PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA
Ministry of Higher Education and Scientific Research



University of Akli Mohand Oulhadj - Bouira
Faculty of Applied Sciences
Department of Mechanical Engineering

METROLOGY Course Handbook
Field: Mechanical Engineering
Specialty: Energy Engineering
Level: Bachelor

METROLOGY

Course Handbook

par

Pr.MAHFOUD Brahim

2025

Foreword

1) Context

In an increasingly data-driven world, the act of measurement forms the bedrock of scientific progress, industrial quality, and technological innovation. The reliability of any data, however, is intrinsically linked to the quality and understanding of the measurement process itself. This is the domain of metrology—the science of measurement. Within this vast field, sensors act as the critical gateway between the physical world and our digital systems, translating phenomena like temperature, pressure, and force into quantifiable electrical signals.

The accuracy, precision, and reliability of these sensors are not merely technical details; they are fundamental to the integrity of every system they support. From ensuring the safety of a pharmaceutical process to optimizing the efficiency of an automotive engine, the correct application and understanding of sensor metrology is paramount. The purpose of this handbook, **"Sensor Metrology: From Fundamentals to Temperature Measurement,"** is to provide a clear, structured, and practical guide to this essential discipline. It is designed to equip students, engineers, and technicians with the foundational knowledge and tools necessary to select, characterize, and understand the behaviour of measurement sensors.

2) Handbook Overview: Objectives and Structure

This handbook is intended for **Bachelor's level students** and is structured to guide the reader logically from the abstract principles of metrology to the concrete application of measuring a specific physical quantity: temperature.

The primary objectives of this work are to:

- Establish a firm understanding of core metrological concepts, including the International System of Units (SI), error analysis, and statistical treatment of data.
- Define and explain the key static and dynamic characteristics that determine sensor performance, such as accuracy, sensitivity, linearity, and response time.
- Provide a detailed exploration of a major sensor family—temperature sensors—illustrating how theoretical principles apply to real-world devices.

The book is organized into two cohesive parts to facilitate this learning journey:

Part 1: Sensor Metrology lays the essential groundwork. It begins with an **Introduction to Metrology** (Chapter 1), establishing basic definitions, the SI system,

and a classification of sensor types. It then delves into **Metrological Characteristics** (Chapter 2), offering a comprehensive look at different types of measurement errors, statistical methods for data analysis, and the concepts of precision and trueness. This part concludes with an examination of **Measurement System Performance** (Chapter 3), detailing the static and dynamic characteristics that define how a sensor behaves in practice.

Part 2: Temperature Sensors applies the foundational knowledge from Part 1 to a critical domain. It systematically covers the main types of temperature sensors, including **Expansion Thermometers** (Chapter 4) like liquid-in-glass and bimetallic strips, **Electrical Thermometers** (Chapter 5) such as Resistance Temperature Detectors (RTDs) and thermistors, and **Thermocouples** (Chapter 6), explaining their principles, practical use, and sensitivities. The part concludes by analyzing the **Dynamic Response of a Temperature Sensor** (Chapter 7), bridging the theoretical dynamics from Part 1 with the practical behaviour of a sensor in a thermal environment.

By seamlessly integrating fundamental theory with practical application, this handbook aims to be an indispensable resource for anyone seeking to master the science of measurement and ensure the validity of the data upon which we all depend.

Table of contents

Foreword i
List of Figures i
List of Tables i
Symbols and notations i
Chapter I. Introduction to metrology 2
I.1) Fundamental Concepts 2
I.1) Key Definitions 2
I.1.2) The International System of Units (SI) and its Symbols 2
I.1.3) Relationships Between SI Units and Anglo-Saxon Units 7
I.2) Sensor Classification 8
I.2.1) Active Sensors 8
I.2.2) Passive Sensors 11
I.2.3) Influence Quantities 12
1.3) The Measurement Chain 12
Chapter II. Metrological Characteristics 15
II.1) Measurement Errors 15
II.1.1 Illegitimate Errors 15
II.1.2 Systematic Errors 15
II.2 Statistical Treatment of Measurements 17
II.2.1 Statistical Characterization of a Distribution 17
II.2.2 Measures of Central Tendency of a Distribution 20
II.2.3 Measures of Distribution Dispersion 20
II.3 Total Errors in a Measurement System 22
II.3.1 Error of a Product 22
II.3.2 Error of a Quotient 22
II.3.3 Error of a Sum 23
II.4 Linear Regression 24
II.5 Trueness, Accuracy and Precision 25
Chapter III: Performance of Measurement Systems 28
III.1 The Ideal Measurement System 28
III.2 Static Characteristics Of A Sensor 28
III.2.1 Measurement Range and Span 28

III.2.2 Static Calibration Curve 29
III.2.3 Accuracy 29
III.2.4 Bias (Zero Drift) 29
III.2.5 Linearity Error 30
III.2.6 Sensitivity 30
III.2.7 Sensitivity Drift 31
III.2.8 Resolution 32
III.2.9 Repeatability (Precision) 32
III.2.10 Reproducibility 32
III.3 Dynamic Characteristics of a Sensor 32
III.3.1 Zero-Order System 32
III.3.2 First-Order System 33
III.3.3 Second-Order Systems 37
Chapter IV: Expansion Thermometers 42
IV.1 Introduction to Temperature Measurement 42
IV.2 Liquid-in-Glass Thermometers 42
IV.2.1 Fundamental Design and Operation 42
IV.2.2 Volume-Temperature Relationship 43
IV.2.3 Thermometric Liquids 44
IV.2.4 Envelope Material Selection 44
IV.2.5 Emergent Stem Correction 44
IV.3 Gas Expansion Thermometers 46
IV.3.1 Operating Principle 46
IV.3.2 Description and Components 46
IV.3 Gas Thermometers 47
IV.3.1 Operating Principle 47
IV.3.2 System Description and Operation 47
IV.4 Vapor Pressure Thermometers 49
IV.4.1 Fundamental Principle 49
IV.4.2 Filling Fluids and Their Operational Ranges 50
IV.5 Solid-Expansion Thermometers 51
IV.5.1 Operating Principle 51
Chapter V: Electrical Thermometers 55

V.1 Introduction 55
V.2 Resistance Temperature Detectors (Rtds) 55
V.3.1 Principle 57
V.3.2 Resistance-Temperature Relationship 58
Chapter VI: Thermocouples 61
VI.1 Principle 61
VI.2 Thermoelectric Effects 62
VI.2.1 The Peltier Effect 62
VI.2.2 The Thomson Effect 62
VI.2.3 The Seebeck Effect 63
VI.3 Practical Principles For Using Thermocouples 63
VI.4 Thermocouple Thermal Sensitivity 68
VI.5 Reference Temperature of A Thermocouple 70
VI.5.1 The Ice-Water Bath 70
VI.5.2 The Electrical Bridge Method 70
VI.5.3 .The Double Furnace Method 72
VI.6 Main Types of Thermocouples and Their Limits of Use 74
VI.7 Comparison between Thermocouples and Electrical Thermometers 75
Chapter VII: Dynamic Response of a Temperature Sensor 77
VII.1 Introduction 77
VII.2 Response to a Step Input 78
VII.3 Response to a Ramp Input 79
General Conclusion 82
Chapter VIII: Measuring Instruments 83
VIII.1. Caliper Gauge or Vernier Caliper 83
VIII.1.2 Types of Vernier Calipers 86
VIII.1.3. Operating Procedure for Vernier Calipers 87
VIII.1.4 Vernier Reading Principle 88
VIII.2. Micrometer or Micrometer Caliper 90
VIII.2. 1.Types of Micrometers and Their Uses 91
questionnaires à ch Les oix multiples (QCM) 95
References 104

List of Figures

FIG.I. 2:ACTIVE SENSORS 10

FIG.II. 1:RANDOM ERRORS 16
FIG.II. 2:HISTOGRAM AND RELATIVE FREQUENCY CURVE 19
FIG.II. 3:CUMULATIVE FREQUENCY DIAGRAM 19
FIG.II. 4:DIFFERENT LEVELS OF DISPERSION 20
FIG.II. 5:LINEAR REGRESSION 24
FIG.II. 6:SYSTEMATIC AND RANDOM 26

FIG.III. 1:MEASUREMENT RANGE AND SPAN 29
FIG.III. 2:BIAS (ZERO DRIFT) 30
FIG.III. 3:SENSITIVITY DRIFT 31
FIG.III. 4:RESPONSE TO A STEP INPUT 33
FIG.III. 5 THE TIME RESPONSE OF A FIRST-ORDER SYSTEM TO A STEP INPUT OF THE MEASURAND. 34
FIG.III. 6:DYNAMIC SENSITIVITY. 36
FIG.III. 7: THE TIME RESPONSE OF A SECOND-ORDER SYSTEM TO A STEP INPUT, FOR DIFFERENT VALUES OF THE DAMPING RATIO Z. 38
FIG.III. 8:THE FREQUENCY RESPONSE OF A SECOND-ORDER SYSTEM FOR DIFFERENT VALUES OF THE DAMPING RATIO Z 39
FIG.III. 9:THE FREQUENCY-DEPENDENT PHASE SHIFT 40

FIG.IV. 1: GGLASS THERMOMETER 43
FIG.IV. 2:GAS THERMOMETERS 48
FIG.IV. 3:VAPOR PRESSURE THERMOMETERS (SINGLE-FILL SENSORS) 49
FIG.IV. 4: VAPOR PRESSURE THERMOMETERS (DUAL-FILL SYSTEMS) 50

FIG.V. 1:RESISTANCE TEMPERATURE DETECTORS (RTDS) WITH PLATINUM 57
FIG.V. 2DIFFERENT THERMISTORS 59

FIG.VI. 1:THERMOCOUPLES 61
FIG.VI. 2:ISOTHERMAL CIRCUIT 62
FIG.VI. 3:HOMOGENEOUS CONDUCTOR 62
FIG.VI. 4:CLOSED CIRCUIT FORMED BY TWO CONDUCTORS A AND B 63
FIG.VI. 5:EMF VERSUS THE TEMPERATURE 68
FIG.VI. 6:VARIATION OF THERMAL SENSITIVITY 69
FIG.VI. 7:THE ICE-WATER BATH 70
FIG.VI. 8:WHEATSTONE BRIDGE 71
FIG.VI. 9:THERMOCOUPLE CIRCUIT WITH REFERENCE FURNACES AT 65.5°C AND 130°C 72

FIG.VII. 1:EVOLUTION OF THE SENSOR TEMPERATURE TC 79
FIG.VII. 2:EVOLUTION OF THE SENSOR TEMPERATURE TC WITH TIME 81

FIG.VIII. 1:VERNIER CALIPER 85
FIG.VIII. 2: TYPES OF VERNIER 87
FIG.VIII. 3:COMPONENTS OF THE MICROMETER 90

List of Tables

TABLE I. 1:BASE UNITS [1] 4
TABLE I. 2:DERIVED UNITS [1] 5
TABLE I. 3:DERIVED UNITS (CONTINUED) [1] 6
TABLE I. 4:SUPPLEMENTARY UNITS 6
TABLE I. 5:MULTIPLES AND SUBMULTIPLES OF UNITS[1] 7
TABLE I. 6:PRIMARY PHYSICAL QUANTITIES 11

TABLE II. 1:FURNACE TEMPERATURE MEASUREMENTS 18
TABLE II. 2:TEMPERATURE DISTRIBUTION FREQUENCY TABLE 18
TABLE II. 3:SUMMARY TABLE OF ERROR 26

TABLE III. 1:THE RESPONSE TIME FOR DIFFERENT TOLERANCE VALUES E(%) 35

TABLE IV. 1:SEVERAL LIQUIDS EMPLOYED IN THERMOMETRIC 44
TABLE IV. 2:FILLING FLUIDS AND THEIR OPERATIONAL RANGES 51

TABLE VI. 1:COEFFICIENTS A_i 69
TABLE VI. 2:MAIN TYPES OF THERMOCOUPLES AND THEIR LIMITS OF USE 74
TABLE VI. 3:COMPARISON BETWEEN THERMOCOUPLES AND ELECTRICAL THERMOMETERS 75

Symbols and notations

Category	Symbol / Abbreviation	Meaning / Description
Fundamental Quantities	l	Length
	m	Mass
	t	Time
	I	Electric Current
	T	Thermodynamic Temperature (K)
	t or θ	Celsius Temperature (°C)
	n	Amount of Substance
	Iv	Luminous Intensity
Derived Quantities	A,S	Area / Surface
	V	Volume
	f	Frequency
	v	Velocity
	a,g	Acceleration
	F	Force
	P	Pressure or Power
	ω	Angular Velocity
	α	Angular Acceleration
	λ	Wavelength
	ρ	Electrical Resistivity
	ε	Dielectric Constant
	μ	Magnetic Permeability
	ϕ	Magnetic Flux
	B	Magnetic Induction / Magnetic Flux Density
	H	Magnetic Field Strength
Mathematical & Statistical Notations	X,Y,Z	Measured Variables or Quantities
	x,y,z	Measured Values
	dx,dy,dz	Absolute Uncertainties
	$\bar{X}$	Arithmetic Mean
	σ	Standard Deviation
	σ^2	Variance
	ρ	Correlation Coefficient
	ε	Error or Tolerance
	Δ	Variation
	τ	Time Constant
	ζ	Damping Ratio
	ω_0	Natural Frequency
	fc	Cut-off Frequency
Errors and Uncertainties	ϵ_m	Mobility Error
	ϵl	Reading Error
	ϵr	Resolution Error
	δ	Bias or Offset
	U	Expanded Uncertainty
	LSB	Least Significant Bit (digital)
Systems & Concepts	SI	International System of Units
	CGPM	General Conference on Weights and Measures
	CIPM	International Committee for Weights and Measures
	RTD	Resistance Temperature Detector (e.g., Pt100)

NTC / PTC	Thermistor (Negative/Positive Temperature Coefficient)
EMF	Electromotive Force
ADC	Analog-to-Digital Converter
FS	Full Scale
Pt100	Pt100 Probe (100 Ω at 0°C)
hp	Horsepower
LSB	Least Significant Bit

Chapter I. Introduction to metrology

Chapter I. Introduction to metrology

I.1) Fundamental Concepts

I.1) Key Definitions

- **Metrology:**

  Defined as the science of measurement.

- **Measurement:**

  Refers to the complete set of experimental operations performed to determine the numerical value of a quantity.

- **Measurand:**

  The specific physical quantity that is the subject of the measurement.

- **Uncertainty:**

  The result x of measuring a quantity X cannot be fully defined by a single number. It must be characterized by a pair (x, ±dx), where dx represents the measurement uncertainty arising from various errors. This defines an interval within which the true value is expected to lie: $x - dx < X < x + dx$.

- **Absolute Error:** This is the difference between the true value of the measurand and its measured value. It is expressed in the unit of measurement.

- **Relative Error:** This is the ratio of the absolute error to the measured result. It is expressed as a percentage of the measured quantity.

I.1.2) The International System of Units (SI) and its Symbols

The International System of Units is built upon seven dimensionally independent base units. From these, derived units and supplementary units are formed. The most commonly used quantities, along with their units, are presented in the following table.

a) Base Units

There are seven base units, which are considered to be dimensionally independent (see Table I.1).

- Unit of Length: the metre (symbol: m)

Definition from the 17th CGPM (1993) - Resolution 1: (So, the phrase “Definition of the 17th CGPM (1993) – Resolution 1” indicates that the following text or formula presents

the official definition of a unit of measurement, as adopted during that session of the CGPM.)"

The metre is the length of the path travelled by light in a vacuum during a time interval of 1/299,792,458 of a second.

- Unit of Mass: the kilogram (symbol: kg)

Definition from the 1st CGPM (1889) and the 3rd CGPM (1901):

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

- Unit of Time: the second (symbol: s)

Definition from the 13th CGPM (1967) - Resolution 1:

The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

- Unit of Electric Current: the ampere (symbol: A)

Definition by the CIPM (1946) - Resolution 1, approved by the 9th CGPM (1948):

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in a vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

- Unit of Thermodynamic Temperature: the kelvin (symbol: K)

Definition from the 13th CGPM (1967) - Resolution 4:

The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

The 13th CGPM (Resolution 3) also decided that the unit kelvin and its symbol K should be used to express an interval or a difference in temperature.

Note: In addition to thermodynamic temperature (symbol T) expressed in kelvins, we also use Celsius temperature (symbol *t*) defined by the expression:

$t = T - T_0$

where $T_0 = 273.15$ K by definition.

- Unit of Amount of Substance: the mole (symbol: mol)

Definition from the 14th CGPM (1971) - Resolution 3:

The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12.

