p = \mu \frac{v}{K} \quad (\text{II.19})$$

Where:

$-\nabla p$ is the pressure gradient [pa/m]

$-\mu$ is dynamic viscosity [$pa \cdot s$]

$-v$ is the Darcy velocity [m/s]

$-K$ is the permeability [m^2]

Forchheimer's Extension

When inertial effects are non-negligible, Forchheimer's extension modifies Darcy's law by adding a nonlinear term:

$$-\nabla p = \mu \frac{v}{K} + \beta \rho v^2 \quad (\text{II.20})$$

Where:

$-\rho$ is the [kg/m^3]

$-\beta$ is the Forchheimer coefficient, which accounts for inertial effects [$1/m$]

$-\beta\rho v^2$ is the nonlinear inertial loss term.

This model is particularly important for moderate to high Reynolds number flows in porous media.

Estimation of the Forchheimer Coefficient — Ergun Equation

The Forchheimer coefficient β can be approximated for packed beds using the empirical **Ergun equation**:

$$\beta = \frac{1.75(1-\varepsilon)}{d_p \varepsilon^3} \quad (\text{II.21})$$

Where:

ε :is the porosity of the medium;

d_p : characteristic particle diameter [m]

This shows that β increase when the porosity decrease or the pore diameter becomes smaller, both of which enhance inertial forces.

Momentum Equation with Darcy–Forchheimer Source Term

In advanced simulations such as CFD, the **volume-averaged Navier–Stokes equation** for flow through porous media includes both Darcy and Forchheimer terms:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla P + \mu \nabla^2 \mathbf{v} - \left(\mu \frac{\mathbf{v}}{K} + \beta \rho \mathbf{v}^2 \right) \quad (\text{II.22})$$

- **Nanofluids properties:**

Simplified equation for thermal conductivity:

$$K_{nf} = K_{bf}(1 + 3\phi_{np}) \quad (\text{II.23})$$

K_{nf} is the thermal conductivity of the nanofluid;

K_{bf} is the thermal conductivity of the base fluid;

ϕ_{np} is the volume fraction of nanoparticles (how much nanoparticles are mixed with the fluid);

Simplified equation for viscosity:

$$\mu_{nf} = \mu_{bf}(1 + 2,5\phi_{np}) \quad (\text{II.24})$$

μ_{nf} is the viscosity of the nanofluid;

μ_{bf} is the viscosity of the base fluid;

ϕ_{np} is the volume fraction of nanoparticles;

Simplified equation for heat transfer (Nusselt number):

$$Nu = (1 + \beta\phi_{np})(Re * Pr) \quad (\text{II.25})$$

Nu is the Nusselt number, representing heat transfer performance.

β is the enhancement factor that adjusts heat transfer in nanofluids.

Re is the Reynolds number (describes fluid flow type).

Pr is the Prandtl number (relates to fluid's thermal properties).

Simplified equation for electrical conductivity:

$$\sigma_{nf} = \sigma_{bf}(1 + \alpha\phi_{np}) \quad (\text{II.26})$$

σ_{nf} is the electrical conductivity of the nanofluid;

σ_{bf} is the electrical conductivity of the base fluid;

α is the conductivity enhancement factor for nanoparticles;

ϕ_{np} is the volume fraction of nanoparticles;

II.3. physical problem and mathematical analysis:

The present physical configuration is inspired by the general model proposed in [Aghbari,2019], which considers a vertical surface immersed in a porous medium subject to convective boundary conditions. While the foundational layout is similar, the current study extends the model by incorporating gyrotactic microorganisms.

We examine a steady, laminar, two-dimensional thermosolutal natural convection flow along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with

water based nanofluid containing spherical nanoparticles and motile gyrotactic microorganism. The plate lies in the (x, z) plane, while the flow develops in the (x, y) plane. The surface of the plate is subjected to convective heating from an external fluid at temperature T_f with heat transfer coefficient h_f . As a result, the temperature at the plate surface is not prescribed but is instead determined by the local convective heat exchange, leading to a temperature distribution that varies along the plate. The surface values of the nanoparticle volume fraction and the motile microorganism density are taken constant denoted by C_w and n_w , respectively. The ambient values attend as y tends to infinity of T , C and n are denoted by T_∞ , C_∞ and n_∞ , respectively.

The Boussinesq approximation is invoked to relate fluid density variations to both temperature and microorganism's concentration, thereby linking the buoyancy force to both thermal and bioconvection effects. The model accounts for multiple transport phenomena, including viscous dissipation, Brownian motion, thermophoresis and bioconvection due to motile microorganism. Additionally, the Soret effect (mass flux induced by temperature gradients) and Dufour effect (heat flux induced by concentration gradient) are included to capture cross-diffusion behavior. A local thermal equilibrium is assumed between the fluid and porous matrix.

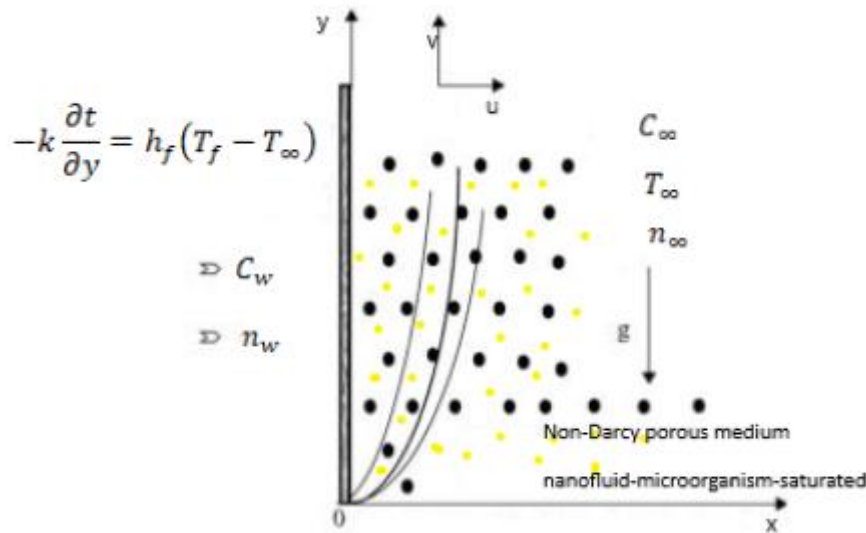


Figure (7): Physical model and coordinate system

- Governing Assumptions for the Mathematical Model:

Assumption Category	Description
Flow regime	Laminar, steady-state, two-dimensional
Fluid properties	Incompressible, Newtonian nanofluid
Buoyancy forces	Driven by both temperature and solute gradients (thermosolutal convection)
Porous medium	Homogeneous, isotropic, non-Darcy (includes Forchheimer inertial effects)
Thermal equilibrium	Local thermal equilibrium between solid matrix and nanofluid
Boundary layer	Thin layer approximation applied; variation mainly in normal direction (y)
Microorganism model	Motile, gyrotactic microorganisms inducing bioconvection
Boussinesq approximation	Density variation only affects the buoyancy term
Wall conditions	No-slip velocity, convective boundary condition for heat and mass transfer
Neglected effects	Thermal radiation and viscous dissipation are negligible

