

People's Democratic Republic of Algeria

Ministry of Higher Education and Scientific Research

Akli Mohand Oulhadj university, Bouira,

Faculty of exact sciences



---

## Course Notes

# Equations of Mathematical Physics

---

By:

*Dr. Amirouche Berkani*

*(Associate Professor)*

For:

**Third year of mathematics licence**

*Academic Year: 2025-2026*

# Contents

<b>1</b>	<b>Generalities about partial differential equations</b>	<b>4</b>
1.1	What is a differential equation? . . . . .	4
1.1.1	What is a partial derivative? . . . . .	5
1.2	What is PDE? . . . . .	5
1.2.1	Where PDEs are coming from? . . . . .	5
1.2.2	Some examples of PDEs . . . . .	6
1.3	Basic concepts and definitions . . . . .	7
1.4	General solutions and auxiliary conditions . . . . .	8
1.4.1	Differentiation of composite functions : the Chain Rule . . . . .	9
1.5	Linear operators and the principle of superposition . . . . .	11
1.6	Boundary conditions . . . . .	14
1.7	Initial conditions . . . . .	15
1.8	Well-posed PDEs . . . . .	15
1.9	Exercises . . . . .	15
<b>2</b>	<b>Partial differential equations of first order</b>	<b>18</b>
2.1	Classification of first-order equations . . . . .	18
2.2	Linear first order equations . . . . .	19
2.3	First-order quasi-linear partial differential equations . . . . .	25
2.3.1	Simultaneous DEs. . . . .	25
2.3.2	Geometrical interpretation of a first-order equation . . . . .	27
2.3.3	Method of characteristics and general solutions . . . . .	28
2.4	Fully-nonlinear first-order equations . . . . .	34
2.4.1	Charpit's method. . . . .	34
2.5	Exercises . . . . .	36

<b>3</b>	<b>Partial differential equations of second order</b>	<b>39</b>
3.1	Classification and canonical forms of equations in two independent variables . . . . .	39
3.2	Change of variables . . . . .	41
3.3	Canonical forms . . . . .	43
3.3.1	Canonical form of hyperbolic equations . . . . .	44
3.3.2	Canonical form of parabolic equations . . . . .	47
3.3.3	Canonical form of elliptic equations . . . . .	49
3.4	General solutions . . . . .	51
3.5	Exercises . . . . .	53
<b>4</b>	<b>Method of separation of variables</b>	<b>55</b>
4.1	Introduction . . . . .	55
4.2	Separation of variables . . . . .	55
4.3	Examples . . . . .	56
4.3.1	Heat equation: homogeneous boundary conditions . . . . .	56
4.3.2	One-dimensional wave equation . . . . .	62
4.4	Exercises . . . . .	66
<b>5</b>	<b>Laplace's Equation</b>	<b>68</b>
5.1	Introduction . . . . .	68
5.2	Harmonic Functions . . . . .	69
5.3	Dirichlet problem in the circle and the Poisson kernel . . . . .	70
5.3.1	Series solution . . . . .	72
5.3.2	Poisson kernel . . . . .	74
5.4	Exercises . . . . .	77
<b>6</b>	<b>Wave equation</b>	<b>79</b>
6.1	The wave equation in one dimension $n = 1$ . . . . .	79
6.2	d'Alembert's formula for the wave equation . . . . .	80
6.2.1	Homogeneous wave equation . . . . .	80
6.2.2	Non homogeneous wave equation . . . . .	82
6.3	Exercises . . . . .	86
<b>7</b>	<b>Heat equation</b>	<b>88</b>
7.1	Introduction . . . . .	88

7.2 Separation of variables . . . . . 90  
    7.2.1 Insulated ends . . . . . 92  
7.3 Exercises . . . . . 93

**Bibliographie** . . . . . **95**