Note: When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

- Unit of Luminous Intensity: the candela (symbol: cd)

Definition from the 16th CGPM (1979) - Resolution 3:

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Table I. 1:Base Units [1]

Quantity	Unit (SI)	Symbol
Length (denoted *l*)	metre	m
Mass (denoted *m*)	kilogram	kg
Time (denoted *t*)	second	s
Electric current (denoted *i*)	Ampere (André-Marie Ampère, 1775-1836)	A
Temperature (denoted *T*)	Kelvin (Lord Kelvin, England, 1824-1907)	K
Amount of substance	mole	mol
Luminous intensity (denoted *I*)	candela	cd

Table I. 2:Derived Units [1]

Quantity	Unit (SI)	Symbol
Area (denoted *A* or *S*)	square metre	m^2
Volume (denoted *V*)	cubic metre	m^3
Frequency (denoted *f*)	Hertz (Heinrich Hertz, Germany, 1857-1894)	Hz
Velocity (denoted *v*)	metre per second	m/s
Force (denoted *F*)	Newton (Isaac Newton, England, 1642-1727)	N
Moment of a force (denoted *M*)	metre-Newton	m·N
Moment of a couple / Torque (denoted *T*)	metre-Newton	m·N
Dynamic viscosity (denoted η)	poiseuille	Pl
Electrical voltage (denoted *U*)	Volt (Alessandro Volta, Italy, 1745-1827)	V
Electromotive force (denoted *E*)	Volt	V
Electrical resistance (denoted *R*)	Ohm (Georg Ohm, Germany, 1789-1854)	Ω
Reactance (denoted *X*)	Ohm	Ω

Table I. 3:Derived Units (Continued) [1]

Quantity	Unit (SI)	Symbol
Impedance (denoted *Z*)	Ohm	Ω
Resistivity (denoted ρ)	Ohm-metre	Ω·m
Electrical capacitance (denoted *C*)	Farad (Michael Faraday, England, 1791-1867)	F
Electrical permeability (denoted μ)	Henry per metre	H/m
Luminous flux	lumen	lm
Illuminance	lux	lx
Wavelength (denoted λ)	metre	m
Angular velocity (denoted ω)	radian per second	rad/s
Acceleration (denoted *a/g*)	metre per second squared	m/s^2
Angular acceleration (denoted α)	radian per second squared	rad/s^2
Energy, Work (denoted *W*)	Joule (James Joule, England, 1818-1889)	J
Power (denoted *P*)	Watt (James Watt, Scotland, 1736-1819)	W
Apparent power (denoted *S*)	Volt-Ampere	VA
Reactive power (denoted *Q*)	Volt-Ampere Reactive	VAR
Pressure (denoted *P*)	Pascal (Blaise Pascal, France, 1623-1662)	Pa
Quantity of electricity, Electric charge (denoted *Q*)	Coulomb (Charles-Augustin de Coulomb, France, 1736-1806)	C
Inductance (denoted *L*)	Henry (Joseph Henry, United States, 1797-1878)	H
Magnetic field strength (denoted *H*)	Ampere per metre	A/m
Magnetic induction / Magnetic flux density (denoted *B*)	Tesla (Nikola Tesla, Serbia, 1856-1943)	T
Magnetic flux (denoted φ)	Weber (Wilhelm Weber, Germany, 1804-1891)	Wb

Table I. 4:Supplementary Units

Quantity	Unit (SI)	Symbol
Plane angle	radian	Rad
Solid angle	steradian	sr

Table I. 5:Multiples and Submultiples of Units[1]

Multiples			Submultiples		
Factor	Prefix	Symbol	Factor	Prefix	Symbol
$\mathbf{10^{24}}$	yotta	Y	10^{-1}	deci	d
$\mathbf{10^{21}}$	zetta	Z	10^{-2}	centi	c
$\mathbf{10^{18}}$	exa	E	10^{-3}	milli	m
$\mathbf{10^{15}}$	peta	P	10^{-6}	micro	µ
$\mathbf{10^{12}}$	tera	T	10^{-9}	nano	n
$\mathbf{10^{9}}$	giga	G	10^{-12}	pico	p
$\mathbf{10^{6}}$	mega	M	10^{-15}	femto	f
$\mathbf{10^{3}}$	kilo	k	10^{-18}	atto	a
$\mathbf{10^{2}}$	hecto	h	10^{-21}	zepto	z
10	deca	da	10^{-24}	yocto	y

I.1.3) Relationships Between SI Units and Anglo-Saxon Units

Distance:

- Inch: 1 in = 2.54 cm
- Foot: 1 ft = 12 in = 30.48 cm
- Mile: 1 mile = 5280 ft = 1.609 km

Volume:

- Pint: 1 pint = 0.94 l
- US gallon: 1 US gal = 4 pints = 3.786 l
- US barrel: 1 bbl = 42 US gal = 159 l

Mass:

- Ounce: 1 oz = 28.35 g
- Pound: 1 lb = 0.454 kg

Power:

- Horsepower: 1 hp = 0.736 kW

I.2) Sensor Classification

A sensor is a device that converts a physical quantity into an usable electrical quantity for measurement or control purposes. This electrical quantity (voltage or current) must represent the measured quantity as accurately as possible. Sensors are classified into active sensors and passive sensors.

I.2.1) Active Sensors

These behave like generators. They are based on a physical effect that transforms the energy of the measured quantity (mechanical, thermal, or radiation energy) into electrical energy. The output response of an active sensor can be a current, voltage, or charge. Among these effects, the most important are:

a) The Thermoelectric Effect:

A circuit formed by two conductors of different chemical natures, with junctions at temperatures T1 and T2, generates an electromotive force e = f(T1,T2). Application example (Fig.I.1a): Measuring e allows determining an unknown temperature T1 when temperature T2 is known (principle of the thermocouple).

b) The Piezoelectric Effect:

Applying mechanical stress to certain materials called piezoelectric materials (such as quartz) causes deformation that generates equal and opposite electrical charges on opposite faces of the material.

Application example (Fig.I.1b): Measuring force, pressure, or acceleration from the voltage generated across a capacitor associated with the piezoelectric element due to variations in its charge.

c) The Photoelectric Effect:

When light, or more generally, electromagnetic radiation with a wavelength below a material-specific threshold, strikes a material, it causes the release of electrical charges. Application Example (Fig.I.1c): The output voltage can be measured to determine the intensity of the radiation flux.

c) The Pyroelectric Effect:

Pyroelectric crystals (e.g., triglycine sulfate) possess a spontaneous electrical polarization that is temperature-dependent. This results in surface charges proportional to the polarization, with opposite signs on opposing faces.

Application Example (Fig.I.1d): The charge across a capacitor connected to a pyroelectric crystal can be measured to determine the intensity of the incident light flux.

d) Electromagnetic Induction:

When a conductor moves within a fixed magnetic field, an electromotive force (EMF) proportional to its velocity is induced. Furthermore, when an electrical circuit is subjected to a varying magnetic flux—due to either its own movement or the movement of the magnetic source—an EMF is induced that is equal in magnitude and opposite in sign to the rate of change of the magnetic flux.

Application Example (Fig.I.1e): Measuring the induced EMF allows for the determination of the velocity of the movement causing it.

e) The Hall Effect:

When a material carrying a current I is subjected to a magnetic field B at an angle θ to the current, a Hall voltage V_H appears in a direction perpendicular to both. This is defined by the formula $V_H = K_H \cdot I \cdot B \cdot sin(\theta)$, where K_H is a constant dependent on the material.

Application Example (Fig.I. 1f): Measuring the Hall voltage V_H enables the determination of the position of an object linked to a magnet.

f) The Photovoltaic Effect:

When light radiation strikes a junction of opposing P-type and N-type semiconductors, it releases electrons (negative charges) and holes (positive charges) near the illuminated junction. Their movement within the junction's electric field alters the voltage across it.

Application Example: The output voltage can be measured to determine the radiation flux.

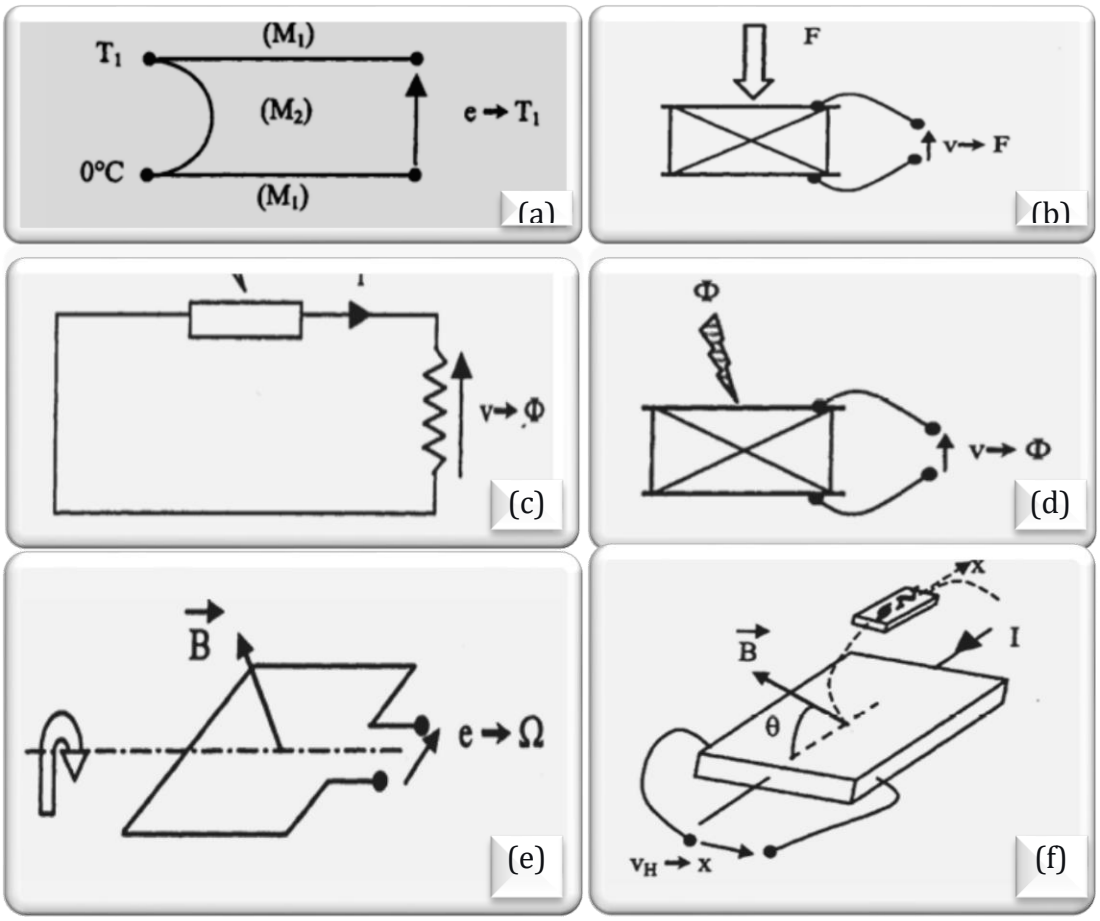


Fig.I. 2:Active Sensors

The fundamental physical principles and application methods of these effects are summarized in the following table:

Physical Effects and Corresponding Sensor Applications

Physical Quantity to Measure	Effect Used	Output Quantity
Temperature	Thermoelectric	Voltage
Radiant Flux	Photoelectric	Voltage
	Photovoltaic	Voltage
	Pyroelectric	Charge
Force	Piezoelectric	Charge
Pressure	Piezoelectric	Charge
Acceleration	Piezoelectric	Charge
Velocity	Electromagnetic Induction	Voltage
Position	Hall	Voltage

I.2.2) Passive Sensors

Passive sensors are impedance-based components integrated into an electrical signal conditioning circuit. A key parameter of this impedance is sensitive to the physical quantity being measured. The impedance variation arises from the measured quantity's effect on:

a) Geometric or Dimensional Properties: This occurs when the sensor incorporates a moving or deformable element.

- In the case of a moving part, each position corresponds to a specific impedance value. Measuring this impedance allows the position to be determined (e.g., the principle behind potentiometers used for displacement or position sensing).
- In the case of a deformable element, an applied strain causes a change in the sensor's impedance (e.g., the principle of strain gauges).

b) Intrinsic Electrical Properties of Materials: The sensor utilizes materials whose electrical properties—such as resistivity (ρ), magnetic permeability (μ), or dielectric constant (ε)—are sensitive to various physical influences like temperature, humidity, or light.

- By varying one specific physical quantity while keeping others constant, a defined relationship is established between that quantity and the sensor's impedance.
- A calibration curve captures this relationship, allowing the value of the physical variable (the measurand) to be deduced from a measurement of the impedance.

The following table provides an overview of the primary physical quantities that can alter the electrical properties of materials used in passive sensors.

Table I. 6:Primary physical quantities

Measurand	Material Types Used	Sensitive Electrical Characteristic
Temperature	Metals, Semiconductors	Resistivity
Radiant Flux	Semiconductors	Resistivity
Strain	Nickel Alloys	Resistivity
	Ferromagnetic Alloys	Magnetic Permeability
Position	Magneto-resistive Materials	Resistivity
Humidity	Lithium Chloride	Resistivity
	Polymers	Dielectric Constant
Level	Insulating Liquids	Dielectric Constant

I.2.3) Influence Quantities

During operation, a sensor is not only affected by the intended measurand but also by other physical quantities. These "influence quantities" can cause changes in the sensor's electrical output that are indistinguishable from the effect of the measurand itself. The most significant influence quantities include:

- **Temperature:** Alters the electrical, mechanical, and dimensional properties of the sensor's components.
- **Pressure, Acceleration, and Vibration:** Can induce mechanical strain and stress that distort the sensor's response.
- **Humidity:** Can modify the sensor's electrical properties and degrade electrical insulation between its components or with the environment.
- **Magnetic Fields:** Can induce unwanted electromotive forces (EMF) that interfere with the useful signal.
- **Power Supply Variations:** Fluctuations in voltage or frequency can disrupt the sensor's output signal.
- **Ambient Light:** Can add to the luminous flux being measured, leading to errors in light sensors.

To ensure accurate measurements, the effects of these influence quantities must be mitigated. This can be achieved by:

- **Physical Protection:** Shielding the sensor through adequate isolation to minimize exposure to influence quantities.
- **Stabilization and Calibration:** Stabilizing the environment to known, constant values and calibrating the sensor specifically under those conditions.
- **Electrical Compensation:** Using circuit designs, such as a Wheatstone bridge with a compensating sensor in an adjacent branch, to cancel out the effects of common influence quantities.

1.3) The Measurement Chain

A measurement chain is a series of devices (including the sensor) designed to determine the value of a measurand as accurately as possible. The process begins with the **sensor**, which is exposed to the measurand. It generates (directly, if active, or via a signal conditioner, if passive) an electrical signal that carries the information about the

measurand. At the end of the chain, the information is presented in a form suitable for use, such as a display reading or a data file for a control system.

While a simple chain might consist of just a sensor and a readout device (e.g., a thermocouple and a voltmeter), modern systems are typically complex electronic chains. A basic measurement chain must perform the following functions through appropriate devices:

[Measurand] -> [Sensor] -> [Signal Conditioners] -> [Display/Processing Unit]

- **Signal Acquisition & Conversion:** The sensor extracts information and translates the physical quantity into an electrical signal.
- **Signal Conditioning:** The signal is processed to prevent degradation from noise or interference. This includes amplification and filtering.
- **Signal Conversion:** The analog signal is converted into a digital format suitable for computational processing, using components like sample-and-hold circuits and analog-to-digital converters (ADC).
- **Visualization & Utilization:** The processed information is displayed for reading and/or used for control purposes, typically handled by a microprocessor or microcontroller.

Chapter II. Metrological Characteristics

Chapter II. Metrological Characteristics

II.1) Measurement Errors

The only quantities with perfectly known values are standards, whose values are established by convention. The value of any other measurand can only be determined after processing by a measurement chain. The difference between the measured value and the true value is the measurement error, primarily caused by imperfections in the measurement chain that degrade the signal information during processing. A rigorous design of the measurement chain can reduce this error and the associated uncertainty of the true value. Different types of measurement errors are described below.

II.1.1 Illegitimate Errors

These result from operator mistakes, improper use, or equipment malfunction. They are faults committed during the measurement process and are generally not considered in the final measurement determination.

II.1.2 Systematic Errors

These are reproducible errors, constant and/or slowly varying relative to the measurement duration. They introduce a constant bias between the true value and the measured value. Common causes include:

- **Reference Value Errors:** Examples include a zero offset in an analog instrument, an incorrect reference temperature for a thermocouple, or an inaccurate supply voltage for a bridge circuit. These can be eliminated through rigorous instrument verification.
- **Calibration Errors:** These are errors in sensor sensitivity or calibration curve. Sensor aging, mechanical fatigue, or chemical degradation of components can alter its initial calibration curve. Frequent sensor recalibration can reduce these errors.
- **Methodological or Usage Condition Errors:**
    - *Speed Error:* Results from taking a measurement before a steady-state condition is reached.
    - *Loading Error:* Caused by the sensor itself altering the value of the measurand it is intended to measure.

- **Data Processing Errors:** These are errors in correction or interpolation, stemming from an incorrect assessment of the adjustments needed to refine the raw measurement result (e.g., deviations from linearity in a non-linear sensor).

II.1.3 Random Errors (Les erreurs accidentelles ou aléatoires)

These are non-reproducible errors; their occurrence and magnitude are considered random. While some causes may be known, the specific error values at the time of measurement are unpredictable. They are quantified using statistical laws. Specific types include:

- *Mobility Error (ε_m):* The maximum change in the measurand that does not produce a detectable change in the sensor's output (e.g., in a wire-wound potentiometer, moving the wiper less than the distance between two turns).
- *Reading Error (ε_l) of an Analog Instrument:* Arises from the operator's skill and the instrument's quality (e.g., fineness of a needle).
- *Resolution Error (ε_r):* The smallest detectable change in the measurand for a given sensor. It combines mobility and reading errors: $\varepsilon_r = \sqrt{(\varepsilon_m^2 + \varepsilon_l^2)}$.
- *Hysteresis Error:* Occurs when a component in the measurement chain (e.g., a spring) exhibits hysteresis, meaning its output depends on its previous state. This error is estimated as half the maximum difference in output values for the same measurand value, depending on whether that value was approached from increasing or decreasing directions.

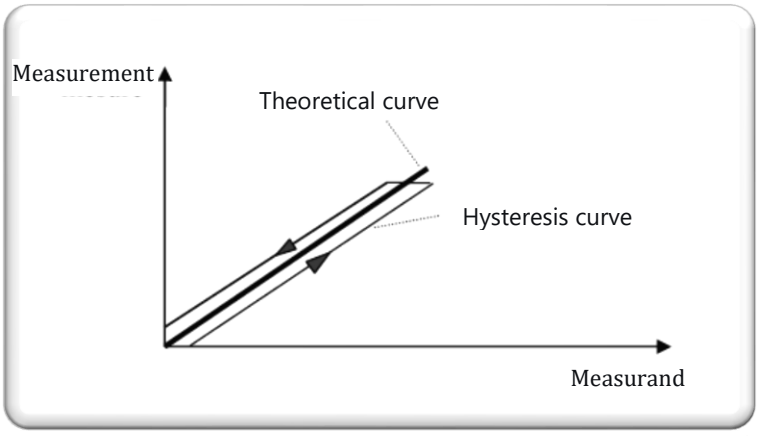


Fig.II. 1:Random Errors

Quantization Error of an Analog-to-Digital Converter:

The quantization process assigns a single digital value to all analog input values within a range corresponding to the Least Significant Bit (LSB). The maximum uncertainty introduced by this process is **±½ LSB**.