II.3.1. Mathematical equations:

A)- Dimensional equations:

Under these assumptions, the governing equations for continuity, momentum, nanoparticle concentration and motile microorganism's density in the porous medium are formulated as follows:

- Continuity equation:

$$\nabla \cdot \vec{V} = 0 \quad (\text{II.27})$$

- Momentum equation:

$$\frac{\mu}{K} v + \frac{C_f \rho_f}{\sqrt{k}} v^2 = -\frac{\partial P}{\partial x} + [(1 - C_\infty) \beta \rho_f g (T - T_\infty) - (\rho_p - \rho_f) g (C - C_\infty) - (\rho_m - \rho_f) g (n - n_\infty)] \quad (\text{II.28})$$

- Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \quad (\text{II.29})$$

- Nanoparticle concentration equation:

$$\frac{1}{\varepsilon} \left(u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (\text{II.30})$$

- Motile microorganism density equation:

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{bw_c}{C_w - C_\infty} \left[\frac{\partial}{\partial y} \left(n \frac{\partial C}{\partial y} \right) \right] = D_n \frac{\partial^2 n}{\partial y^2} \quad (\text{II.31})$$

Thus, the Darcy velocity component in the x and y direction are denoted by u and v , respectively. The temperature is represented by T , the pressure by P , while the nanoparticle concentration and motile microorganism's density is denoted by C and n , respectively. The permeability of the porous medium with porosity ε is represented by K , the fluid, the particle and the microorganisms densities are represented by ρ_f , ρ_p and ρ_m , respectively; the dynamic viscosity by μ . α and β are defined as thermal diffusivity of porous medium and volumetric expansion coefficient of fluid, with $\alpha = \frac{k}{(\rho c)_f}$ where k is thermal conductivity of the porous medium, τ is the ratio between the effective heat capacity of the nanoparticles and the base fluid defined by $\tau = \frac{\varepsilon(\rho c)_p}{(\rho c)_f}$ where $(\rho c)_p$ is the effective heat capacity of the nanoparticle material and $(\rho c)_f$ is the effective heat capacity of the base fluid. D_B, D_T, D_m and D_n are respectively the Brownian diffusion coefficient, the thermophoretic diffusion coefficient, the chemical molecular diffusivity and the diffusivity of the microorganisms; b and w_c defined as the chemotaxis constant and the maximum cell swimming speed and the product (bw_c) is the swimming velocity of the cells.

B)- Boundary conditions and non-dimensionalisation:

The boundary conditions are taken to be:

$$\begin{aligned} v = 0, -k \frac{\partial t}{\partial y} = h_f (T_f - T_\infty), C = C_w, n = n_w \text{ at } y = 0 \\ u = v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty, n \rightarrow n_\infty \text{ as } y \rightarrow \infty \end{aligned} \quad (\text{II.32})$$

In order to reduce the system of governing partial differential equation (PDE_s) into a set of ordinary differential equations (ODE_s), the following transformation is introduced:

The local Rayleigh number:

$$Ra_x = \frac{(1 - C_\infty) \rho_f \beta g K x}{\mu \alpha} \quad (\text{II.33})$$

And similarity variable

$$\eta = \frac{y}{x} Ra_x^{1/2} \quad (\text{II.34})$$

The introduction of this similarity variable η is rooted in classical boundary layer theory, which assumes that flow quantities—such as velocity, temperature, and concentration—vary rapidly in the normal direction (y) but slowly in the streamwise direction (x). This justifies reducing the governing partial differential equations to a simpler set within the thin boundary layer region adjacent to the surface.

The characteristic boundary layer thickness $\delta(x)$ represents the distance from the wall over which significant changes in velocity and temperature occur. From the similarity variable, it follows that:

$$\eta = 1 \Rightarrow y = \delta(x) \approx \frac{x}{Ra_x^{1/2}} \quad (\text{II.35})$$

Therefore, the boundary layer thickness scales inversely with the square root of the local Rayleigh number:

$$\delta(x) \sim \frac{x}{Ra_x^{1/2}} \quad (\text{II.36})$$

This scaling implies that higher buoyancy forces (larger Ra_x) result in thinner boundary layers, due to stronger convection near the surface.

These assumptions hold provided that:

- The flow is laminar and steady;
- The boundary layer remains thin relative to the streamwise length ($\delta(x) \ll x$)
- There is no significant pressure gradient along the wall;
- The flow exhibits self-similarity, allowing reduction of PDEs to ODEs using η

Understanding the physical interpretation of $\delta(x)$ is essential for justifying the similarity solution framework and ensuring that the boundary layer approximation is valid under the problem's thermophysical conditions

Also the dimensionless variables S, θ, γ and χ are defined by :

$$S(\eta) = \frac{\psi}{\alpha Ra_x^{1/2}} ;$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} ; \gamma(\eta) = \frac{C-C_\infty}{C_w-C_\infty} ; \chi(\eta) = \frac{n-n_\infty}{n_w-n_\infty} \quad (\text{II.37})$$

The stream function ψ is defined by :

$$u = \frac{\partial \psi}{\partial y} ; \quad v = -\frac{\partial \psi}{\partial x} \quad (\text{II.38})$$

Using equations (II.28-II.31) with the similarity variables above, we obtain the following system of ordinary differential equations:

$$s''(1 + Nds') - \theta' + Nr f' + Rb \chi' = 0 \quad (\text{II.39})$$

$$\theta'' + \frac{1}{2}s\theta' + Nb\gamma'\theta' + Nt(\theta')^2 + Du\gamma'' = 0 \quad (\text{II.40})$$

$$\gamma'' + \frac{1}{2}Lns\gamma' + \theta''\left(\frac{Nt}{Nb} + SrLe\right) = 0 \quad (\text{II.41})$$

$$\chi'' + \frac{1}{2}Lbs\chi' - Pe[\chi'\gamma' + \gamma''(\chi + bi)] = 0 \quad (\text{II.42})$$

Where the various parameters are defined as follows:

$$Nd = \frac{C_f \sqrt{K} \rho_f \alpha Ra_x}{\mu x} : \text{the non-Darcy parameter;}$$

$$Nr = \frac{(\rho_p - \rho_f)(C_w - C_\infty)}{(1 - C_\infty)\beta \rho_f (T_f - T_\infty)} : \text{the buoyancy ratio parameter;}$$

$$Rb = \frac{(\rho_m - \rho_f)\gamma(n_w - n_\infty)}{(1 - C_\infty)\beta \rho_f (T_f - T_\infty)} : \text{the bioconvection Rayleigh number;}$$

$$Nb = \frac{\tau_{DB}(C_w - C_\infty)}{\alpha} : \text{the Brownian motion parameter;}$$

$$Nt = \frac{\tau_{DT}(T_f - T_\infty)}{T_\infty \alpha} : \text{the thermophoresis motion parameter;}$$

$$Du = \frac{D_m k (C_w - C_\infty)}{c_s c_p \alpha (T_f - T_\infty)} : \text{Dufour number;}$$

$$Ln = \frac{\alpha}{\varepsilon D_B} : \text{nanofluid Lewis number;}$$

$$Le = \frac{\alpha}{D_B} : \text{traditional Lewis number;}$$

$$Sr = \frac{D_m k (T_f - T_\infty)}{\alpha T_m (C_w - C_\infty)}; \text{ Soret number};$$

$$Lb = \frac{\alpha}{D_n}; \text{ bioconvection Lewis number};$$

$$Pe = \frac{bw_c}{D_n}; \text{ bioconvection Péclet number};$$

$$bi = \frac{n_\infty}{(n_w - n_\infty)}; \text{ bioconvection constant}.$$

The boundary conditions (II.32) becomes:

$$\eta = 0 : S(0) = 0; \theta(0) = -h(1 - \theta(0)); \gamma(0) = 1; \chi(0) = 1. \quad (\text{II.43})$$

$$\eta \rightarrow \infty : S'(\infty) = 0; \theta(\infty) = 0; \gamma(\infty) = 0; \chi(\infty) = 0. \quad (\text{II.44})$$

The physical quantities of interest that characterize the rates of heat and mass transfer are defined in terms of the local Nusselt number Nu_x and the regular local Sherwood number Sh_x and are given by:

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)}$$

$$Sh_x = \frac{xq'_w}{D_B(C_w - C_\infty)}$$

Here, q_w represents the surface heat flux and q'_w the surface mass flux, where:

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$q'_w = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

By using the dimensionless variables, we obtain:

$$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(0)$$

$$\frac{Sh_x}{Ra_x^{1/2}} = -\gamma'(0)$$

Chapter III: Numerical Method and Algorithm

III.1. Analytical resolution:

The system of coupled ordinary differential equations (II.28-II.31) exhibits significant nonlinear behavior, which makes finding an analytical solution highly challenging. When these equations are coupled with the boundary conditions (II.43-II.44), they collectively form a two-point boundary value problem that can be tackled numerically for selected parameter values.

bvp4c is a finite-difference collocation solver designed to handle nonlinear, stiff, and coupled systems of ODEs. It approximates the solution over a mesh by constructing piecewise cubic polynomials and enforcing the differential equations at selected collocation points.

Specifically, bvp4c uses the **Lobatto IIIa collocation** method, a fourth-order implicit Runge-Kutta scheme that:

- Evaluates the ODE system at three points per subinterval (including both endpoints),
- Provides adaptive mesh refinement based on local error estimates.

This method is highly effective for boundary layer-type problems, where steep gradients (near the wall) require fine resolution and high numerical stability.

The user must supply three main components:

1. **ODE function:** Defines the system $\frac{dy}{dx} = f(x, y)$
2. **Boundary condition function:** Specifies residuals $R(y(a), y(b)) = 0$
3. **Initial guess:** Provides a reasonable starting profile for $y(x)$, required for convergence.
4. The transformed system involves 8 coupled first-order ODEs.
5. Boundary conditions are specified at the plate surface and at infinity
6. A **sufficiently large finite domain** is used to simulate the far-field conditions.
7. The solver iteratively refines the mesh and updates the solution until all boundary conditions and tolerance criteria are met.

The system solved by bvp4c corresponds to the following dimensionless ODEs:

$$\frac{dy}{d\eta} = \begin{bmatrix} y_2 \\ S_2(y) \\ \vdots \\ y_8 \end{bmatrix}$$

Subject to the boundary conditions:

$$\text{At } \eta = 0: y_1 = 0, y_3 = 0, y_5 = 1, y_7 = 1$$

$$\text{As } \eta \rightarrow \infty: y_2 \rightarrow 0, y_4 \rightarrow 0, y_6 \rightarrow 0, y_8 \rightarrow 0$$

An initial mesh and profile are constructed using trial functions that satisfy the boundary conditions approximately. Once solved, the outputs provide profiles of:

- **Velocity** $S'(\eta)$
- **Temperature** $\theta(\eta)$
- **Nanoparticle concentration** $\gamma(\eta)$
- **Microorganism density** $\chi(\eta)$

along the similarity variable η .

In practice, the domain is segmented into smaller subintervals via a mesh of discrete points. The MATLAB solver `bvp4c` is used to compute the numerical solution by solving a corresponding set of nonlinear algebraic equations, while ensuring that both the boundary and collocation conditions are fulfilled across all subintervals. The solver dynamically estimates the numerical error on each subinterval, and if predefined tolerance is not met, it automatically adjusts the mesh and recomputes the solution. For the algorithm to initiate, the user must supply an initial distribution of mesh points along with an estimated initial solution over those points.

- **Comparison with the shooting method:**

As an alternative, the **shooting method** transforms the BVP into an initial value problem (IVP), integrating from one boundary using guessed initial conditions and adjusting them iteratively until the far-end boundary conditions are satisfied. Although simpler in concept, the shooting method often suffers from:

- Instability in stiff or highly nonlinear systems,
- Sensitivity to initial guesses,
- Difficulty when multiple dependent variables and coupled equations are involved.

For these reasons, **bvp4c** is **preferred** in this study, as it directly handles boundary conditions at both ends, avoids numerical instability, and offers mesh control for resolving sharp gradients typical of natural convection and nanofluid flows.

III.2. program validation:

As previously described, the system of nonlinear ordinary differential equations (II.28-II.31), coupled with the boundary conditions (II.43-II.42), was solved numerically using the finite difference method. To evaluate the accuracy of the method used, the corresponding results are compared with those obtained by Nield [34]. The reduced Nusselt number has linear regression coefficients and an error margin. C_r, C_b and C_t are the coefficient in the linear regression estimate $\frac{Nu_{est}}{Ra_x^{1/2}} = 0.444 + C_r Nr + C_b Nb + C_t Nt$, and $\hat{\varepsilon}$ is the maximum relative error

defined by : $\hat{\varepsilon} = \left| \frac{Nu_{est} - Nu}{Nu} \right|$, applicable for Nr, Nb and Nt each in $[0, 0.5]$.

LN	C_r	C_b	C_t	$\hat{\varepsilon}[34]$	$\hat{\varepsilon}_{present}$
1	-0.309	-0.060	-0.166	0.154	0.1538
2	-0.230	-0.129	-0.162	0.147	0.1457
5	-0.148	-0.209	-0.152	0.126	0.1259
10	-0.111	-0.245	-0.150	0.119	0.1202
20	-0.086	-0.268	-0.149	0.114	0.1141
50	-0.064	-0.288	-0.149	0.110	0.1107
100	-0.053	-0.298	-0.148	0.108	0.1069
200	-0.045	-0.304	-0.148	0.107	0.1036
500	-0.039	-0.310	-0.148	0.106	0.1044

Table (1): Comparison of the maximum relative error $\hat{\varepsilon}$

Chapter IV: Results and disscusion

IV. Result and discussion:

A comprehensive parametric analysis was performed in accordance with methodology described in the previous section. This analysis involved varying several key dimensionless parameters, including the modified Darcy number (Nd), the thermophoresis parameter (Nt), the Brownian motion parameter (Nb), the Dufour number (Du), the Soret number (Sr), the bioconvection Rayleigh number (Rb), the nanoparticle and traditional Lewis numbers (Ln, Le) and the bioconvection Lewis number (Lb)

In addition, the influence of the convective boundary conditions was examined through the convective parameter h . These parameters were systematically altered within physically realistic ranges to generate a representative set of graphical result. The outcomes of this analysis are presented in terms of the dimensionless velocity profile $S'(\eta)$, the temperature distribution $\theta(\eta)$, the nanoparticle concentration profile $\gamma(\eta)$ and the distribution of the microorganism density $\chi(\eta)$. These graphical representations allow for detailed understanding of how each parameter influences the behavior of the flow, heat, mass and microorganism transport in the system under investigation.