II.1.3.2 Errors from Random Noise in the Measurement Chain:

This category includes errors caused by random parasitic signals, such as:

Thermal noise generated by the random motion of charge carriers in resistors, creating voltage fluctuations that interfere with the useful signal.

Power supply voltage fluctuations that affect instrument performance and alter signal amplitude.

II.1.3.3 Errors from Influence Quantities:

When the effects of influence quantities (e.g., temperature, humidity) are not accounted for during sensor calibration, their impact can be considered random. These errors occur when the measuring instrument is used under environmental conditions different from those during its calibration.

II.2 Statistical Treatment of Measurements

Repeated measurements of the same quantity yield scattered results due to inherent errors. Statistical analysis is therefore required to determine the most probable value of the measured quantity and to establish uncertainty limits. This analysis involves several steps:

*a)Establishing the Data Distribution***:** Creating a graphical representation (e.g., histogram) of the data for an initial assessment.

*b)Characterizing the Statistical Distribution***:** Identifying the central tendency using measures like the mean, mode, and median.

*c)Determining the Distribution Dispersion***:** Quantifying the spread of the measurements around the mean value using variance and standard deviation.

II.2.1 Statistical Characterization of a Distribution

Suppose a quantity X*X* is measured n*n* times, yielding results $x_1,x_2,\ldots,x_n$. If the value x_1 occurs n_1 times, x_2 occurs n_2 times, ..., and x_n occurs n_n times, then:

- The total number of observations is $n=n_1+n_2+\cdots+n_n$
- The relative frequency $f_n(x_i)$, which corresponds to the empirical probability of the value x_i occurring, is given by:

$f_n(x_1) = \frac{n_1}{n}$; $f_n(x_2) = \frac{n_2}{n}$; $f_n(x_n) = \frac{n_n}{n}$ et $\sum_{i=1}^{n} f_n(x_i) = 1$

The distribution can be visualized using a histogram, a relative frequency curve, or a cumulative frequency diagram. As the number of observations increases, the random fluctuations in these graphical representations decrease.

Example: The temperature of a furnace was measured every 30 minutes over a 10-hour period. The recorded values are summarized in the table below.

Table II. 1:Furnace Temperature Measurements

Measurement No.	Temperature (°C)	Measurement No.	Temperature (°C)
1	209	11	212
2	195	12	205
3	212	13	225
4	225	14	214
5	216	15	216
6	228	16	216
7	231	17	205
8	212	18	193
9	237	19	220
10	200	20	230

You must begin by ordering these values and dividing them into groups to determine the distribution frequency and graphically represent the distribution:

Table II. 2:Temperature Distribution Frequency Table

Group Intervals (°C)	Observations in Group	Relative Frequency	Cumulative Frequency
190-199	2	0.1	0.1
200-209	4	0.2	0.3
210-219	7	0.35	0.65
220-229	4	0.2	0.85
230-239	3	0.15	1
Total	**20**	**1**	

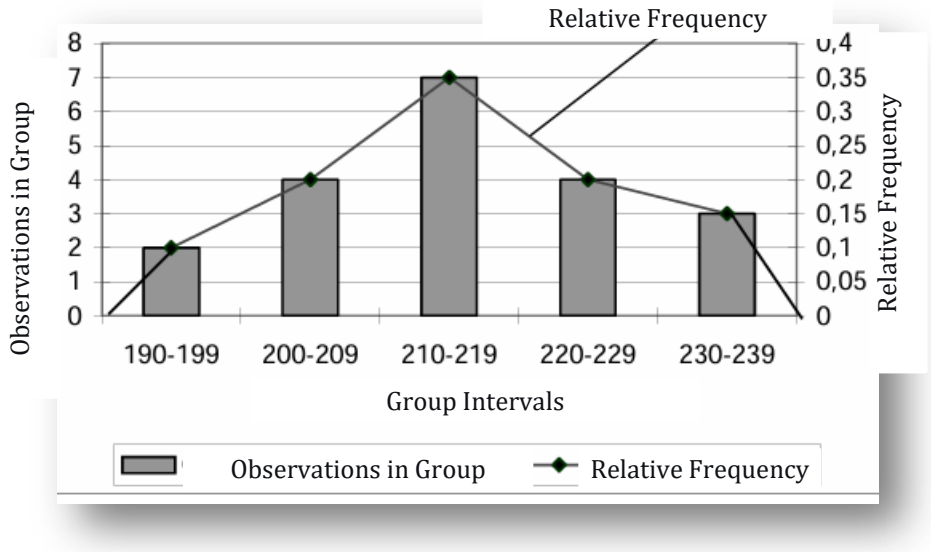


Fig.II. 2: Histogram and Relative Frequency Curve

A cumulative frequency diagram can also be represented:

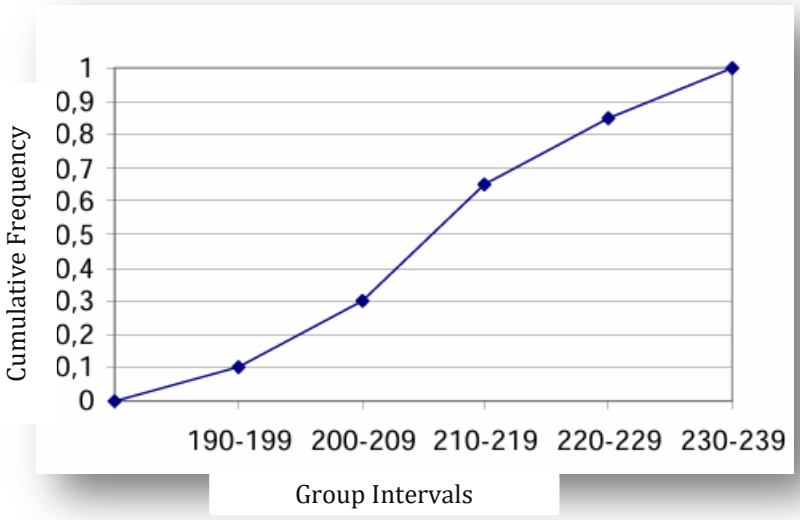


Fig.II. 3:Cumulative Frequency Diagram

II.2.2 Measures of Central Tendency of a Distribution

II.2.2.1 The Mean

When a measurement of the same quantity X is repeated n times, yielding results $x_1, x_2, \ldots, x_n$, the mean value $\bar{x}$ is defined by the formula: $\bar{X} = \frac{\sum_{i=1}^{n} X}{n}$

The mean is the most commonly used measure of central tendency. The value of $\bar{x}$ approaches the true value of the quantity as the number of measurements increases.

II.2.2.2 The Median

The median represents the central value within an ordered dataset.

II.2.2.3 The Mode

The mode is the most frequently occurring value in a dataset, corresponding to the peak of the relative frequency distribution.

Note:

In the example we presented, the three measures of central tendency yield different results:

- Mean: 215.05 °C
- Median: 215 °C
- Mode: 214.5 °C

These three values do not coincide because the number of measurements is limited and the distribution is non-Gaussian.

II.2.3 Measures of Distribution Dispersion

Two different datasets can share the same mean while having different levels of dispersion.

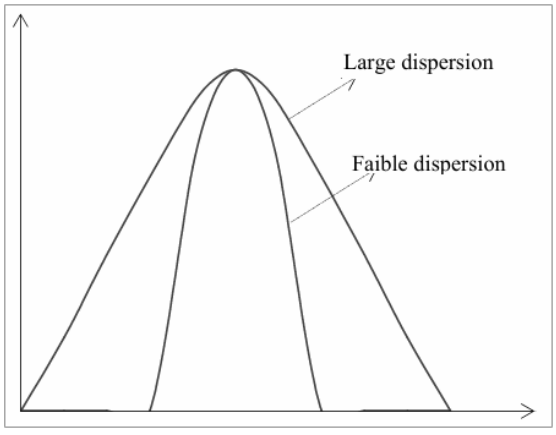


Fig.II. 4:different levels of dispersion

There are several measures of dispersion:

II.2.3.1 The Range

$$R = X_{max} - X_{min}$$

II.2.3.2 The Mean Deviation

$$d_x = \frac{1}{n}(\sum_{i=1}^{n}|X_i - \bar{X}|)$$

II.2.3.3 The Variance

$$\sigma^2 = \frac{1}{n-1}(\sum_{i=1}^{n}|X_i - \bar{X}|^2)$$

II.2.3.4 The Standard Deviation

This is the most widely used measure of dispersion: $\sigma = \left[\frac{1}{n-1}(\sum_{i=1}^{n}|X_i - \bar{X}|^2)\right]^{1/2}$

II.2.3.5 The Coefficient of Variation

This is a normalized parameter that indicates data variability relative to the mean value.

$$C_V = \frac{\sigma}{X}\ 100(\%)$$

II.2.3.6 The Standard Error of the Mean

The standard error of the mean is given by:

$S_x = \frac{\sigma}{\sqrt{n}}$, $S_x \to 0$ when $n \to 0$

When random errors affecting measurements are independent, the probability distribution of measurement results follows the normal distribution, also known as the Gaussian distribution.

The probability $P(x_1, x_2)$ of obtaining a measurement result between two values $x_1$and x_2 can be expressed as:

$P(x_1, x_2) = \int_{x_1}^{x_2} P(x)dx$ where p(x) is the probability density function for the measured value x.

For the Gaussian distribution:

$$P(x) = \left[\frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(X_i - \bar{X})^2}{2\sigma^2})\right]$$

- The most probable value of x*x* is the mean
- The probability of a measurement result falling within specified limits is:

- P($\bar{X}$±σ)=68.27%
- P($\bar{X}$±2σ)=95.45%
- P($\bar{X}$±3σ)=99.73

Typically, an uncertainty equal to 3 times the standard deviation (3σ) is used.

II.3 Total Errors in a Measurement System

A measurement instrument often consists of multiple components, each subject to systematic and random errors. We can evaluate errors from each component and combine them to determine the total system error.

II.3.1 Error of a Product

Consider a quantity X obtained by measuring Y and Z such that $X=Y\cdot Z$, with Y and Z being positive numbers. If measurement of Y gives $y\pm dy$ and measurement of Z gives $z\pm dz$:

$(y-dy)(z-dz)<X<(y+dy)(z+dz)$

or:

$$(y-dy)(z-dz)=yz-ydz-zdy+dydz=yz\left[1-\left(\frac{dz}{z}+\frac{dy}{y}-\frac{dydz}{yz}\right)\right]$$

and

$$(y+dy)(z+dz)=yz+ydz+zdy+dydz=yz\left[1+\left(\frac{dz}{z}+\frac{dy}{y}+\frac{dydz}{yz}\right)\right]$$

Neglecting second-order errors (since measurement errors are typically small, around 1-2%):

$$X=yz\pm yz\left(\frac{dz}{z}+\frac{dy}{y}\right)\rightarrow\frac{dX}{X}=\frac{dz}{z}+\frac{dy}{y}$$

For products, relative errors add up.

II.3.2 Error of a Quotient

Consider a quantity X obtained by measuring Y andZZ such that X=ZY, with Y and Z being positive numbers.

If measurement of Y gives $y\pm dy$ and measurement of Z gives z±dz:

$$\frac{y-dy}{z+dz} < X < \frac{y+dy}{z-dz}$$

or :

$$\frac{y-dy}{z+dz} = \frac{(y-dy)(z-dz)}{(z+dz)(z-dz)} = \frac{yz\left(1-\left(\frac{dy}{y}+\frac{dz}{z}-\frac{dydz}{yz}\right)\right)}{z^2-dz^2}$$

And

$$\frac{y+dy}{z-dz} = \frac{(y+dy)(z+dz)}{(z-dz)(z+dz)} = \frac{yz\left(1+\left(\frac{dy}{y}+\frac{dz}{z}+\frac{dydz}{yz}\right)\right)}{z^2-dz^2}$$

Neglecting second-order errors:

$$X = \frac{y}{z} \pm \frac{y}{z}\left(\frac{dz}{z}+\frac{dy}{y}\right) \Rightarrow \frac{dx}{x} = \frac{dz}{z}+\frac{dy}{y}$$

For quotients, relative errors add up.

II.3.3 Error of a Sum

Consider a quantity *X* obtained by measuring *Y* and *Z* such that *X*=*Y*+*Z*, with Y*Y* and *Z* being positive numbers. If measurement of *Y* gives *y*±*dy* and measurement of *Z* gives *z*±*dz*:

$$(y\text{-}dy)+(z\text{-}dz)<X<(y+dy)+(z+dz)$$

$$X=(y+z) \pm (dy \pm dz) \Rightarrow dx=dy+dz$$

For sums, absolute errors add up.

Consider a quantity *X* obtained by measuring *Y* and *Z*, where X=Y−Z, and both *Y* and *Z* are positive numbers.

If the measurement of *Y* yields *y*±*dy* and the measurement of *Z* yields *z*±*dz*, then the range of possible values for *X* is given by:

$$(y-dy)-(z+dz)<X<(y+dy)-(z-dz)$$

This simplifies to:

$$X=y-z\pm(dy+dz) \Rightarrow dx=dy+dz$$

In the case of a difference, the absolute errors add up.

II.4 Linear Regression

Regression analysis provides a statistical approach to correlate experimental data that depend on several measured variables. If a variable y*y*, which describes the behavior of a process and depends on several independent variables $x_1, x_2, \ldots, x_n$, is measured, the **least squares method** can be used to fit a straight line through scattered data points:

$Yi=mxi+b$

where $yi=f(xi)$.

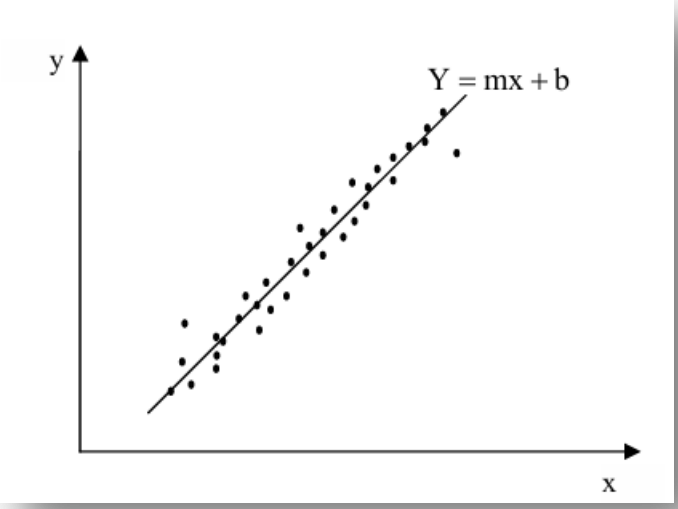


Fig.II. 5: Linear Regression

The parameters m*m* and b*b* are determined such that they minimize the deviation between the data points and the regression line:

$\Delta^2=\sum(\boldsymbol{y_i} - \boldsymbol{Y_i})^2$

By setting the partial derivatives to zero:

$\frac{\partial \Delta^2}{\partial m}=0$ and $\frac{\partial \Delta^2}{\partial b}=0$

we obtain:

$$m = \frac{\sum x \sum y - n \sum xy}{(\sum x)^2 - n \sum x^2} \text{ and } b = \frac{\sum y - m \sum x}{n} = \frac{\sum x \sum xy - n \sum x^2 \sum y}{(\sum x)^2 - n \sum x^2}$$

Interpretation of Dispersion and Correlation

The dispersion of *y* serves as an indicator of the suitability of regression analysis:

- If the dispersion is **low**, regression analysis is **well-suited** to describe the variation of *y*.
- If the dispersion is **high**, regression analysis is **not appropriate**.

The Correlation Coefficient (ρ)

The square of the correlation coefficient, ρ^2, quantifies the strength of the linear relationship:

$$\rho = 1 - \frac{n-1}{n-2}\frac{[y^2] - m[xy]}{[y^2]}$$

With $[y^2] = (\sum y^2) - \frac{(\sum y)^2}{n}$; and $[xy] = \sum xy - \frac{(\sum x)(\sum y)}{n}$

- If ρ^2=1, variables *x* and *y* are perfectly correlated.
- If ρ2=0, variables *x* and *y* are not correlated.

II.5 Trueness, Accuracy and Precision

- **Precision:** Low random error. Measurements are repeatable and closely grouped (low standard deviation), regardless of how close they are to the true value.
- **Trueness:** Low systematic error (bias). The average of the measurements is very close to the true value.
- **Accuracy:** The combination of high precision and high trueness. Measurements are both repeatable and correct, falling very close to the true value. It is often expressed as a margin of error or uncertainty interval around a measured result.