IV.1. Dimensionless velocity profile $S'(\eta)$:

The behavior of the dimensionless velocity distribution is analyzed with respect to the similarity variable η . This relationship is illustrated in figures (7-9), which were generated by varying key dimensionless parameter Nd, Nt, Nb, Du, Sr and the convective parameter h .

- Effects of the convective parameter h and the modified Darcy parameter Nd on the dimensionless velocity profile:

The figure (8) illustrate the impact of the convective parameter h and the non-Darcy modified parameter Nd on the dimensionless velocity profile $S'(\eta)$. It is observed that increasing h enhances the velocity near the wall due to stronger convective heating, which amplifies buoyancy forces and promote fluid motion within the boundary layer. For all cases of Nd, the velocity profiles corresponding to $h=100$ consistently lie above those for $h=1$, confirming the role of h in accelerating the flow. On the other hand, as Nd increase, an observable reduction in the velocity profile is observed, especially near the wall. This is attributed to the non-Darcy drag force, which introduce additional resistance to the flow. Therefore, while increasing h enhances the convective motion, increasing Nd suppresses it due to intensified inertial resistance within the porous structure.

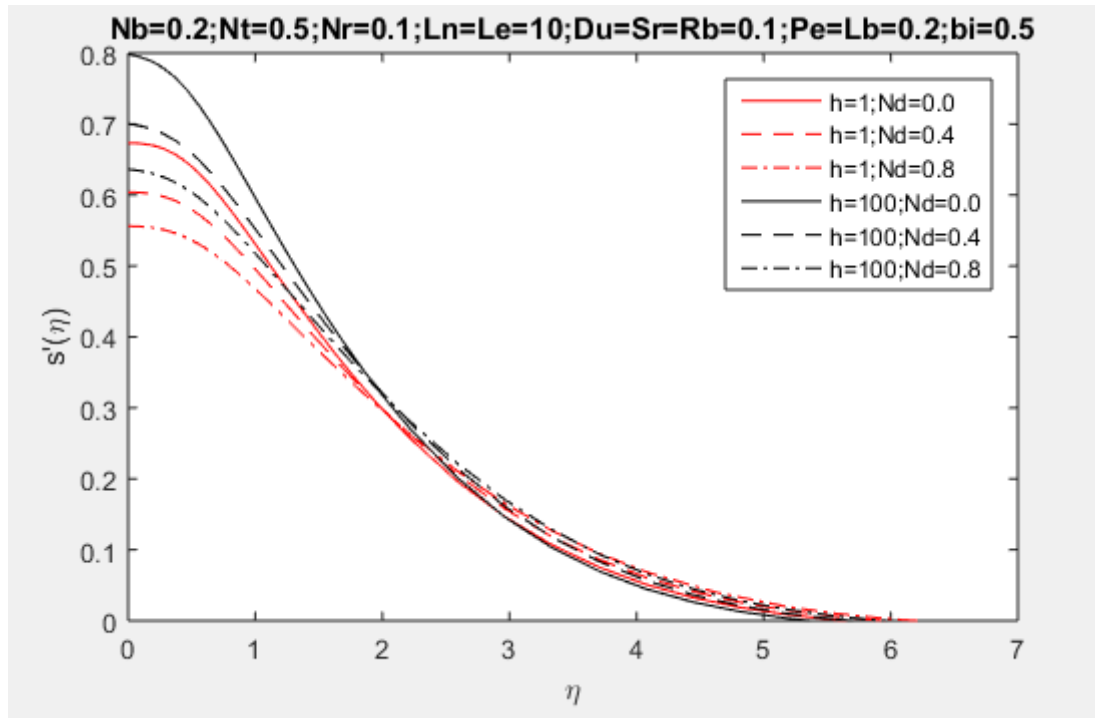


Figure (8): Effects of the convective parameter h and the modified Darcy parameter Nd on the dimensionless velocity profile.

- Effects of the convective parameter h and the bioconvection Rayleigh number Rb on the dimensionless velocity profile:

This figure (9) highlights the influence of the convective parameter h and the bioconvection Rayleigh number Rb on the dimensionless velocity profile. It is observed that increasing the convective parameter leads to an enhancement in the fluid velocity near the boundary, indicating stronger convective effects promote momentum transfer. Additionally, the presence of bioconvection, represented by the Rayleigh number, contribute to an increase in the velocity profile. This suggests that the upward movement of motile microorganisms enhances the buoyancy-driven flow. Altogether, both stronger surface convection and bioconvection effects act to intensify the velocity within the boundary layer.

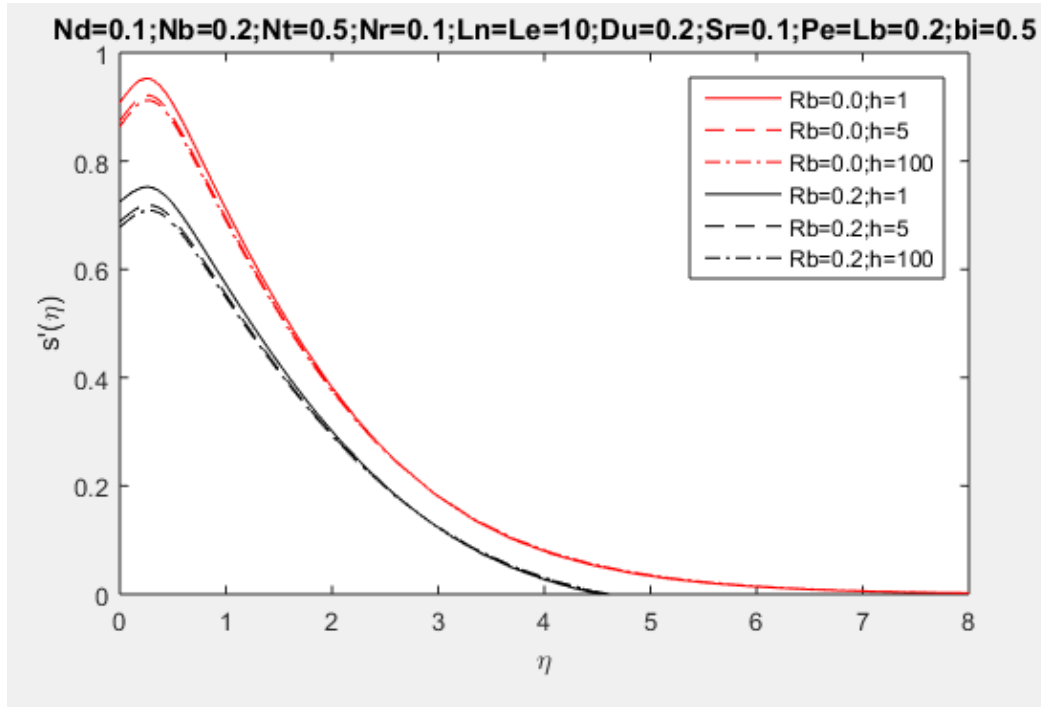


Figure (9): effect of convective parameter h and the bioconvection Rayleigh number Rb on the dimensionless velocity profile

IV.2. Dimensionless temperature profile $\theta(\eta)$:

The following figures (10-11) depict the variation of the dimensionless temperature distribution with respect to the similarity variable η , for the specific parameter values Nb , Nt , Du and the convective parameter h .

- **Effects of the convective parameter h , the Brownian motion parameter Nb and the thermophoresis parameter Nt on the dimensionless temperature profile:**

This figure (10) presents the effects of the convective parameter and the nanoparticle parameters (Brownian motion and thermophoresis) on the dimensionless temperature profile. As the Brownian motion Nb and thermophoresis parameter Nt increase, the temperature profile rises, showing that nanoparticle movement contributes to greater heat retention within the fluid. on, increasing the convective parameter h leads to a noticeable decrease in temperature near the surface, signifying more efficient heat removal due to stronger convective interaction. In essence, while nanoparticle dynamics enhance thermal diffusion, stronger surface convection works to lower the fluid temperature by intensifying heat transfer away from the boundary layer.

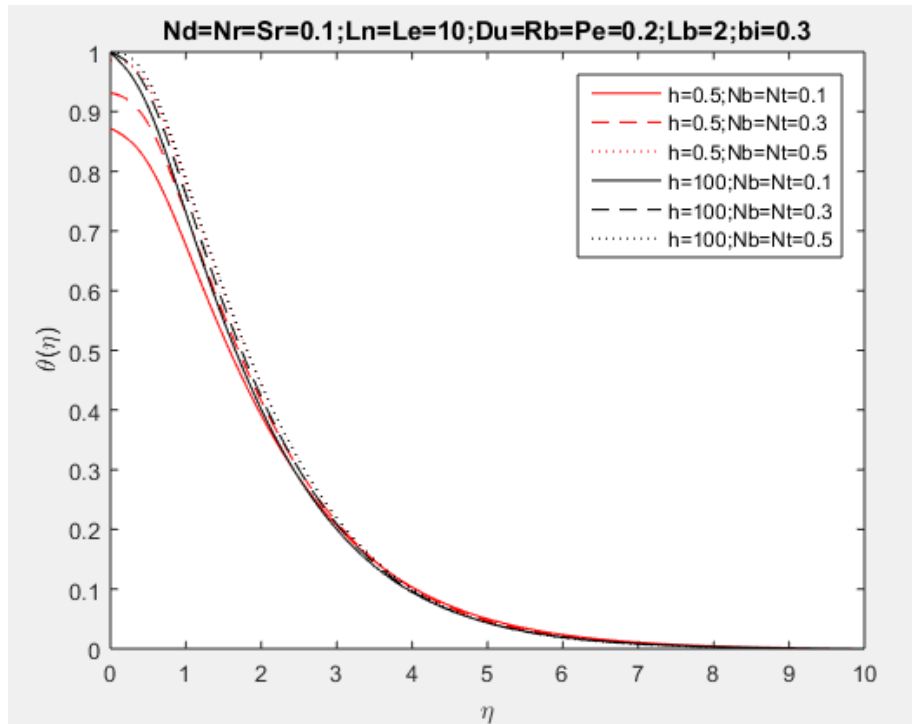


Figure (10): Effects of the convective parameter h , the Brownian motion parameter Nb and the thermophoresis parameter Nt on the dimensionless temperature profile

Effects of the convective parameter h and the Dufour number Du on the dimensionless temperature profile:

This figure (11) demonstrates the influence of the Dufour number Du and the convective parameter h on the dimensionless temperature profile. It is evident that an increase in the Dufour number results in a higher temperature distribution within the boundary layer, highlighting the contribution of concentration gradients to thermal diffusion. In contrast, increasing the convective parameter leads to a reduction in temperature near the surface, indicating more effective heat removal due to stronger convective effects. Thus, while the Dufour effect acts to enhance thermal energy within the fluid, a higher convective parameter intensifies cooling at the boundary.

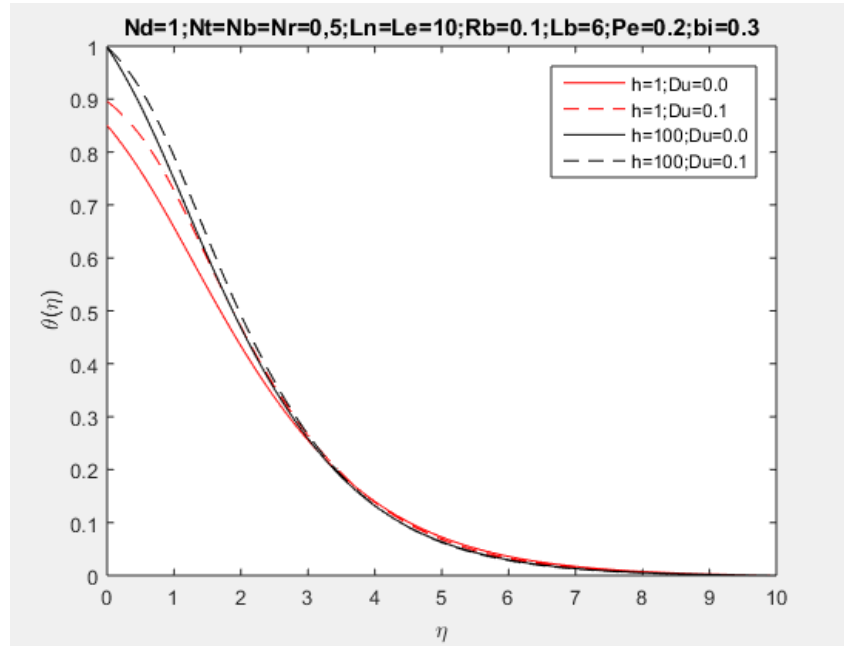


Figure (11): Effects of the convective parameter h and the Dufour number Du on the dimensionless temperature profile

IV.3. Dimensionless nanoparticle concentration profile $\gamma(\eta)$:

The dimensionless nanoparticle concentration profile as a function of the similarity variable η is shown in figure (12) corresponding the selected values Ln , Le , Sr and the convective parameter h .

- **Effects of the convective parameter h , the Soret number Sr , nanoparticle and traditional Lewis numbers Ln and Le on the dimensionless nanoparticle concentration profile:**

The figure (12) demonstrates the combined effects of the nanoparticle Lewis number Ln , the traditional Lewis number Le and the Soret number Sr on the dimensionless nanoparticle concentration profile $\gamma(\eta)$. It is evident that increasing $Ln=Le$ leads to a notable decrease in concentration throughout the boundary layer. This occurs because higher Lewis numbers signify stronger mass diffusivity relative to thermal diffusivity, which promotes faster depletion of nanoparticle concentration from the wall region. Additionally, the presence of Soret effect consistently enhances the concentration profiles compared to the case without it, particularly at higher η values. This enhancement is attributed to thermodiffusion, where temperature gradients induce additional nanoparticle migration, resulting in thicker concentration boundary layers. The influence of Sr is more pronounced at lower Ln , where mass diffusivity is weaker, allowing the thermodiffusive effect to dominate.