Precision + Trueness ⇔ Accuracy

Large systematic errors and low random errors: a precise but not true instrument. (Fig.II.6a): *The measurements are consistent and closely grouped (precise) but are consistently offset from the true value (not true).*

Low systematic errors and large random errors: a true but not precise instrument. (Fig.II.6b): *Explanation: The average of the measurements is close to the true value (true), but the individual measurements are scattered and not repeatable (not precise).*

Large systematic and random errors: an instrument that is neither true nor precise, (Fig.II.6c)*: The measurements are scattered (not precise) and their average is far from the true value (not true).*

Low systematic and random errors: a true and precise instrument, therefore ACCURATE, (Fig.II.6d): *Explanation: The measurements are closely grouped around the true value, meaning the instrument is both true and precise, which defines accuracy.*

Table II. 3: Summary Table of Error

Systematic Error (Trueness)	Random Error (Precision)	Result
Large	Low	Precise, but not True
Low	Large	True, but not Precise
Large	Large	Neither True nor Precise
Low	Low	True and Precise

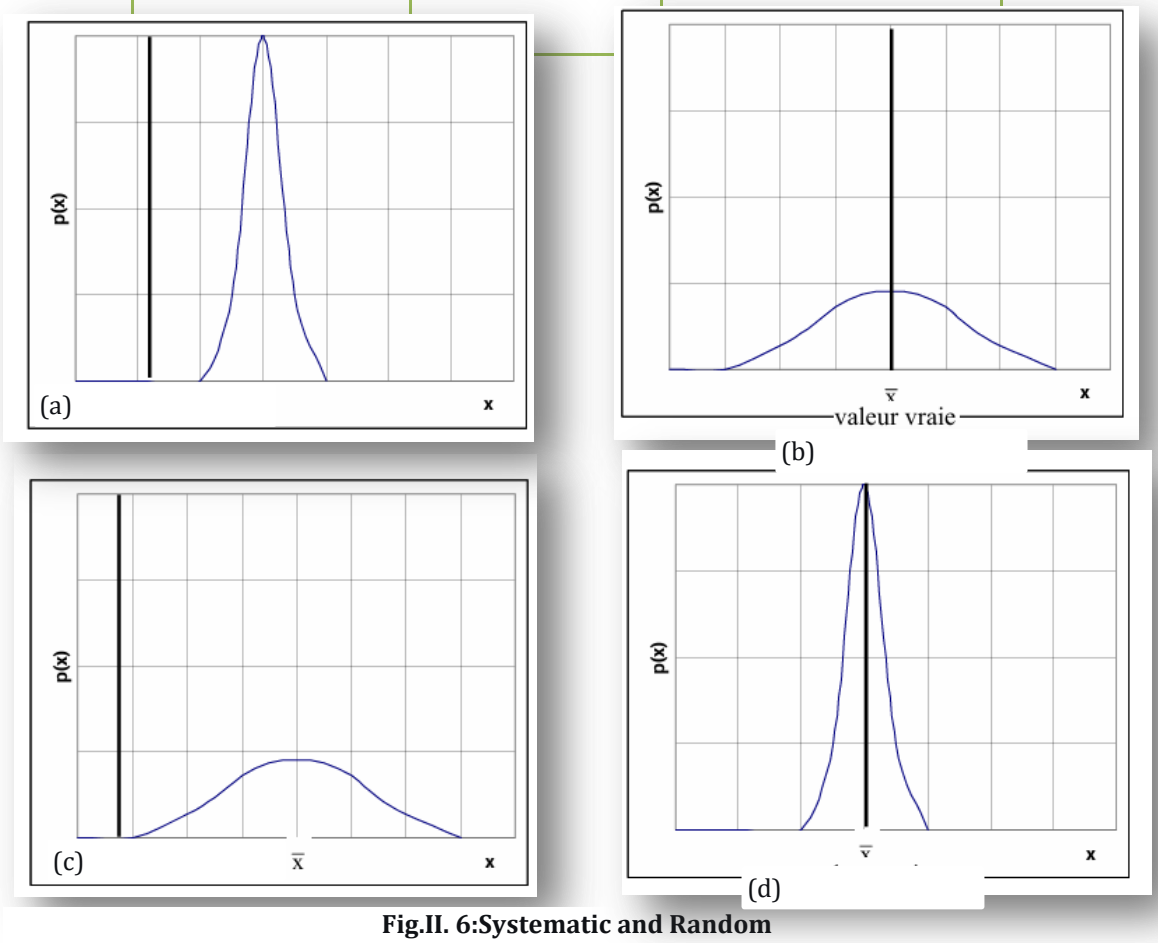


Fig.II. 6:Systematic and Random

Chapter III. Performance of Measurement Systems

Chapter III: Performance of Measurement Systems

III.1 The Ideal Measurement System

An ideal measurement system is defined by an output signal that is a perfect, linear function of the quantity being measured (the measurand). This means the output signal replicates the input signal exactly, irrespective of how the input may change over time.

Since such an ideal system is a theoretical construct and does not exist in practice, it is used as a reference model. The performance of any real measurement system is assessed by comparing its output to the output that an ideal system would produce under the same operating conditions.

This evaluation is divided into two categories:

- **Static Performance:** This is determined by applying a constant or steady-state input signal. The system's output is then compared against the ideal, unchanging response.
- **Dynamic Performance:** This is determined by applying a time-varying input signal. The system's ability to follow these changes is evaluated by comparing Its Output To The Ideal Response.

III.2 Static Characteristics Of A Sensor

III.2.1 Measurement Range and Span

- The **Measurement Range** specifies the complete interval of values of the measurand over which the instrument is designed to operate and provide correct measurements. It is defined by its lower and upper limits.
- The **Span** is a numerical value representing the total width of the measurement range. It is calculated as the difference between the maximum value and the minimum value of the range.
- For instruments that have an adjustable measurement range, the maximum value of the span is specifically designated as the **Full Scale (FS)** value.

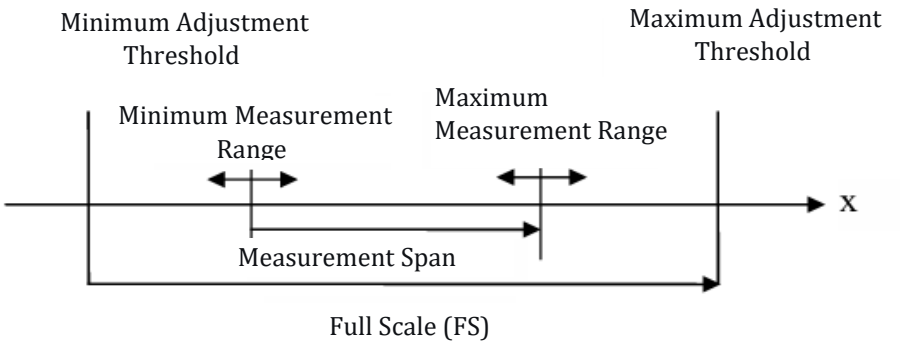


Fig.III. 1:Measurement Range and Span

III.2.2 Static Calibration Curve

Each measuring instrument possesses its own unique static calibration curve. This curve is used to convert a raw, unprocessed measurement reading into a corrected value. The calibration process involves subjecting the sensor to a series of known, constant input values (the measurand) and precisely recording the corresponding steady-state electrical output values once the system has stabilized. The resulting data is typically presented in the form of a table or a graphical plot (a calibration curve or chart).

III.2.3 Accuracy

Accuracy refers to the closeness of agreement between a measured value and the true value of the measurand. It can be quantified relative to the true value or to the instrument's full-scale (FS) reading, and is usually expressed as a percentage:

- **Accuracy relative to the true value:**
  Accuracy (%) = [(Measured Value - True Value) / True Value] × 100
- **Accuracy relative to full scale:**
  Accuracy (%) = [(Measured Value - True Value) / Full Scale Value] × 100

III.2.4 Bias (Zero Drift)

Bias, or zero drift, is a persistent, constant deviation or offset in the instrument's output signal over a period of time. This shift can be caused by various factors, including changes in environmental conditions (such as temperature or humidity), fluctuations in

electrical supply characteristics, or the natural aging and degradation of the sensor components

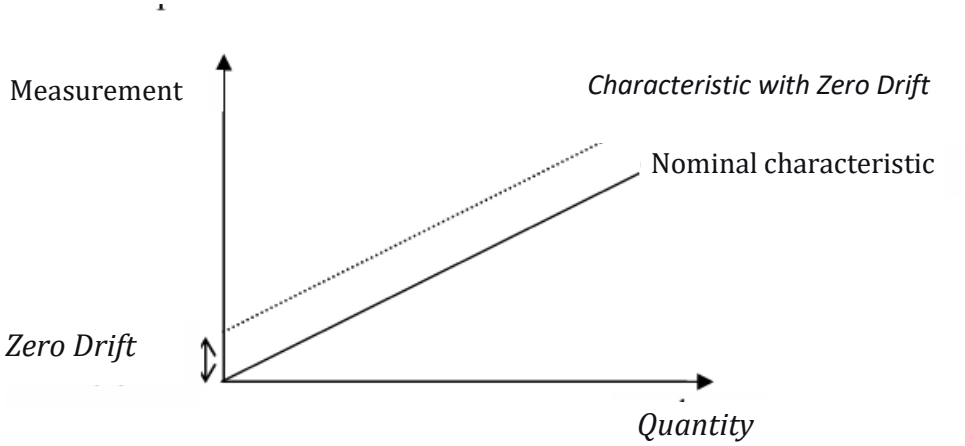


Fig.III. 2:Bias (Zero Drift)

III.2.5 Linearity Error

This is the maximum deviation of a measuring instrument's output from its calibration curve. It indicates that the sensor's characteristic response is not a straight line.

III.2.6 Sensitivity

Sensitivity is a critical factor in sensor selection. It is defined as the ratio of the change in the output signal to the change in the input measurand, around a specific operating point. Under specified conditions, the manufacturer typically provides this value. It allows the user to estimate the sensor's response magnitude based on the expected variations in the measurand, ensuring the entire measurement chain meets the required conditions.

Let m be the measurand and s be the output signal from the sensor, such that s = f(m). The sensitivity S at a specific point m is the derivative:

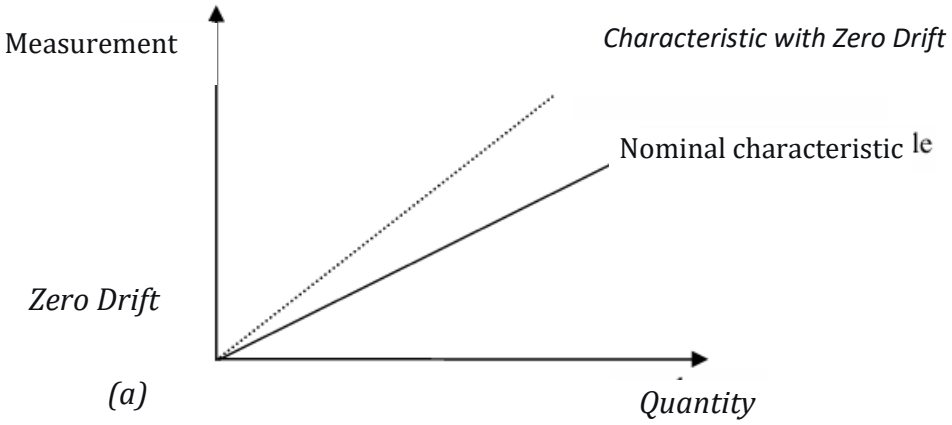
$S = ds / dm$

If the function is linear, the sensor's sensitivity is constant and equal to the slope of the calibration curve:

$S = \Delta s / \Delta m$

III.2.7 Sensitivity Drift

This is the variation in a sensor's sensitivity due to changes in environmental conditions. It exhibits a linear dependence on the measured value. A measuring instrument can experience zero drift, sensitivity drift, or both simultaneously.



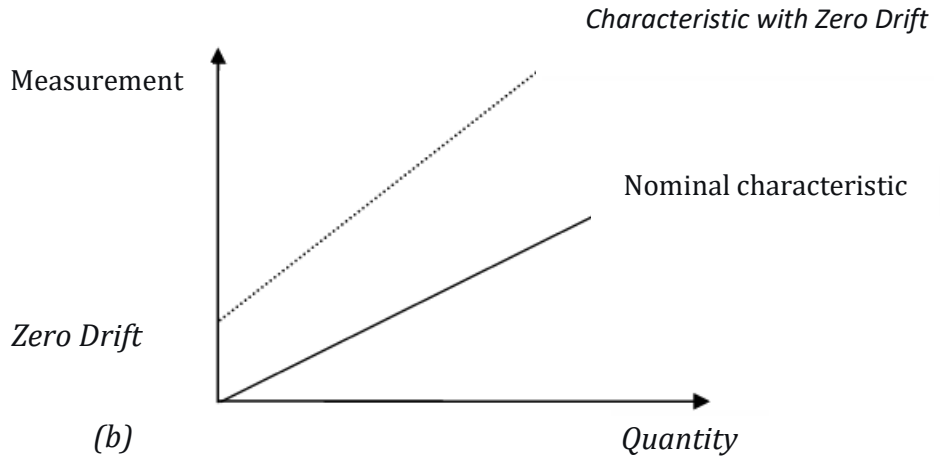


Fig.III. 3:Sensitivity Drift

III.2.8 Resolution

The resolution of a measuring instrument is the smallest change in the measurand that produces a detectable change in the sensor's output. It represents the minimum input variation the system can discern. For digital instruments, resolution is defined as the ratio of the measurement range to the number of measurement points.

III.2.9 Repeatability (Precision)

Repeatability describes the closeness of agreement between successive measurements of the same measurand, carried out under identical conditions (same procedure, operator, instrument, location, and within a short period of time).

Note: Good repeatability does not guarantee sensor accuracy, only that the process is consistent. Conversely, poor repeatability typically indicates a low-quality sensor, as influencing variables are unlikely to change significantly over a short duration.

III.2.10 Reproducibility

This concept should not be confused with repeatability. Reproducibility refers to the closeness of agreement between measurements of the same measurand performed under different conditions (e.g., different instruments, operators, time, or location). Reproducibility is a more significant indicator of sensor quality than repeatability, as it accounts for various potential error sources like instrument calibration drift or operator skill.

III.3 Dynamic Characteristics of a Sensor

The dynamic characteristics of a measuring instrument describe its behavior in a transient state. The dynamic performance of such a system is evaluated based on its response to standard test input signals: step, ramp, and sinusoidal.

III.3.1 Zero-Order System

This is an ideal measurement system where the output is directly proportional to the input. It is governed by the equation:

$s = K \cdot m$

where:

-K is the system sensitivity,

-s is the output quantity,

-m is the input quantity (measurand).

In practice, a potentiometer, which outputs a voltage proportional to the cursor displacement, closely approximates a zero-order system.

III.3.2 First-Order System

Its behavior is governed by a first-order differential equation of the form:

$A \cdot (ds/dt) + B \cdot s = m(t)$

where A and B are constants and m(t) is the value of the measurand at time t.

III.3.2.1 Response to a Step Input

When the measurand undergoes an abrupt change (a step input), the system's transition to a steady-state is preceded by a transient response. Analyzing the step response of a first-order system allows for the determination of its time constant.

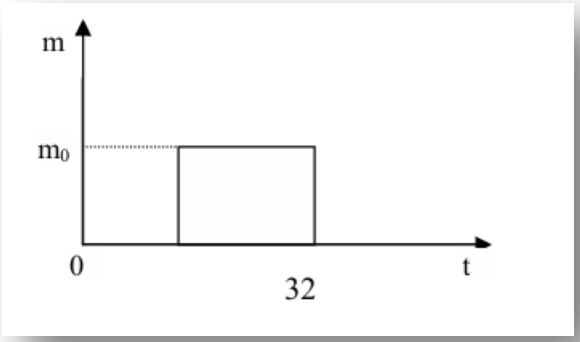


Fig.III. 4:Response to a Step Input

For a step input of the measurand, and assuming an initial output of s=0 at t=0, the solution to the previous differential equation is:

$$s = s_0\left\{1 - exp\left(-\frac{1}{\tau}\right)\right\}$$

Where:

- $\mathbf{s_0}$ is the steady-state output value (s_0 = M/B, where M is the magnitude of the step input).
- $\boldsymbol{\tau}$ is the system's time constant (τ = A/B).

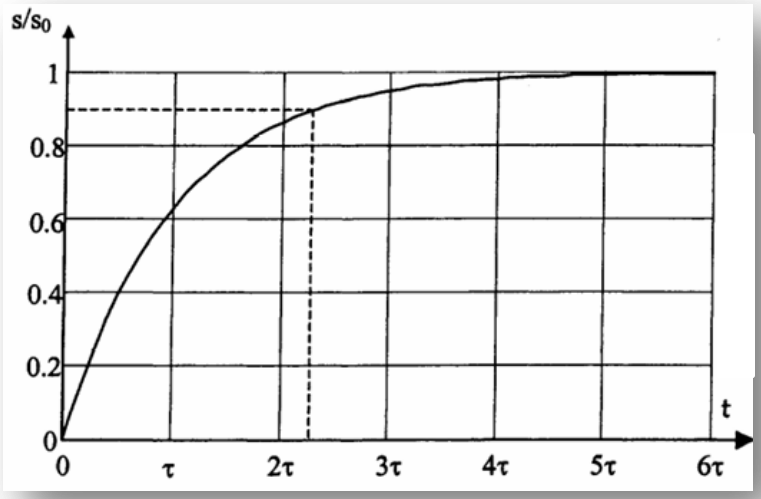


Fig.III. 5 The time response of a first-order system to a step input of the measurand.

The parameter used to characterize a system's speed is the **response time, t_r**. This is the time interval after a sudden change in the measurand (a step input) until the output variation remains within a conventionally defined tolerance band, **ε(%)**, of its final value. Therefore, a response time must always be specified with its corresponding **ε(%)** tolerance.

The response time t_r can be determined either graphically from the step response curve or calculated using the mathematical expression:

$t_r(\varepsilon == 2.3\ (2 - log\varepsilon)\tau$

The response time for different tolerance values ε(%) is given in the table below:

Table III. 1:The response time for different tolerance values ε(%)

Tolerance ε(%)	Response Time $t_r(\varepsilon)$
10	2.3τ
5	3.0τ
2	3.9τ
1	4.6τ
0.1	6.9τ

III.3.2.2 Response to a Sinusoidal Signal

The response of a system to a sinusoidal signal characterizes its **dynamic sensitivity**.

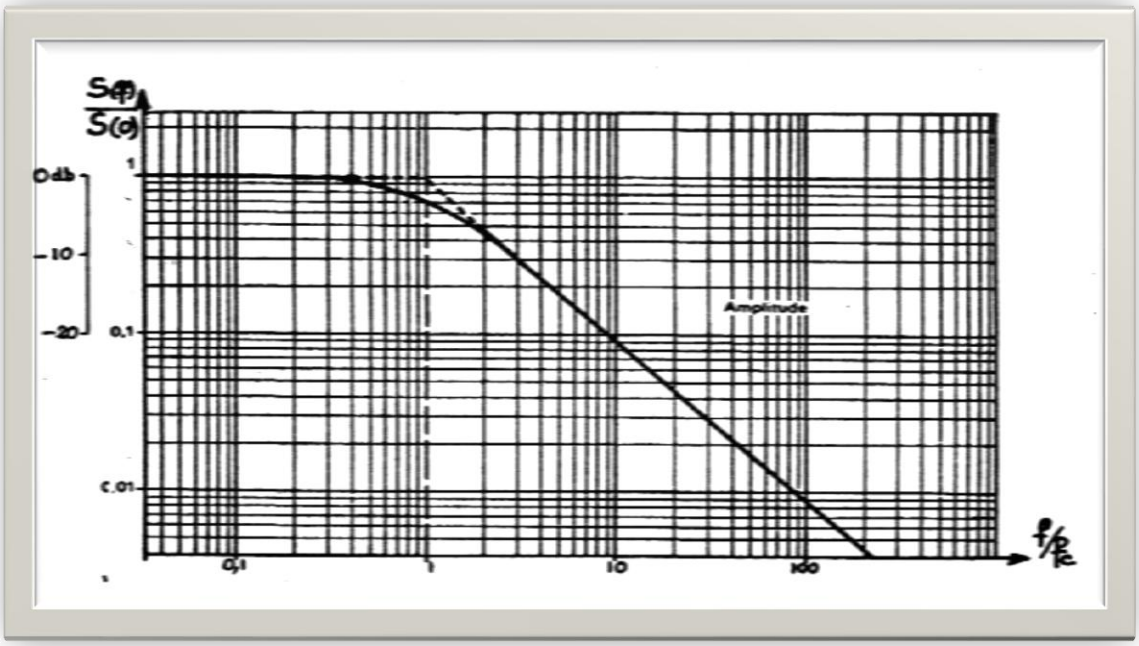
For a sinusoidal measurand defined as $m(t) = m_t \cos(\omega t)$, where:

- m_t is the signal amplitude,
- ω is the angular frequency (related to the period T by $T = 2\pi/\omega$ and to the frequency f by $f = \omega/2\pi$),

the sensor's output response takes the form:
$s(t) = s_t \cos(\omega t + \psi)$

Starting from the governing differential equation $A\frac{ds}{dt} + Bs = m(t)$, the **dynamic sensitivity** S(f) and the **phase shift** ψ are given by: $S(f) = s_1/m_1 = 1/B\left[\frac{1}{\sqrt{1+(\frac{f}{f_c})^2}}\right]$;and $\psi = -Arc\ tan\left(\frac{f}{f_c}\right)$, Here, f_c is the **cut-off frequency**, defined as:$f_c = \frac{B}{2\pi A}$

Key Insight:
When the frequency f approaches zero ($f \to 0$), the dynamic sensitivity S(f) converges to the **static sensitivity** $S(0) = 1/B$.



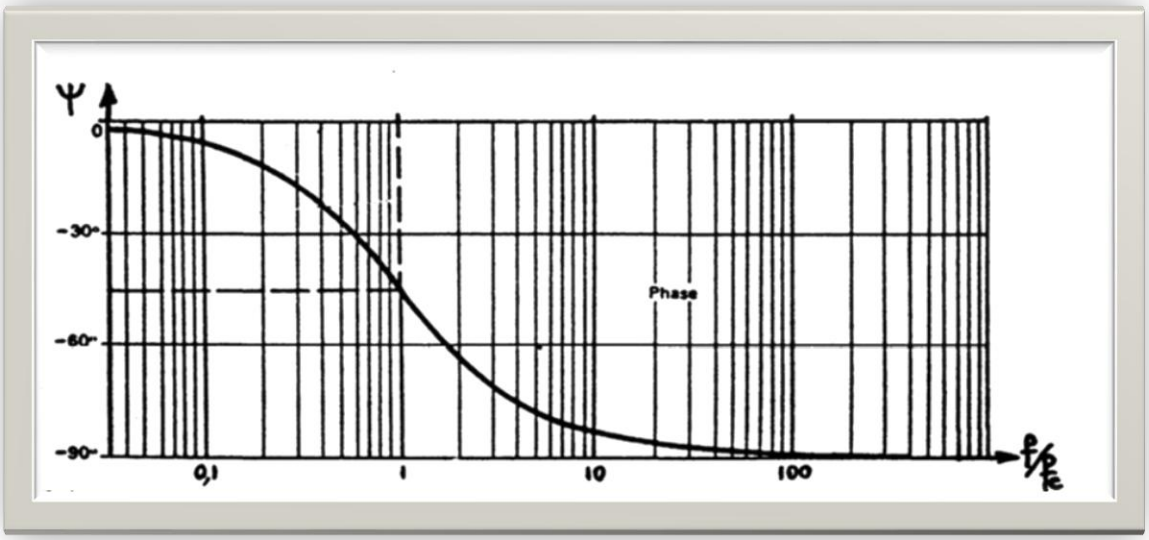


Fig.III. 6:Dynamic sensitivity.

III.3.3 Second-Order Systems

The behavior of a second-order system is governed by a second-order differential equation of the form:

$$A\frac{d^2s}{dt^2} + B\frac{ds}{dt} + C.s = m(t)$$

where A, B, and C are constants.

This equation can be rewritten in the standard form:

$$\frac{d^2s}{dt^2} + \frac{B}{A}\frac{ds}{dt} + \frac{C}{A}.s = \frac{1}{A}m(t)$$

A more common and useful representation is:

$$\frac{d^2s}{dt^2} + 2\omega_0\zeta\frac{ds}{dt} + \frac{C}{A}.s = \frac{1}{A}m(t)$$

Where : **ζ** is the **damping ratio** of the system, defined as $\zeta = \frac{B}{2\sqrt{CA}}$

ω_0 is the **natural (angular) frequency** of the system, defined as $\omega_0 = \sqrt{\frac{C}{A}}$

f_0 is the **natural frequency**, defined as $f_0 = \frac{\omega_0}{2\pi}$

K is the static sensitivity (often 1/A in this formulation).

III.3.3.1 Response to a Step Input

When a step input m_0 is applied to the system, the differential equation becomes:

$$A\frac{d^2s}{dt^2} + B\frac{ds}{dt} + C.s = m_0$$

Assuming initial conditions of s = 0 and ds/dt = 0 at t = 0, the shape of the system's response (output s(t)) depends critically on the value of the **damping ratio (ζ)** and the product ω_0 t.

The temporal response of a second-order system to a step input, for different values of the damping ratio ξ, is characterized in Fig III.7:

To minimize the sensor's **response time** (which is inversely proportional to ω_0), the optimal value for the damping ratio **ζ is between 0.6 and 0.8**. This corresponds to

an **underdamped** system ($\xi < 1$), which exhibits a transient oscillatory response that settles quickly to the final value without excessive overshoot.

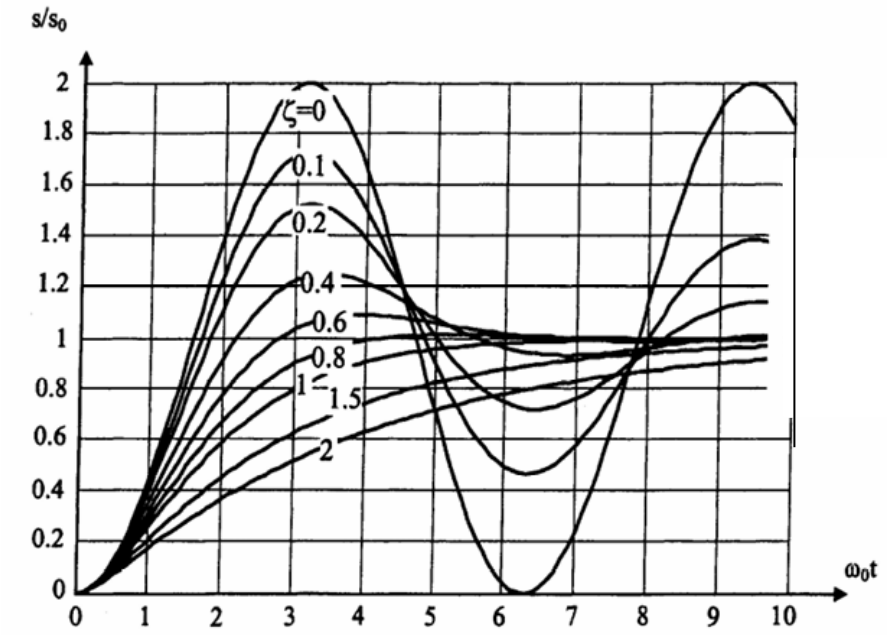


Fig.III. 7: The time response of a second-order system to a step input, for different values of the damping ratio ζ.

III.3.3.2 Response to a Sinusoidal Signal

When the measurand is sinusoidal, the frequency-dependent sensitivity $S(f)$ is given by: $S(f) = s_1/m_1 = S(0)\left[\frac{1}{\sqrt{\left(1+(\frac{f}{f_0})^2\right)^2+4\zeta\left(\frac{f}{f_0}\right)^2}}\right]$

where $S(0)=1/C$ represents the **static sensitivity**.

The frequency-dependent phase shift $\psi(f)$ is expressed as:

$\psi(f) = -\text{Arctan}\left[\frac{2\zeta(\frac{f}{f_0})}{\left(1-\frac{f}{f_0}\right)^2}\right]$.

The frequency response of a second-order system for different values of the damping ratio ζ is illustrated in the Figs. III.(8-9). The most uniform frequency response was obtained for $\zeta = 0.7$. For damping ratio values between 0.6 and 0.7, the signal transit time remains most constant across an extended frequency range.

A sensor damping coefficient between 0.6 and 0.7 ensures optimal performance by providing:

- A nearly flat amplitude response across a relatively wide frequency band
- Minimal phase distortion
- The fastest possible response time

This optimal damping range represents the best compromise between frequency response uniformity, signal fidelity, and system responsiveness

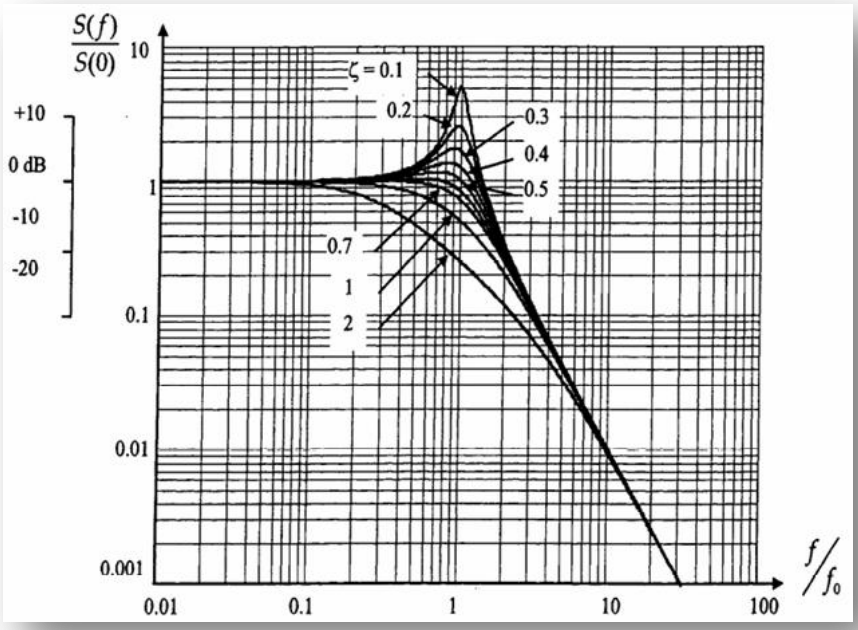


Fig.III. 8:The frequency response of a second-order system for different values of the damping ratio ζ

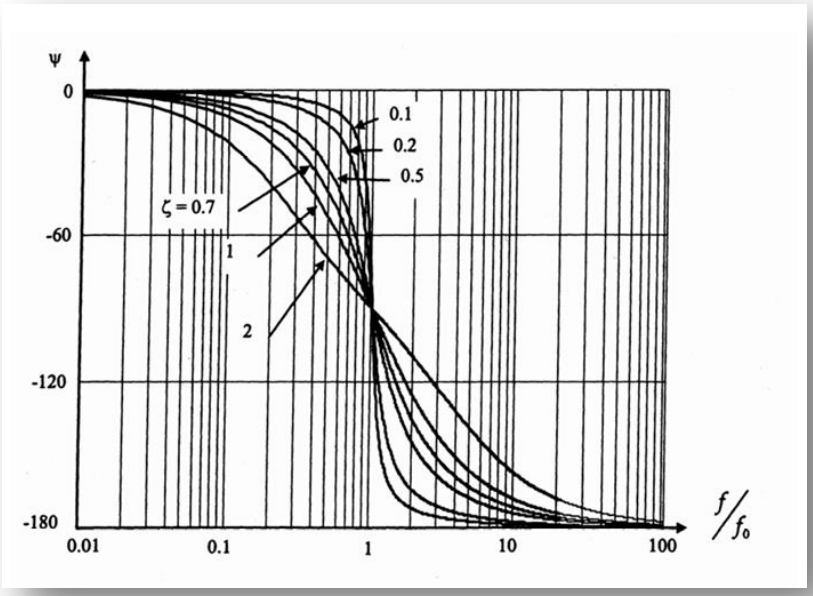


Fig.III. 9:The frequency-dependent phase shift

Chapter IV. Expansion Thermometers

Chapter IV: Expansion Thermometers

IV.1 Introduction to Temperature Measurement

Temperature represents a fundamental physical property distinct from extensive quantities like length or mass. While extensive quantities can be directly quantified through comparison to standardized references, temperature is an intensive property - its physical significance cannot be established through simple multiplicative operations.

As an abstract thermodynamic parameter, temperature is operationally defined through observable changes in material properties. Various physical characteristics of substances exhibit predictable variations with temperature fluctuations, enabling multiple measurement approaches. This chapter specifically examines thermal expansion as the fundamental principle for temperature measurement.

Key Characteristics:

- Thermal expansion provides a reversible, reproducible phenomenon for temperature quantification
- The expansion behavior manifests differently across the three states of matter
- Three primary thermometer types utilize this principle: liquid-expansion, gas-expansion, and solid-expansion devices

IV.2 Liquid-in-Glass Thermometers

IV.2.1 Fundamental Design and Operation

The liquid-in-glass thermometer employs a simple yet effective design based on volumetric thermal expansion:

Structural Components:

- **Reservoir/Bulb:** Contains the primary liquid volume
- **Capillary Tube:** Precision-bore glass tubing (typically 0.2-0.3 mm diameter)
- **Safety Bulb:** Expansion chamber preventing pressure buildup
- **Graduated Scale:** Etched or mounted temperature reference

Operating Principle:

The thermometer utilizes the differential expansion between the working liquid and glass envelope. When temperature increases, the liquid expands significantly more than the glass container, causing the liquid column to rise in the capillary. Common working liquids include mercury (-39°C to 357°C) and alcohol (-115°C to 78°C), selected based on their thermal expansion coefficients and measurement ranges.

Measurement Characteristics:

- Direct visual reading through liquid meniscus alignment
- Limited to point temperature measurements
- Requires sufficient immersion depth for accuracy
- Response time depends on bulb size and thermal conductivity

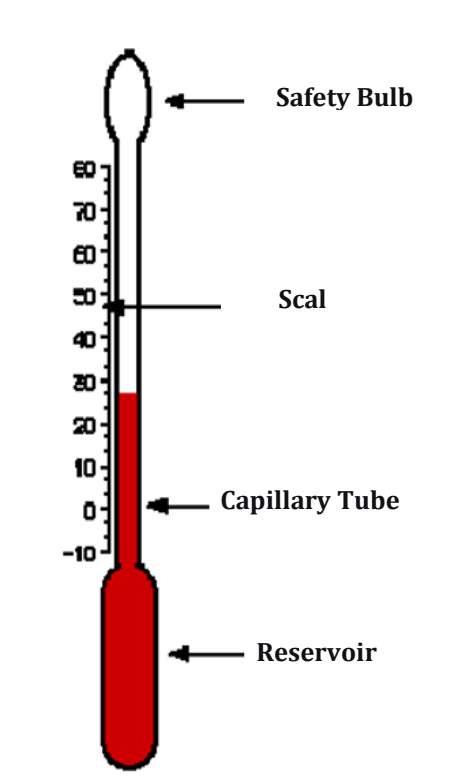


Fig.IV. 1: Gglass thermometer

IV.2.2 Volume-Temperature Relationship

The mathematical relationship describing the volume variation of the liquid with temperature is given by:

$V=V_0(1+aT)$

where:

- V_0 = liquid volume at 0°C
- V = liquid volume at T(°C)
- a = coefficient of volumetric expansion of the liquid ($°C^{-1}$)

IV.2.3 Thermometric Liquids

While mercury is the most commonly used liquid, several other liquids are also employed:

Table IV. 1:Several liquids employed in Thermometric

Liquid	Temperature Range (°C)	Coefficient a ($°C^{-1}$)
Pentane	-200 to +20	-
Ethyl Alcohol	-110 to +100	1.17
Toluene	-90 to +100	1.03
Mercury	-38 to +650	0.182

Important Notes:

- The space above the liquid column may be evacuated, but to prevent column separation and enable high-temperature measurements, it's typically filled with an inert gas (nitrogen or argon) under controlled pressure.
- For a mercury thermometer designed to measure 600°C, the nitrogen pressure reaches approximately 20 bars.
- The expansion chamber (safety bulb) serves to mitigate excessive pressure variations.

IV.2.4 Envelope Material Selection

The choice of glass material for the thermometer envelope depends on the intended temperature range:

- Up to 450°C: Jena glass
- Up to 630°C: Supremax glass
- Up to 1000°C: Fused silica

IV.2.5 Emergent Stem Correction

Temperature measurement requires correction when the thermometer cannot be fully immersed. As shown in the accompanying figure, in cases (a) and (c) the thermometric

liquid column is completely immersed in the medium whose temperature is being measured, allowing full liquid expansion to be measured accurately. When partial immersion occurs, an emergent stem correction must be applied to compensate for the unheated portion of the liquid column. However, in configuration (b), the liquid column is only immersed up to the graduation mark n within the medium of unknown temperature x. The portion of the column between graduations n and h is exposed to the ambient temperature T.