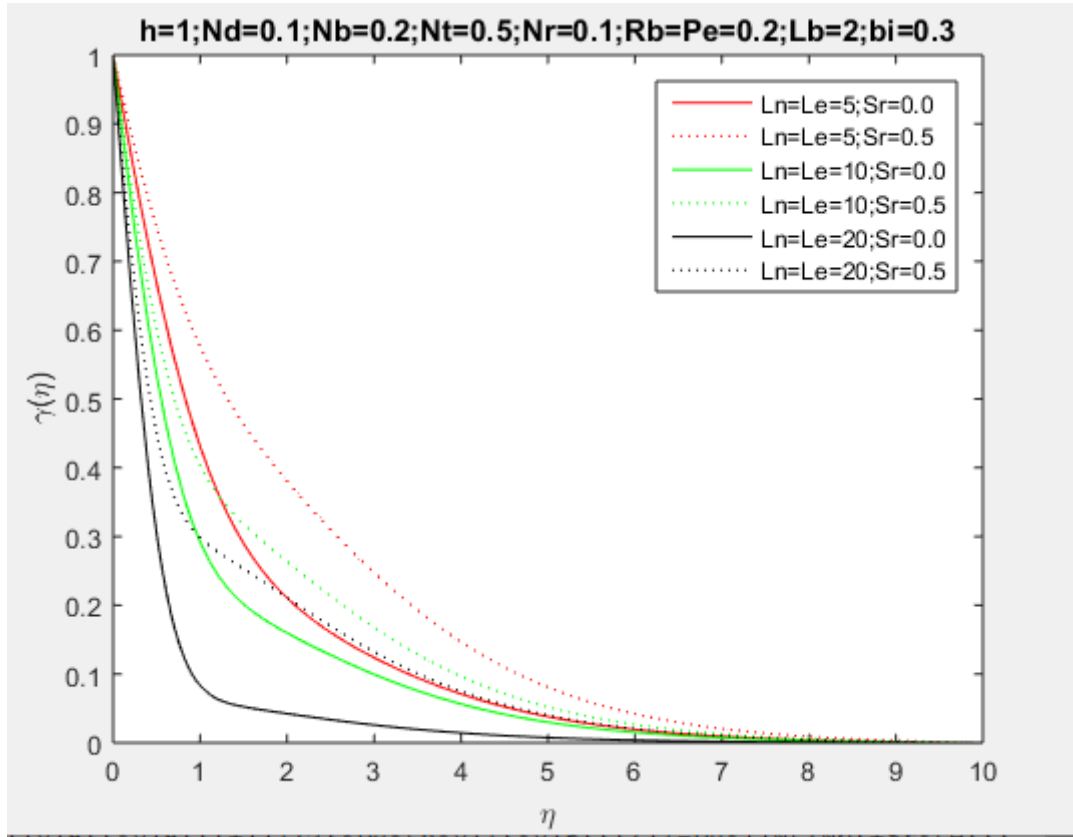


Figure (12): Effects of the Soret number Sr , nanoparticle and traditional Lewis numbers Ln and Le on the dimensionless nanoparticle concentration profile

IV.4. Dimensionless microorganism density distribution $\chi(\eta)$:

Figures (13-14) presents the fluctuation of the dimensionless microorganism density distribution depending on the similarity variable η , using defined values for key parameters Nb , Nt , Pe and the convective h .

- Effects of the bioconvection Lewis number Lb and the Péclet number Pe on the dimensionless microorganism density distribution:

The plot in the figure (13) illustrates how variations in the bioconvection Lewis number Lb and Péclet number Pe affect the distribution of microorganism density in the fluid. It is evident that increasing Lb results in a substantial reduction in microorganism concentration throughout the domain. This is because a higher Lb signifies enhanced diffusion of microorganisms relative to the thermal diffusivity, leading to a thinner concentration boundary layer. Additionally, for a fixed Lb , increasing Pe further reduces the concentration. This occurs due to the strengthened advective transport, which accelerates the migration of microorganisms away from the wall region. Overall, both parameters contribute to a faster decay of the microorganism density profile, with the combined influence of high Pe and high Lb yielding the most pronounced thinning effect.

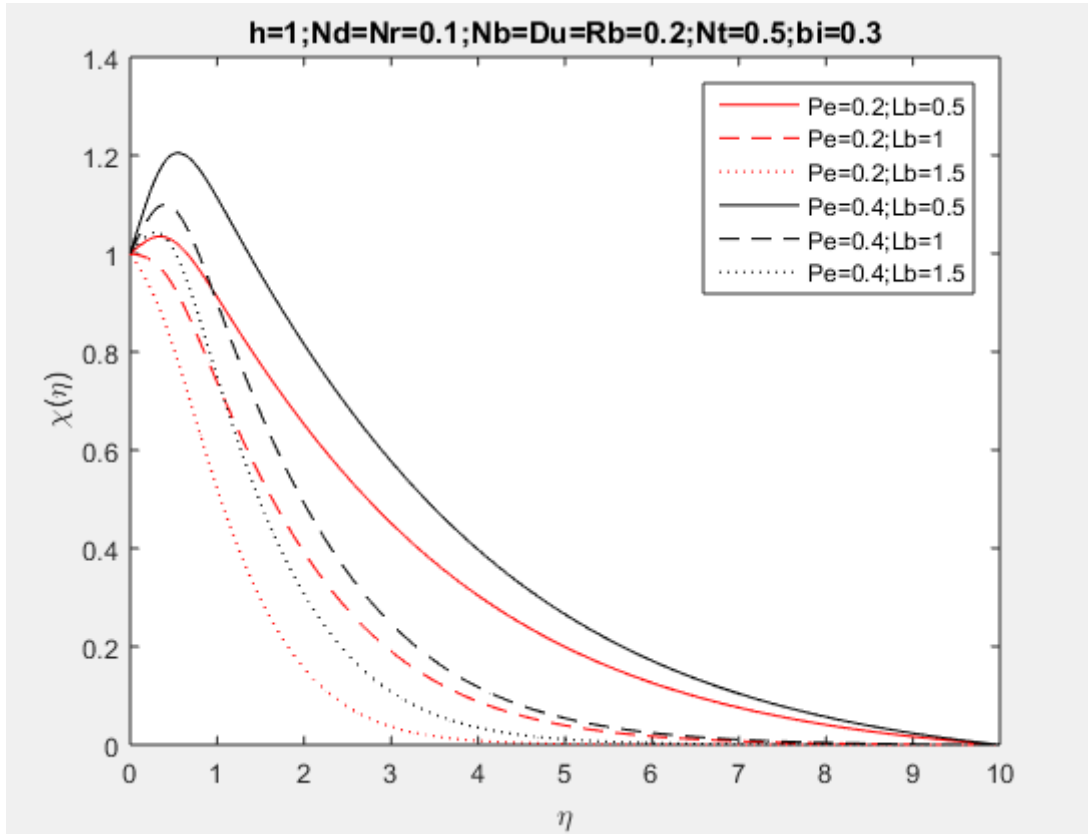


Figure (13): Effects of the bioconvection Lewis number L_b and the Péclet number Pe on the dimensionless microorganism density profile.