If we let v represent the volume corresponding to one graduation mark, the volume V of the emergent stem at ambient temperature is:

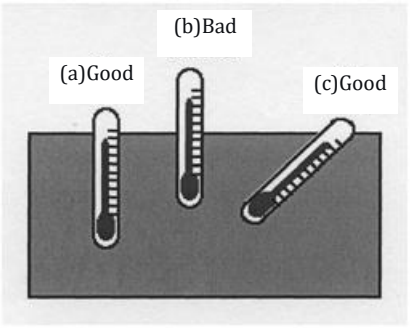
$$V=(h-n)v$$

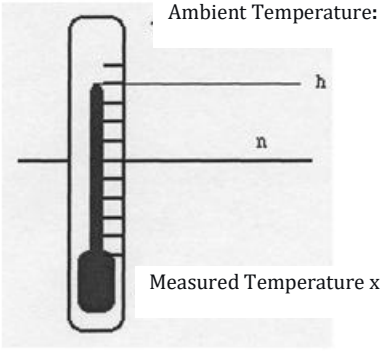
The required correction is therefore:

$$\Delta h=\Delta V/v=a(h-n)(x-T)$$

Practical Consideration:

In reality, the emergent stem is not entirely at ambient temperature. Standard practice applies **70% (7/10) of the calculated correction**. Given the inherent imprecision of this method, optimal thermometer positioning (configurations a or c) is recommended whenever possible.





Assessment of Liquid-in-Glass Thermometers:

These thermometers are valued for their simplicity and potential precision. However, they present two main limitations:

1. They have a relatively **slow response time**.

2. They require **direct visual reading**, necessitating operator presence and making them unsuitable for automated data logging, acquisition, long-term monitoring, or control applications. This second limitation is common to all expansion-based thermometers.

IV.3 Gas Expansion Thermometers

IV.3.1 Operating Principle

The principle is based on the Ideal Gas Law:

PV=nRT$PV=nRT$

where:

- P = Pressure (Pa)
- V = Volume (m^3)
- n = Number of moles of gas
- R = Ideal gas constant (8.31 $J \cdot mol^{-1} \cdot K^{-1}$)
- T = Absolute Temperature (K)

If a fixed quantity of gas is confined within a **constant volume (V)**, the pressure (P) developed by the gas becomes directly proportional to its absolute temperature (T). This is the fundamental principle of the gas expansion thermometer.

IV.3.2 Description and Components

A gas thermometer consists of the following key components:

1. ***Probe (A):*** A bulb that contains the thermometric gas and is placed in the measurement environment.

2. ***Capillary Connection Tube*****:** A narrow-bore tube (typically up to 100 meters long) connecting the probe to the measuring element.

3. ***Bourdon Tube (Spiral Gauge):*** A fixed-end tube (B) that flexes in response to pressure changes.

4. Indicator Dial**:** A graduated scale that displays the temperature corresponding to the pointer's position.

Operation: A temperature change at the probe alters the gas pressure. This pressure change is transmitted via the capillary to the Bourdon tube, causing its free end to deflect. This mechanical movement is converted into a rotary motion of a pointer over the calibrated dial.

Common Gases: Helium, Hydrogen, Nitrogen, and Carbon Dioxide.

Advantages and Disadvantages:

- *Advantage: High precision* (around 1% in industrial measurements). They are particularly suited for measuring very low temperatures (e.g., down to -268°C with Helium, -240°C with Hydrogen). Some, like the hydrogen thermometer, serve as primary standards for defining reference temperatures.
- *Disadvantage:* The probe tends to be bulky.

IV.3 Gas Thermometers

IV.3.1 Operating Principle

The fundamental principle is based on the Ideal Gas Law:

PV=nRT$PV=nRT$

where:

- P = Pressure (Pascals)
- V = Volume (m^3)
- n = Number of moles of gas
- R = Ideal gas constant (8.31 $J \cdot mol^{-1} \cdot K^{-1}$)
- T = Absolute temperature (Kelvin)

When a fixed quantity of gas is confined within a constant volume enclosure, the pressure developed by the gas becomes directly proportional to its absolute temperature. This pressure-temperature relationship forms the operational basis of gas expansion thermometers.

IV.3.2 System Description and Operation

A gas thermometer system comprises three main components:

1. *Thermometric Probe (A):* A gas-filled bulb that serves as the temperature-sensing element
2. *Capillary Connection Tube***:** A small-diameter linking tube (maximum length: 100 meters)
3. *Bourdon Tube Measurement System***:** A pressure-measuring element consisting of:
    - Fixed end (B) connected to the capillary

- Free end that moves in response to pressure changes
- Indicator needle and calibrated temperature dial

Operational Sequence:

- Temperature changes at the probe modify the gas pressure
- Pressure variations transmit through the capillary to the Bourdon tube
- The Bourdon tube's free end deflects proportionally to pressure changes
- Mechanical linkage converts this motion to indicator rotation
- Temperature reading displays on the calibrated scale

Common Thermometric Gases: Helium, Hydrogen, Nitrogen, Carbon dioxide

- *Performance Characteristics*

Advantages:

- High measurement precision (approximately 1% accuracy in industrial applications)
- Exceptional capability for cryogenic measurements:
  - Helium: down to -268°C
  - Hydrogen: down to -240°C
- Suitable as reference standards for temperature calibration (e.g., hydrogen thermometers)

Primary Limitation:

- Relatively large physical dimensions of the sensing probe

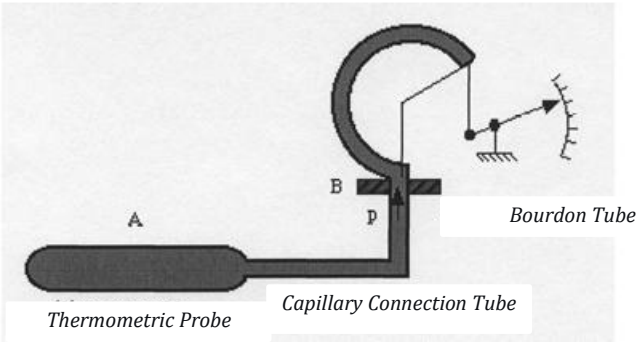


Fig.IV. 2:Gas Thermometers

IV.4 Vapor Pressure Thermometers

IV.4.1 Fundamental Principle

The vapor pressure of a pure liquid is defined as the pressure at which the liquid exists in equilibrium with its saturated vapor. For any given liquid, the vapor pressure depends solely on temperature. Therefore, measuring a liquid's vapor pressure provides an indirect method for temperature measurement through pressure detection.

The simplest implementation is the **single-fill vapor pressure thermometer** (shown in the schematic below). This system features:

- **Sensing Bulb:** Similar to a gas thermometer probe but with the capillary tube immersed within it
- **Capillary Connection Tube**
- **Pressure Measurement Element**

The entire system (bulb, capillary, and pressure gauge) is filled with a volatile liquid. When the sensing bulb is exposed to heat, partial vaporization occurs, establishing a liquid/vapor equilibrium that depends on the temperature. The system pressure rises to match the liquid's vapor pressure at that specific temperature, and this pressure is transmitted to the pressure gauge for measurement.

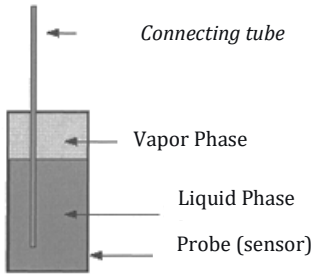


Fig.IV. 3:Vapor Pressure Thermometers (single-fill sensors)

Single-Fill Configuration:

*Important Limitation***:** Since vaporization always occurs at the hottest point in the closed system, single-fill sensors only function properly when the sensing bulb temperature

exceeds the ambient temperature. Otherwise, vaporization would occur in the pressure sensor itself, causing measurement errors.

Dual-Fill Configuration:

To overcome this limitation and enable temperature measurements both above and below ambient conditions, **dual-fill systems** are employed. These systems feature:

- *Volatile Liquid:* Located in the central portion of the sensing bulb
- *Transmitter Liquid:* A non-evaporating liquid (oil or glycerin) filling the bottom of the bulb and the entire capillary-pressure sensor assembly

The transmitter liquid serves as a hydraulic medium to transfer the vapor pressure from the volatile liquid to the pressure gauge without itself evaporating, ensuring reliable operation across wider temperature ranges.

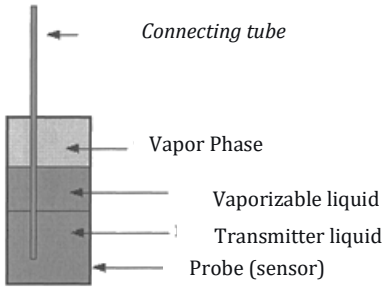


Fig.IV. 4: Vapor Pressure Thermometers (Dual-fill systems)

Advantage and Disadvantage of Liquid Thermometers

The primary advantage of these liquid-filled thermometers is their high sensitivity. A key disadvantage, however, is that the position of the probe relative to the sensor can influence the measurement accuracy.

IV.4.2 Filling Fluids and Their Operational Ranges

Different liquids are used as the thermometric fluid, each suitable for specific temperature and pressure ranges, as detailed in the table below:

Table IV. 2:Filling Fluids and Their Operational Ranges

Fluid	Temperature Range (°C)	Pressure Range (bar)
Ammonia	-20 to +60	2 to 25
Propane	0 to 100	5 to 45
Butane	+20 to +140	2 to 30
Ethyl Chloride	+30 to +180	2 to 50
Methyl Alcohol	+60 to +220	1 to 53
Benzene	+80 to +280	1 to 43

IV.5 Solid-Expansion Thermometers

IV.5.1 Operating Principle

This type of thermometer is based on the physical principle that a metal rod expands or contracts with changes in temperature. This relationship is defined by the formula:

$$L=L_0(1+\alpha T)$$

Where:

- L: Final length of the rod at temperature T (in °C)
- L_0: Original length of the rod at 0°C
- α: Coefficient of linear expansion for the metal (in $°C^{-1}$)

By measuring this change in length (L-L_0), the temperature can be determined.

The linear expansion of the metal can therefore be used as a thermometric property. This is the principle upon which solid-expansion thermometers are based. The most commonly used metals are platinum ($\alpha = 9\times10^{-6}\ °C^{-1}$), zinc ($\alpha = 30\times10^{-6}\ °C^{-1}$), and Invar, which is an alloy of iron and nickel ($\alpha = 2\times10^{-6}\ °C^{-1}$).

4.5.2 The Bimetallic Strip (Bi-metallic-strip thermometer)

The bimetallic strip thermometer is made of two strips of metals with very different coefficients of expansion, bonded together across their entire flat surface. The bonding of the two components must be done in such a way that the contact is perfect and the junction zone is mechanically as strong as each of the two strips.

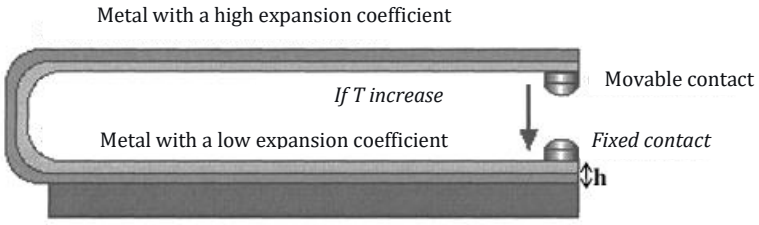
When the bimetallic strip is subjected to a temperature change, the differential expansion of the two strips creates internal stress, resulting in a curvature of the entire assembly.

- Metal with a high coefficient of expansion

- If T increases →
- Metal with a low coefficient of expansion
- Mobile Contact
- Fixed Contact

Metale à coefficient de delitation élevé (or faible) ; contact mobile ; contact fixe

Metal with a high (or low) expansion coefficient; movable contact; fixed contact.



If we denote h_1 as the thickness of the strip with the high coefficient of expansion α_1, and h_2 as the thickness of the strip with the low coefficient of expansion α_2, the radius of curvature of the bimetallic strip of thickness h (h=h_1+h_2) is given by:

$$\rho = \frac{h(3(1+r_h)^2 + (1+r_h r_e)({r_e}^2 - \frac{1}{r_h r_e}))}{6(\alpha_1 + \alpha_2)(1+r_h)\Delta T}$$

Where:

- r_h= h_2/h_1 is the thickness ratio
- r_e=E2/E1 is the ratio of the elastic moduli

The sensitivity of bimetallic strips is improved by minimizing the total thickness h and increasing the difference between α_1 and α_2, which means using metals with coefficients of expansion that are as different as possible.

Bimetallic strips are widely used as thermostats: when the desired temperature is reached, the fixed contact touches the mobile contact, which then acts as a current switch. This is the case, for example, with the thermostat in an iron.

How It Works:

1. When the temperature changes, both metal strips try to expand or contract.

2. The metal with the higher coefficient of expansion (α) will change its length more than the metal with the lower coefficient.
3. Because the two strips are bonded together, this differential expansion creates internal stress, forcing the entire strip to **bend into a curve**.
4. The strip will always bend **towards the side of the metal with the lower expansion coefficient**.

Key Design Factors:

The curvature of the strip for a given temperature change (ΔT) depends on a complex formula involving:

- The **thickness** of each metal strip (h_1, h_2).
- The **difference in their expansion coefficients** ($\alpha_1 - \alpha_2$).
- The **ratio of their elastic moduli** (E), which is a measure of the metals' stiffness.

Maximizing Sensitivity:

To make the thermometer more sensitive (i.e., to get more bending for a small temperature change), engineers:

1. **Maximize the difference** ($\alpha_1 - \alpha_2$) by choosing very dissimilar metals.
2. **Minimize the total thickness** (h) of the strip.

Primary Application: Thermostats

This bending motion is harnessed for practical use, most commonly in **thermostats**. One end of the strip is fixed, and the other end moves. A electrical contact is placed so that when the strip bends enough (at a specific temperature), it either makes or breaks a circuit, acting as an automatic switch. A common everyday example is the thermostat that controls the temperature of a clothes iron.

Chapter V. Electrical Thermometers

Chapter V: Electrical Thermometers

V.1 Introduction

Electrical thermometers offer significant advantages in terms of versatility and practicality. The electrical signal they produce can be easily transmitted and recorded, making them highly flexible for both industrial and laboratory use, all while maintaining excellent accuracy.

Operating Principle & Sensor Types

These devices work by exploiting the fact that the electrical resistance of a material changes predictably with temperature. This consistent relationship allows temperature to be determined by measuring electrical resistance. Based on the core material used, there are two main families of sensors:

1. **Resistance Temperature Detectors (RTDs)**: Use pure metals.
2. **Thermistors**: Use semiconductor materials.

The fundamental difference between them lies in how their resistance changes with temperature.

V.2 Resistance Temperature Detectors (Rtds)

V.2.1 Principle

In RTDs, the electrical resistance of a metal conductor increases reliably as the temperature rises. This effect is perfectly repeatable and reversible.

Defining the Relationship

The precise relationship between resistance (R) and temperature T in °C is given by a high-order polynomial:

$$R=R_0(1+aT+bT^2+cT^3)$$

Where:

- R_0 is the resistance of the sensor at 0°C.
- a,b,c are positive calibration coefficients specific to the metal used (e.g., platinum, nickel).

Once these coefficients are determined through precise calibration at known temperatures, any unknown temperature can be found simply by measuring the sensor's resistance.

Simplification for Small Temperature Ranges

For small temperature variations (ΔT) around a specific operating point (T), the relationship can be simplified to a linear approximation:

$$R(T+\Delta T) \approx R(T) \cdot (1+\alpha \Delta T)$$

Here, $\alpha\alpha$ (the thermal sensitivity at temperature T) is defined as:

$$\alpha = \frac{1}{R(T)} \frac{dR}{dT}$$

This linear model is often sufficient and useful for control systems and understanding behavior over limited ranges.

V.2.2 Metal Selection Criteria

Depending on the application field and the specific qualities required, resistors are made from platinum, nickel, and more rarely from copper or tungsten:

- **Platinum** can be obtained with very high purity (99.999%), which gives it very stable mechanical and electrical properties.
- **Nickel** has high thermal sensitivity, but its much less stable electrical and mechanical properties limit its use to temperatures below 250°C.
- The advantage of **copper** is the linearity of its resistance variation law, but it cannot be used beyond 180°C for stability reasons.
- The thermal sensitivity of **tungsten** is superior to that of platinum above 100K, and it is usable at higher temperatures than platinum with superior linearity, but its stability is inferior to that of platinum.

Metal	Resistivity at 0°C (μΩ·cm)	Melting Point (°C)	Application Range
Copper	7	1083	-190 to +150°C
Nickel	6.38	1453	-60 to +180°C
Platinum	9.81	1769	-250 to +1100°C
Tungsten	1.72	3380	-269 to +27°C

The most commonly used material is **platinum**, which is typically encapsulated with ceramic and placed in a sealed steel sheath. The resistance is 100Ω at 0°C, which is why these thermometers are commonly referred to as **Pt 100 probes**.



Fig.V. 1:Resistance Temperature Detectors (Rtds) with platinum

V.3 Thermistors

V.3.1 Principle

Thermistors are temperature sensors with very high thermal sensitivity—approximately ten times greater than resistance temperature detectors (RTDs). Their temperature coefficients are generally negative and highly dependent on temperature.

They are made from mixtures of semiconducting metal oxides, such as nickel, cobalt, or manganese oxides. Unlike metals, the resistance of semiconductors decreases as temperature increases.

Thermistors are available in various forms: discs, small cylinders, rings, beads, with the sensing element either unprotected or protected by coating or encapsulation. The high resistivity of the materials used allows for appropriate resistance values with small amounts of material, resulting in compact sizes (on the order of millimeters). This compactness enables near-point measurement of temperature.

V.3.2 Resistance-Temperature Relationship

The resistance of a thermistor as a function of temperature is given by:

$$R(T) = R_0 exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where:

- R_0 is the resistance at temperature T0$T0$
- β is a temperature-dependent coefficient, typically between 3000 and 5000 K.