- Effects of the convective parameter h , the Brownian motion parameter N_b and the thermophoresis parameter N_t on the dimensionless microorganism density distribution:

The figure (II.8) illustrates how variations in the thermophoresis parameter N_t , Brownian motion parameter N_b , and the convective parameter h impact the distribution of the dimensionless microorganism density $\chi(\eta)$. When both N_t and N_b are increased, the profiles exhibit a noticeably sharper descent, indicating a more efficient dispersion of microorganisms from the wall into the fluid domain. This behaviour highlights the role of thermal gradients and random motion in enhancing the migratory tendencies of microorganisms, leading to a decrease in their near-wall concentration. On the other hand, adjusting the convective parameter h primarily influences the boundary layer characteristics. Higher values of h slightly suppress the density near the surface due to stronger convective transfer, but this effect is subtler compared to the dominant influence of N_t and N_b . The interaction between these parameters governs the balance between accumulation near the wall and outward diffusion, shaping the spatial structure of the microorganism population. These results emphasize the importance of microscale thermophysical forces in controlling bio-convection and microorganism transport in convective-diffusive systems.

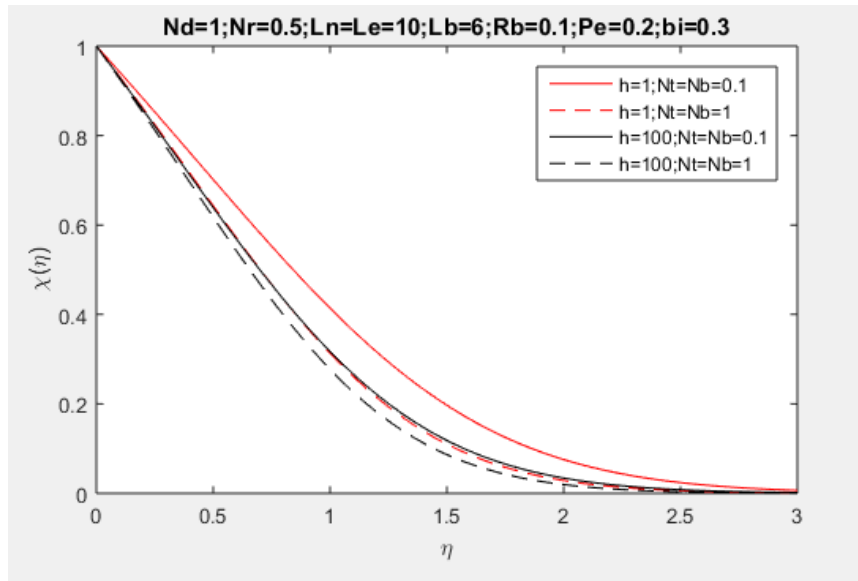


Figure (14): Effects of the convective parameter h , the Brownian motion parameter Nb and the thermophoresis parameter Nt on the dimensionless microorganism density profile.

IV.5. Heat and mass transfer rates:

In order to demonstrate the effect of the relevant parameters on the heat and mass transfer rates in terms of local quantities such as: the local Nusselt number Nu_x the regular local Sherwood number Sh_x for different values of h , Nd , Nb , Nt , Du and Sr .

Table (2) : values of $\frac{Nu_x}{Ra_x^{1/2}}$ and $\frac{Sh_x}{Ra_x^{1/2}}$ for selected values of h , Nd and Nb

with $Nt=Nr=0.5$, $Ln=Le=10$, $Lb=6$, $Pe=0.2$, $bi=0.3$

H	Nd	$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(0)$			$\frac{Sh_x}{Ra_x^{1/2}} = -\gamma'(0)$		
		Nb			Nb		
		0,1	0,5	1	0,1	0,5	1
5	0	0,2580	0,2043	0,1391	1,0349	1,3439	1,4014
	0,5	0,2465	0,1935	0,1312	0,9854	1,2694	1,3189
	1	0,2380	0,1858	0,1257	0,9487	1,2161	1,2609
50	0	0,2771	0,2157	0,1443	1,0694	1,3850	1,4300
	0,5	0,2631	0,2033	0,1356	1,0136	1,3025	1,3417
	1	0,2530	0,1946	0,1296	0,9732	1,2446	1,2804
1000	0	0,2793	0,2170	0,1448	1,0733	1,3895	1,4331
	0,5	0,265	0,2044	0,1361	1,0167	1,3061	1,3441
	1	0,2547	0,1956	0,1300	0,9759	1,2477	1,2825

The table (2) illustrates how the heat and mass transfer rates, represented by the local Nusselt number and the local Sherwood number, are influenced by variation in the Brownian motion parameter Nb , the convective parameter h and the non-Darcy modified number Nd . Increasing Nb consistently reduces the Nusselt number, indicating weaker heat transfer due to thermal disturbances caused by enhanced nanoparticle movement. On the other hand, the Sherwood number increases with higher Nb , reflecting stronger mass diffusion. As Nd increase, both and decrease, suggesting that greater resistance due to porous medium effects (represented by higher Nd) suppresses momentum transfer, weakening both thermal and solutal gradients at the wall.

The convective parameter h has a generally positive effect on both heat and mass transfer. Higher values of h enhance boundary layer convection, leading to increased wall gradients and thus higher Nusselt and Sherwood numbers. This trend is more evident at lower values of Nb and Nd , where convective effects are less hindered by nanoparticle diffusion or porous medium resistance. However, when Nd is large, the influence of h is somewhat diminished, as the overall flow resistance counteracts the convective enhancement. In summary, Brownian motion and higher porous medium resistance reduce heat transfer, while enhanced surface convection tends to improve both heat and mass transfer.

Table (3): values of $\frac{Nu_x}{Ra_x^{1/2}}$ and $\frac{Sh_x}{Ra_x^{1/2}}$ for selected values of h , Nd and Nt

with $Nb=Nr=0.5$, $Ln=Le=10$, $Lb=6$, $Pe=0.2$, $bi=0.3$

H	Nd	$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(0)$			$\frac{Sh_x}{Ra_x^{1/2}} = -\gamma'(0)$		
		Nt			Nt		
		0,1	0,5	1	0,1	0,5	1
5	0	0,2359	0,2043	0,1723	1,3162	1,3439	1,3991
	0,5	0,2240	0,1935	0,1628	1,2444	1,2694	1,3203
	1	0,2155	0,1858	0,1560	1,1927	1,2161	1,2643
50	0	0,2528	0,2157	0,1795	1,3621	1,3850	1,4365
	0,5	0,2386	0,2033	0,1688	1,2814	1,3025	1,3504
	1	0,2286	0,1946	0,1614	1,2246	1,2446	1,2903
1000	0	0,2548	0,2170	0,1803	1,3673	1,3895	1,4405
	0,5	0,2403	0,2044	0,1695	1,2855	1,3061	1,3537
	1	0,2301	0,1956	0,162	1,2281	1,2477	1,2932

The table (3) summarizes the influence of the thermophoresis parameter Nt , the non-Darcy parameter Nd , and the convective parameter h on heat and mass transfer characteristics. It is observed that increasing Nt reduces the heat transfer rate (Nusselt number) while enhancing the mass transfer rate (Sherwood number), due to the thermophoretic movement of nanoparticles away from the surface. Higher values of Nd generally lead to a decline in heat and mass transfer, reflecting the added resistance in non-Darcy porous media. Conversely, increasing the convective parameter h significantly improves both thermal and mass transfer, emphasizing the role of surface convection in enhancing transport processes.

Table (4): values of $\frac{Nu_x}{Ra_x^{1/2}}$ and $\frac{Sh_x}{Ra_x^{1/2}}$ for selected values of h , Nd and Du

with $Nb=0.2$, $Nr=0.1$, $Nt=0.5$, $Ln=Le=10$, $Lb=6$, $Pe=0.2$, $bi=0.3$

H	Nd	$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(0)$			$\frac{Sh_x}{Ra_x^{1/2}} = -\gamma'(0)$		
		Du			Du		
		0,0	0,1	0,2	0,0	0,1	0,2
5	0	0,2819	0,1781	0,0132	1,4789	1,7077	2,0783
	0,5	0,2648	0,1678	0,0152	1,3723	1,5774	1,9082
	1	0,2531	0,1607	0,0160	1,3023	1,4936	1,8015
50	0	0,3023	0,1892	0,0139	1,5193	1,7357	2,0807
	0,5	0,2818	0,1772	0,0159	1,4029	1,5985	1,9104
	1	0,2682	0,1691	0,0166	1,3281	1,5113	1,8035
1000	0	0,3046	0,1905	0,0140	1,5239	1,7389	2,0810
	0,5	0,2837	0,1783	0,0160	1,4063	1,6008	1,9106
	1	0,2700	0,1701	0,0167	1,3309	1,5133	1,8037

The table (4) illustrates the effects of the Dufour number Du , modified non-Darcy number Nd , and convective parameter h on local heat and mass transfer rates. An overall decline in both the Nusselt and Sherwood numbers is observed as Du increase. This behaviour is expected since the Dufour effect introduces energy flux due to concentration gradients, which can disturb the thermal boundary layer and reduce the effectiveness of pure heat conduction exacerbates the disruptive influence of the Dufour effect. Additionally, higher Nd values consistently reduce transfer rates due to increased resistance in the porous medium. While increasing h generally improves both heat and mass transfer by strengthening convective motion, its impact becomes more noticeable at lower Du values, where cross-diffusion effects are less dominant.

Conclusion:

In this chapter, a comprehensive mathematical model was established to study the coupled transport phenomena involving nanofluids and gyrotactic microorganisms flowing through a non-Darcy porous medium under natural convection. Using similarity transformations, the governing partial differential equations including continuity, momentum, energy, nanoparticle concentration and microorganism's density were reduced to a nonlinear system of ordinary differential equations, which were numerically solved using the BVP4C solver in MATLAB.

The resulting analysis was supported by a series of parametric plots that examined the influence of key dimensionless numbers, such as the modified non-Darcy parameter (Nd), convective heat transfer parameter (h), Brownian motion number (Nb), thermophoresis number (Nt), Dufour number (Du), Soret number (Sr), nanoparticle Lewis number (Ln), traditional Lewis number (Le), bioconvection Lewis number (Lb), bioconvection Rayleigh number (Rb) and bioconvection Péclet number (Pe).

The numerical results revealed that:

- A higher Nd implies **greater resistance** beyond the linear Darcy regime. This suppresses the buoyancy-driven motion, thereby **reducing the fluid velocity** within the boundary layer. The suppression is more prominent near the wall, where the inertial drag is strongest.
- An increase in Du enhances the **heat transfer due to solute diffusion**, resulting in an **increase in the temperature** within the thermal boundary layer. The fluid gains additional energy from the mass flux, leading to a thicker temperature profile and elevated wall temperatures.
- Higher Sr leads to stronger **thermophoretic motion**, causing nanoparticles to migrate along temperature gradients. As a result, the **concentration boundary layer becomes thinner**, and the **peak concentration shifts away from the wall**, sometimes even decreasing near the surface.
- A larger Lb means **lower motile microorganism diffusivity** relative to solutes, which causes the **microorganism profile to become more concentrated near the wall**. Since

their movement is slower, they accumulate and create denser bioconvective zones, amplifying local instability.

- An increase in either Nb or Nt causes **thermal and solutal coupling** that strengthens the nonlinear interaction between heat and mass transfer. This results in **slower decay of temperature and concentration profiles**, influencing both velocity and bioconvection patterns.

This chapter successfully links theoretical modeling with numerical results, offering a comprehensive understanding of how multiple interdependent parameters control velocity, heat, mass and microorganism transport in porous nanofluid systems.

General conclusion

V. General conclusion:

This thesis has presented a detailed theoretical and numerical investigation of natural convection flow of a nanofluid containing gyrotactic microorganisms over a vertical plate embedded in a non-Darcy porous medium. The study focused on analyzing the combined effects of heat and mass transfer mechanisms, nanoparticle dynamics and bioconvection behavior arising from motile microorganisms.

The mathematical model incorporated important transport phenomena such as Brownian motion and thermophoresis to describe the behavior of nanoparticles, along with the bioconvection effects associated with microorganisms. The momentum, energy, nanoparticle concentration and microorganism density equations were transformed into a coupled system of nonlinear ordinary differential equations using similarity transformations. The numerical solution of the system was obtained via the BVP4C solver in MATLAB, which provided accurate and stable solutions over a range of physical parameters.

A thorough parametric study was conducted to examine the influence of various dimensionless parameters on velocity, temperature, concentration and microorganism density distribution. Key observations include the significant impact of the non-Darcy number N_d and convective parameter h on the velocity field, with increasing N_d reducing fluid motion due to enhanced flow resistance in the porous medium. The Rayleigh number for bioconvection R_b was also found to enhance the flow due to buoyancy-driven effects caused by microorganisms.

The thermal field was strongly affected by Brownian motion N_b and thermophoresis N_t , both contributing to increased heat transfer near the surface. Meanwhile, the nanoparticle concentration was primarily influenced by the Lewis numbers Le and Ln as well as the Soret number S_r , which controlled the mass diffusion process. The microorganism distribution responded notably to variations in the Péclet number Pe and the bioconvection Lewis number L_b , underlining the sensitivity of bioconvective transport to microbial motility and diffusion characteristics.

Overall, the results of this study highlight the complex interactions between nanofluid behavior, porous media characteristics and microorganism activity in thermosolutal convection systems. The insight gained contribute to a better understanding of transport phenomena in Nano-bioconvection flows and offer potential applications in bioengineering, thermal management and porous media technology.

Futures extensions of this work could include the study of other physical effects such as viscous dissipation, thermal radiation or chemical reaction. Moreover, investigating different geometrical configuration (such as radial domains, wavy surfaces or enclosures) could provide additional insight. Experimental validation and stability analysis would also strengthen the applicability of the model for real-world thermal systems involving nanofluids and biological flows.

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