The thermal sensitivity is expressed as: $\alpha_R = -\frac{\beta}{T^2}$

The following figures show the variation of resistance and thermal sensitivity of a thermistor as a function of temperature on a semi-logarithmic scale. They indicate that both the resistance and thermal sensitivity vary inversely with temperature. The sensitivity of thermistors is very high at low temperatures, making them particularly suitable for detecting and measuring very small temperature variations (from 10^{-4} to 10^{-3} K). They can be used without major stability issues up to about 200°C; beyond this, special materials such as silicon carbide must be used. Since the variation of resistance with temperature is very significant, a given thermistor is typically used for a measurement range of 50 to 100°C. For broader measurement ranges, different thermistors with appropriately chosen resistances must be used successively.

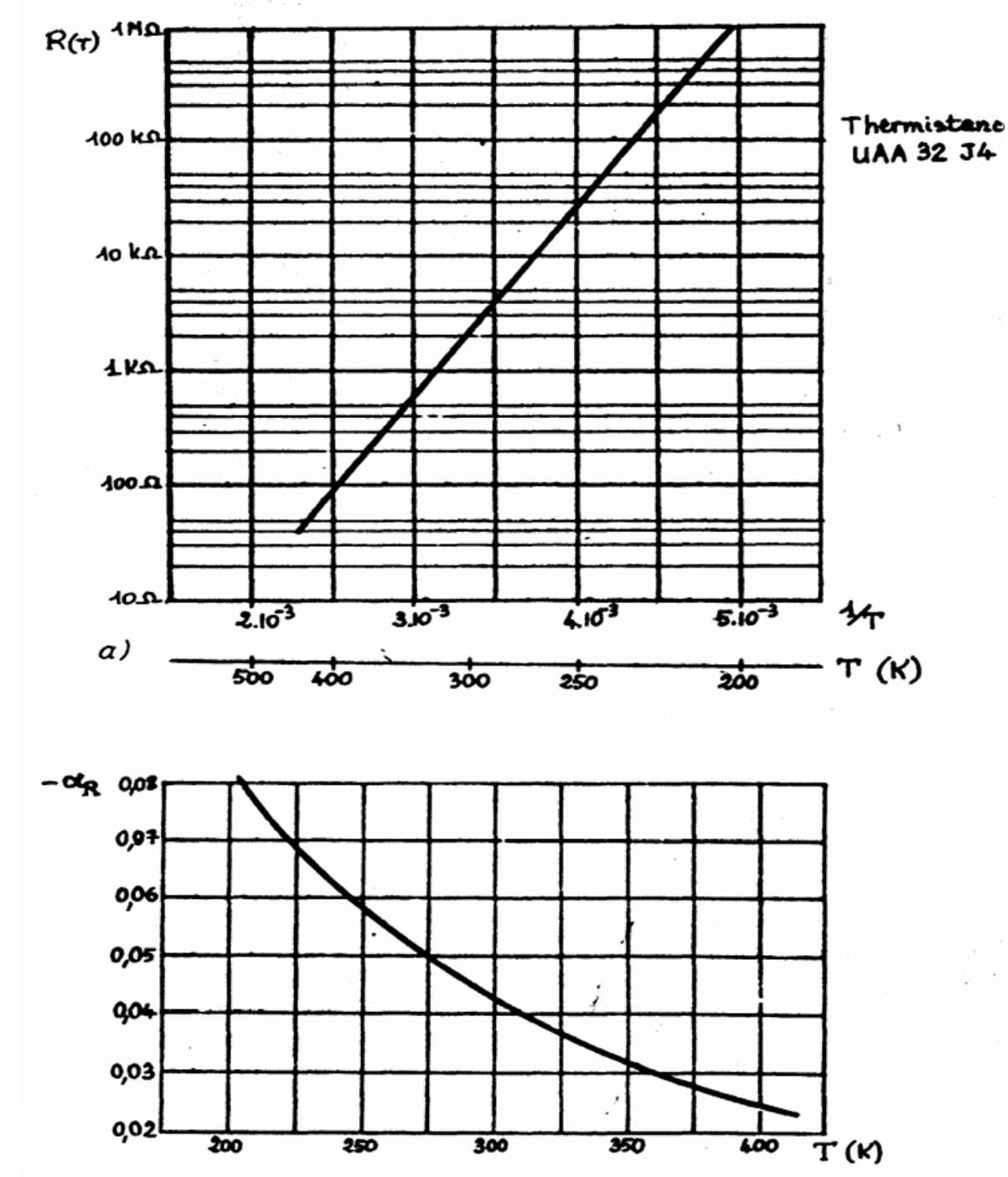


Fig.V. 2Different thermistors

Chapter VI. Thermocouples

Chapter VI: Thermocouples

VI.1 Principle

A thermocouple consists of two conductors, A and B, made of different materials. Their junctions are maintained at temperatures T1 and T2, and the device generates an electromotive force (EMF), V_0, which depends on both the materials A and B and the temperatures of the two junctions.

Typically, the temperature of one junction is fixed, known, and used as a reference (T1 = T_{ref}), while the other junction is placed in the medium whose temperature is to be measured. The junction at temperature T1 is referred to as the **cold junction**, and the junction at temperature T2 is referred to as the **hot junction**.

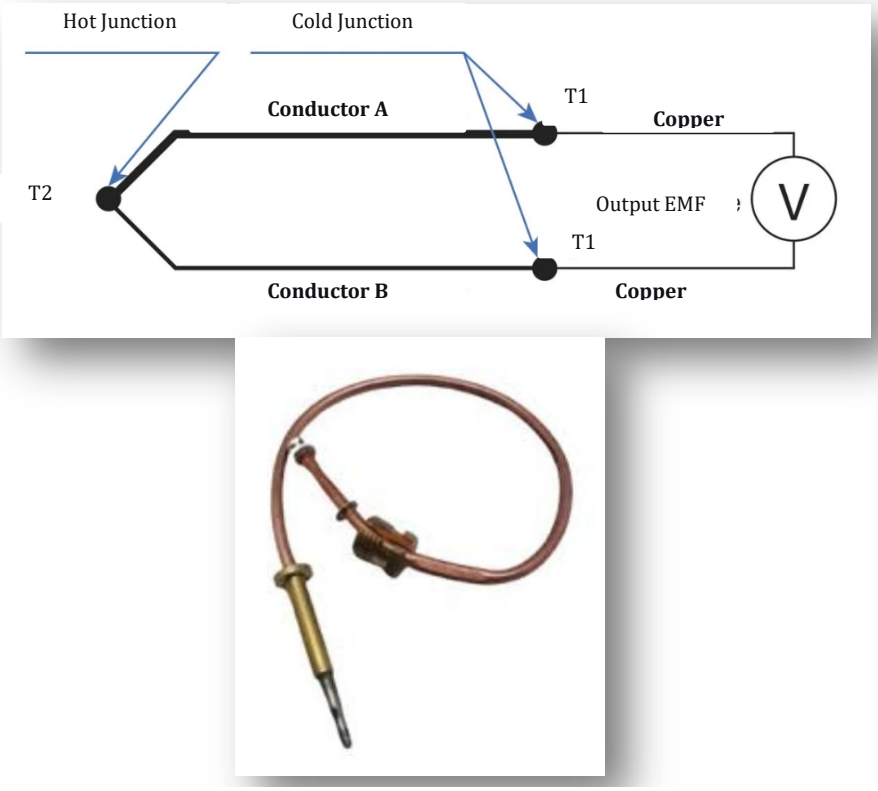


Fig.VI. 1:Thermocouples

VI.2 Thermoelectric Effects

The operation of thermocouples relies on several thermoelectric phenomena.

VI.2.1 The Peltier Effect

When two different conductors A and B are joined at a point and this junction is held at a uniform temperature T*T*, a potential difference develops across the junction. This voltage depends solely on the materials used and the junction temperature. It is known as the Peltier EMF:

$V_M - V_N = P_{A/B}^T$

Volta's Law: In an isothermal circuit (a circuit where all parts are at the same temperature) made from different conductors, the sum of all Peltier EMFs around the loop is zero. Therefore, the net EMF is: $V_M - V_N = P_{A/B}^T$

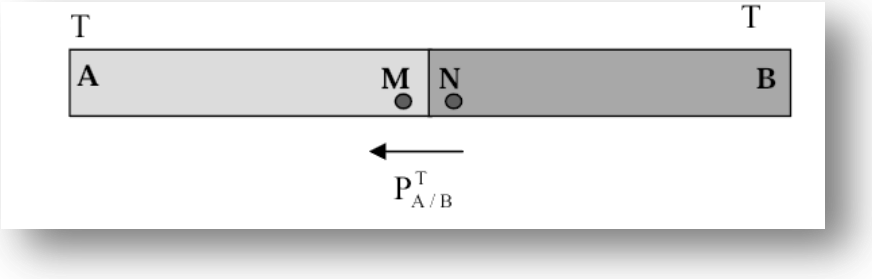


Fig.VI. 2:Isothermal circuit

VI.2.2 The Thomson Effect

When there is a temperature difference between two points M and N (at temperatures T_1 and T_2) within a homogeneous conductor A, an electromotive force (EMF) develops. This EMF depends solely on the material of the conductor and the temperatures T_1 and T_2. This is known as the Thomson EMF:

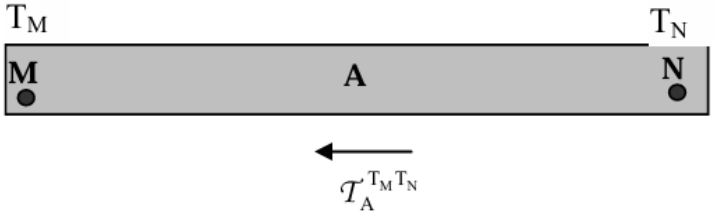


Fig.VI. 3:Homogeneous conductor

$$V_{MN}=\int_{T1}^{T2} \mu_A dT$$

where μ_A is the Thomson coefficient of conductor A, which is temperature-dependent.

VI.2.3 The Seebeck Effect

Consider a closed circuit formed by two conductors A and B, with their junctions maintained at temperatures T_1 and T_2. This circuit forms a thermocouple and generates a net EMF known as the Seebeck EMF ($S_{A/B}$). This EMF results from the combined Peltier and Thomson effects occurring within the circuit:

$$S_{A/B} = \Pi_{A/B}(T_2) - \Pi_{A/B}(T_1) + \int_{T1}^{T2} (\mu_A - \mu_B) dT$$

If one temperature (e.g., T_1) is fixed at a known reference value (typically 0°C), the Seebeck EMF depends only on the other temperature (T_2) for a given pair of conductors A and B.

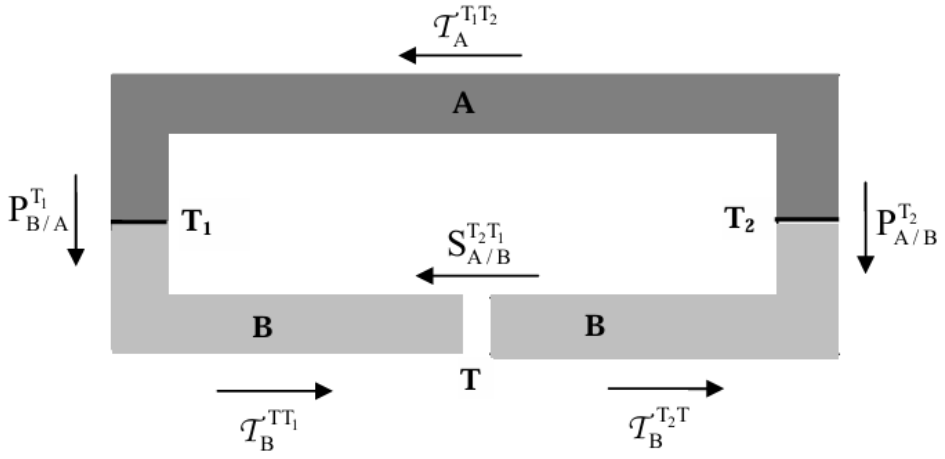


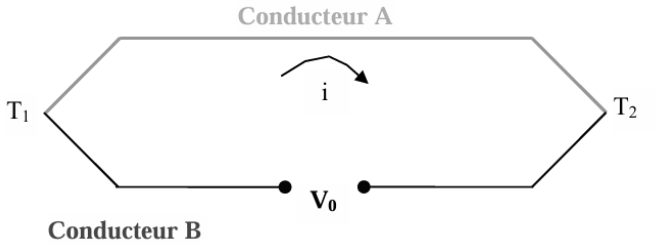
Fig.VI. 4:Closed circuit formed by two conductors A and B

VI.3 Practical Principles For Using Thermocouples.

The practical application of thermocouples is based on the following six principles:

a) A thermocouple must consist of at least two different materials and two junctions.

The output voltage V_0 is a function of the temperature difference (T_1 - T_2) between the two junctions:



$$V_0 = e_{A/B}(T_1) + e_{B/A}(T_2)$$

where:

$\mathbf{e_{B/A}}$ is the junction potential per unit temperature when current flows from B to A

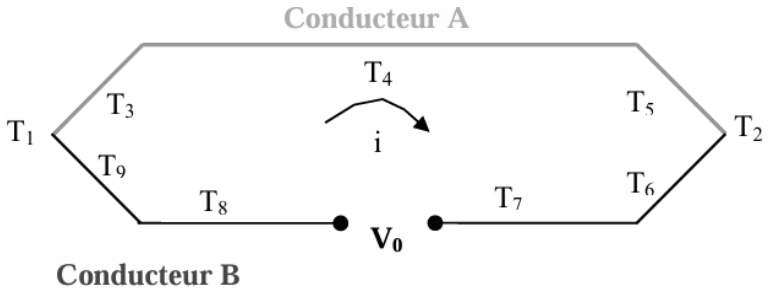
$\mathbf{e_{A/B}}$ is the junction potential per unit temperature when current flows from A to B

Given that $e_{B/A} = -e_{A/B}$, we get:

$$V_0 = e_{A/B}(T_1 - T_2)$$

Important Note: This relationship is **not linear**, as $e_{A/B}$ $e_{A/B}$ itself depends on temperature.

b) V_0 depends solely on the materials of conductors A and B and the temperature difference (T_1 - T_2) between the junctions.

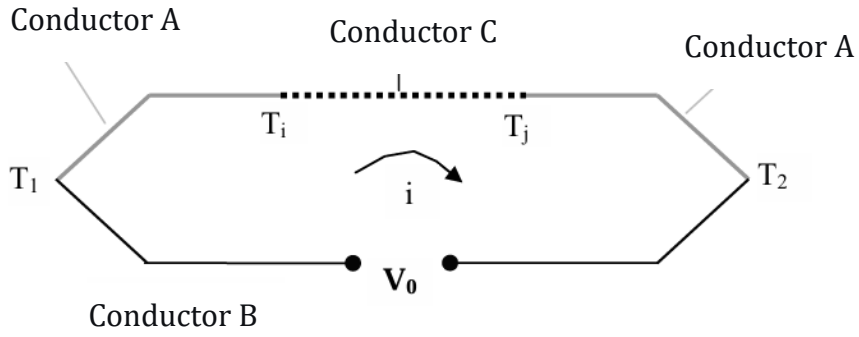


$$V_0 = f(T_1 - T_2)$$

c) Law of Intermediate Metals

When a third conductor C is introduced into a thermocouple circuit composed of conductors A and B, the electromotive force (EMF) generated by the circuit remains unchanged ***provided that both ends of the added conductor are maintained at the same temperature****.*

Circuit Diagram:



Mathematical Proof:
The output voltage can be expressed as:

$$V_0 = e_{B/A}(T_1) + e_{A/C}(T_2) + e_{C/A}(T_2) + e_{A/B}(T_2)$$

Given the fundamental identities:

$e_{A/B} = -e_{B/A}$; $e_{A/C} = -e_{C/A}$

The equation simplifies to:

$$V_0 = e_{B/A}(T_1 - T_2) + e_{A/C}(T_i - T_j)$$

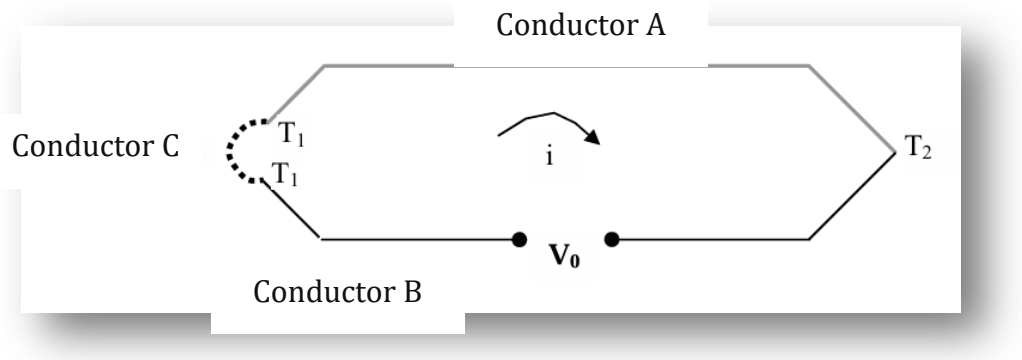
Key Conclusion:

The influence of the inserted conductor is completely eliminated when $T_i = T_j$. This principle remains valid regardless of how many additional conductors are introduced, as long as each pair of connection points is maintained at the same temperature.

d) Junction Insertion Principle

Inserting a conductor directly into a thermocouple junction will not affect the output voltage V_0 ***if both ends of the inserted conductor are maintained at the junction temperature****.*

Circuit Diagram:



Mathematical Proof:

The circuit EMF can be described as:

$$V_0 = e_{B/A}(T_1) + e_{C/A}(T_1) + e_{A/B}(T_2)$$

Using the identity relationships:

$$e_{C/A} = e_{C/B} + e_{B/A}$$

$$e_{B/A} = e_{B/C} + e_{C/A}$$

The expression reduces to:

$$V_0 = e_{B/A}(T_1 - T_2)$$

The output voltage remains identical to the original thermocouple configuration. This principle holds true for any number of conductors inserted into the junction, provided all connection points are maintained at the junction temperature.

e) Law of Successive Temperatures:

When the reference temperature T_1 of a thermocouple changes to a new value T_1', the Seebeck EMF of the A/B couple changes from $S_{A/B}^{T_2T_1}$ to $S_{A/B}^{T_2T_1'}$

This law means that if you know the Seebeck EMF at one reference temperature, you can calculate it for a different reference temperature using the relationship between these values.

Lorsque la température de référence T1 d'un thermocouple prend une nouvelle valeur T'1, la f.e.m de Seebeck du couple A/B passe de la valeur

$$S_{A/B}^{T_2T_1'} = \left(\mathcal{T}_B^{T_2T_1'} - \mathcal{T}_A^{T_2T_1'}\right) + \left(P_{A/B}^{T_2} - P_{A/B}^{T_1'}\right)$$

which can be written as

$$S_{A/B}^{T_2T_1'} = \left(P_{A/B}^{T_2} - P_{A/B}^{T_1}\right) + \left(\mathcal{T}_B^{T_2T_1} - \mathcal{T}_A^{T_2T_1}\right) - \left(P_{A/B}^{T_1'} - P_{A/B}^{T_1}\right) - \left(\mathcal{T}_B^{T_1'T_1} - \mathcal{T}_A^{T_1'T_1}\right)$$

which gives

$S_{A/B}^{T_2T_1'} = S_{A/B}^{T_2T_1} - S_{A/B}^{T_1'T_1}$,

or

$$S_{A/B}^{T_2T_1} = S_{A/B}^{T_2T_1'} + S_{A/B}^{T_1'T_1}$$

This relation is used particularly when a measurement was taken with a reference temperature T_1', yielding the EMF from $S_{A/B}^{T_2T_1'}$, and we wish to find the value of from $S_{A/B}^{T_2T_1}$ for T_1=0°C. This conversion allows us to determine the temperature T_2 being measured using the standard reference table for the specific thermocouple type.

f) Law of Successive Metals:

If we consider a first thermocouple made of conductors A and B, and a second thermocouple made of conductors B and C, with both couples having their junctions at the same temperatures T_1 and T_2, we can determine the EMF of a thermocouple made of materials A and C as follows:

For the thermocouple A/B $S_{A/B}^{T_2T_1} = \left(\mathcal{T}_B^{T_2T_1} - \mathcal{T}_A^{T_2T_1}\right) + \left(P_{A/B}^{T_2} - P_{A/B}^{T_1}\right)$

For the thermocouple B/C : $S_{B/C}^{T_2T_1} = \left(\mathcal{T}_C^{T_2T_1} - \mathcal{T}_B^{T_2T_1}\right) + \left(P_{B/C}^{T_2} - P_{B/C}^{T_1}\right)$

For the thermocouple A/B

$$S_{A/C}^{T_2T_1} = \left(\mathcal{T}_C^{T_2T_1} - \mathcal{T}_A^{T_2T_1}\right) + \left(P_{A/C}^{T_2} - P_{A/C}^{T_1}\right) = \left(\mathcal{T}_C^{T_2T_1} - \mathcal{T}_A^{T_2T_1} + \mathcal{T}_B^{T_2T_1} - \mathcal{T}_B^{T_2T_1}\right) + \left(P_{A/B}^{T_2} + P_{B/C}^{T_2} - P_{A/B}^{T_1} - P_{B/C}^{T_1}\right)$$

or $$S_{A/C}^{T_2T_1} = S_{A/B}^{T_2T_1} + S_{B/C}^{T_2T_1}$$

From a practical standpoint, if metal B is a reference metal (such as Pb or Pt) for which various thermocouple reference tables exist when paired with different other conductors, then knowing the Seebeck EMF values (from these tables) for its combinations

with conductors A and C enables us to determine the EMF output of the A/C thermocouple pair, even if a specific reference table for the A/C combination is not available.

VI.4 Thermocouple Thermal Sensitivity

Over large temperature ranges, the EMF output of a thermocouple is a non-linear function of temperature. For example, the figure below shows the variation of EMF as a function of temperature for different commonly used thermocouple types. However, this variation is generally considered linear over limited temperature intervals, where the acceptable interval size depends on the required measurement accuracy.

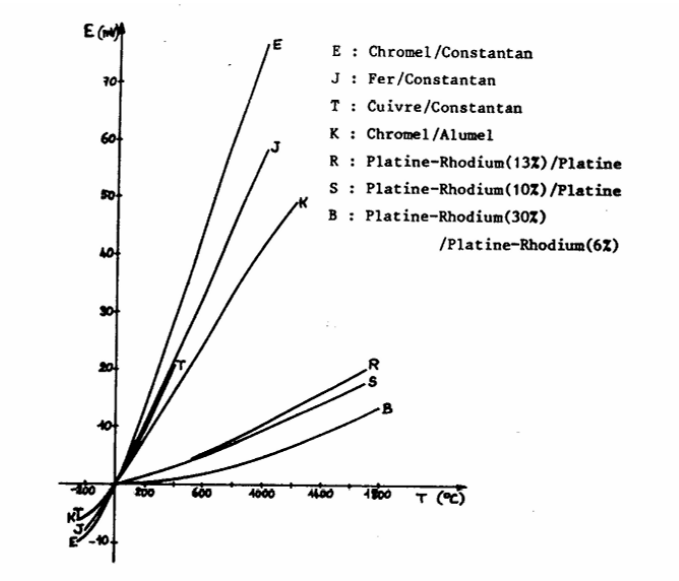


Fig.VI. 5:EMF versus the temperature

The relationship between the EMF, E (in μV), of a thermocouple and the temperature T (in °C) is described by a polynomial equation of the form:

where: $$E = \sum_{i=1}^{n} a_i T^i$$

- n is the highest order of the polynomial (generally less than 9)
- a_i are coefficients specific to the thermocouple type.

For example, for a Type B thermocouple (Platinum-30% Rhodium / Platinum-6% Rhodium), n=8 and the coefficients a_i are as follows:

Table VI. 1:coefficients a_i

$a_0 = 0$	$a_1 = -2.4674601620 \times 10^{-1}$	$a_2 = 5.9102111169 \times 10^{-3}$
$a_3 = -1.4307123430 \times 10^{-6}$	$a_4 = 2.1509149750 \times 10^{-9}$	$a_5 = -3.175800720 \times 10^{-12}$
$a_6 = 2.4010367459 \times 10^{-15}$	$a_7 = -9.0928148159 \times 10^{-19}$	$a_8 = 1.3299505137 \times 10^{-22}$

The thermal sensitivity S (in µV/°C) of a thermocouple, also called the thermoelectric power, is defined at a temperature T (with Tref = 0°C) by: $S(T)\frac{dS_{A/B}^{T\,0°C}}{dT}$

The figure below shows the variation of thermal sensitivity as a function of temperature for the different thermocouples mentioned previously.

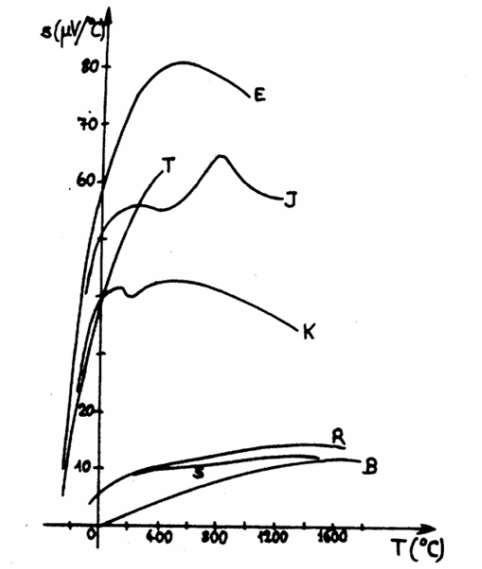


Fig.VI. 6:Variation of thermal sensitivity

For example, for a Type J thermocouple (Iron/Constantan), S(0°C) = 52.9 µV/°C and S(700°C) = 63.8 µV/°C, while for a Type S thermocouple (Platinum-10% Rhodium / Platinum), S(0°C) = 6.4 µV/°C and S(700°C) = 11.3 µV/°C.

The order of magnitude of thermocouple sensitivities is much lower than that of temperature sensors using resistive elements.

VI.5 Reference Temperature of A Thermocouple

Accurate temperature measurement using a thermocouple requires precise knowledge of the reference junction temperature. Indeed, any uncertainty in the value of T_ref directly leads to uncertainty in the measured temperature. Several methods are used to maintain this temperature constant at 0°C.

VI.5.1 The Ice-Water Bath

The reference junction is immersed in a bath consisting of a mixture of water and ice at normal atmospheric pressure, housed within an adiabatic (thermally insulated) enclosure. The water and ice must be of high purity, and the ice should be finely crushed. This ice must be replenished frequently to maintain a constant reference temperature. A temperature of 0°C is achieved with an accuracy of ±0.1°C.

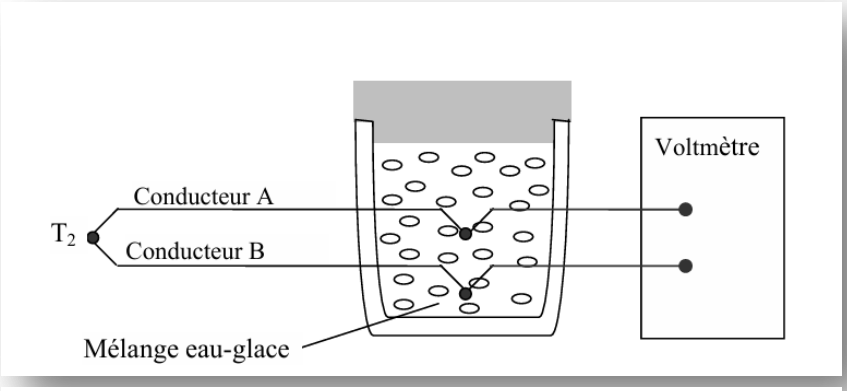


Fig.VI. 7:The Ice-Water Bath

VI.5.2 The Electrical Bridge Method

It is possible to maintain the reference temperature at 0°C using electrical circuits, known as cold junction compensation circuits.

If the ambient temperature T_a at the moment of measurement is known, and T_c is the temperature to be measured, the following relationship holds: $S_{A/B}^{T_c\,0°C} = S_{A/B}^{T_c\,T_a} + S_{A/B}^{T_a\,0°C}$

$$S_{A/B}^{T_c\,0°C} = S_{A/B}^{T_c\,T_a} + S_{A/B}^{T_a\,0°C}$$

The cold junction compensation circuit is designed to automatically generate a voltage v(T_a) equal to $S_{A/B}^{T_a\ 0°C}$ This EMF, when added to the thermocouple's EMF $SS_{A/B}^{T_c\ T_a}$, allows the measurement of the EMF $S_{A/B}^{T_a\ 0°C}$ across the measuring circuit terminals.

The voltage v(T_a) is determined based on the resistance R(T_a) of a resistance temperature detector (RTD) maintained at ambient temperature. The device used for this purpose is a **Wheatstone bridge**. The basic circuit configuration is shown in the following figure.

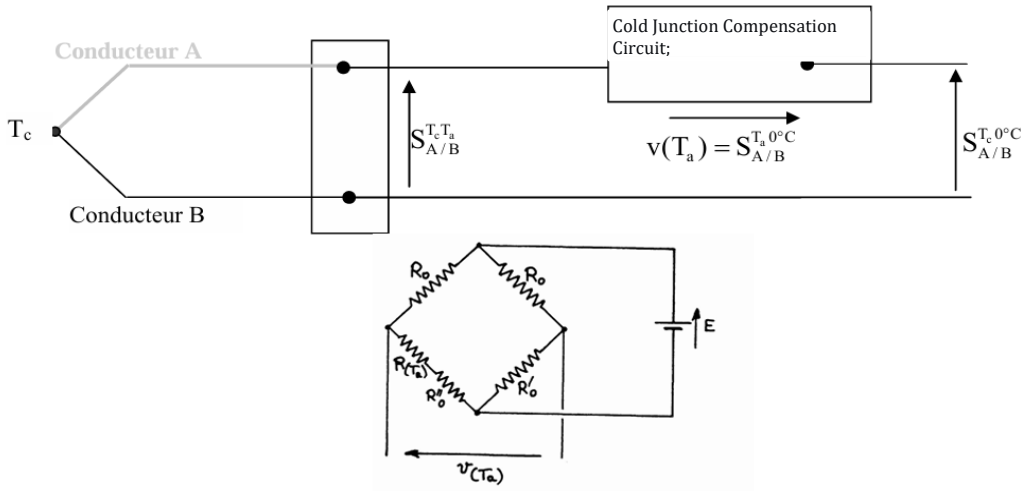


Fig.VI. 8:Wheatstone bridge

The Wheatstone bridge enables cold junction compensation for variations in ambient temperature around 0°C. The resistors R_0, R_0', and R_0'' are independent of temperature, whereas the resistance R(T_a) of the sensor is a linear function of ambient temperature, with its thermal sensitivity coefficient α being the slope.

The bridge is balanced at 0°C. At an ambient temperature T_a, its imbalance voltage is expressed as:

$$v(T_a) = E\frac{R_0(R_0' - R_0'')}{(R_0 + R_0')^2}\alpha_R T_a$$

The numerical values of the bridge components are selected to ensure:

$$S_{A/B}^{T_a\,0°C} = E\frac{R_0(R_0' - R_0'')}{(R_0 + R_0')^2}\alpha_R T_a$$

Assuming the thermocouple sensitivity remains constant over the variation range of T_a, we obtain:

$$S = E\frac{R_0(R_0' - R_0'')}{(R_0 + R_0')^2}\alpha_R$$

VI.5.3 .The Double Furnace Method

This method uses two furnaces at different temperatures to simulate the 0°C reference temperature.

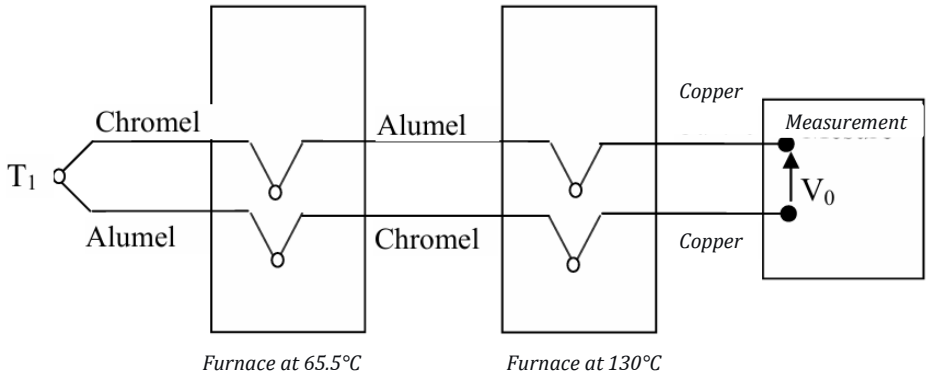


Fig.VI. 9:Thermocouple Circuit with Reference Furnaces at 65.5°C and 130°C

- **Key Idea**: The voltage V_0 is the sum of individual thermoelectric voltages from each junction in the circuit. By carefully choosing the temperatures (130°C and 65.5°C) and leveraging the properties of the metal pairs, the circuit cancels out the effects of the actual cold junction, making the output voltage depend only on the temperature T_1 relative to 0°C.
- **Simplification**:

The terms $e_{Cuivre/Alumel·130}$, and $e_{Chromel/Cuivre·130}$ represent voltages from Copper-Alumel and Chromel-Copper junctions at 130°C.

- The terms $e_{Alumel/Chromel·65.5}$ appear twice, meaning two Alumel-Chromel junctions at 65.5°C.

- The term $e_{Chromel/Alumel}.T1$ is the voltage from a Chromel-Alumel junction at temperature T1.
- Using the fact that $e_{Alumel/Chromel}=-e_{Chromel/Alumel}$ (since reversing the metals reverses the voltage polarity), and assuming the Seebeck coefficients are constant, the equation simplifies to $V_0=e_{Chromel/Alumel}(T_1-0\circ C)$. This means the circuit behaves like a single Chromel-Alumel thermocouple with its cold junction at 0°C.
- **Why It Works**: The voltages from the junctions at 130°C and 65.5°C combine in such a way that they offset any cold junction effects, effectively "referencing" the measurement to 0°C. This allows accurate temperature measurement of T1*T*1 without needing a physical ice bath or other reference.

Simplified Rewr

Explanation of the Equation and Circuit

The equation describes the output voltage V_0 in a thermocouple circuit that uses multiple junctions to simulate a cold junction at 0°C. This is part of the "double furnace method" where two furnaces at different temperatures (130°C and 65.5°C) are used to create a reference equivalent to 0°C. The circuit involves metals like Copper (Cuivre), Alumel, and Chromel, which are common in thermocouples (e.g., Type K thermocouple uses Chromel and Alumel).

iting

The output voltage V_0 is given by:

$V_0=e_{Cu/Al}\cdot 130+e_{Al/Ch}\cdot 65.5+e_{Ch/Al}\cdot T1+e_{Al/Ch}\cdot 65.5+e_{Ch/Cu}\cdot 130$

where:

- $e_{Cu/Al}$: Seebeck coefficient for Copper-Alumel
- $e_{Al/Ch}$: Seebeck coefficient for Alumel-Chromel
- $e_{Ch/Al}$: Seebeck coefficient for Chromel-Alumel (note: $e_{Ch/Al}=-e_{Al/Ch}$)
- $e_{Ch/Cu}$: Seebeck coefficient for Chromel-Copper

After simplification:

$V_0=2e_{Al/Ch}\cdot 65.5+e_{Ch/Al}\cdot 130+e_{Ch/Al}\cdot T_1=e_{Ch/Al}(T_1-0°C)$

Thus, V_0 is proportional to the temperature difference between T_1 and 0°C, as if the cold junction were at 0°C.

VI.6 Main Types of Thermocouples and Their Limits of Use

The following table presents, for different types of thermocouples, their operating temperature ranges and accuracy for the specified wire diameters. Tables showing the electromotive force (e.m.f.) of these thermocouples as a function of temperature are provided in the appendix.

Table VI. 2:Main Types of Thermocouples and Their Limits of Use

Thermocouples	Temperature Range	EMF (mV)	Accuracy
Copper / Constantan Type T (1.63 mm diameter)	-270 to 370°C	-6.258 to 19.027	± 2% from -100°C to -40°C ± 0.8% from -40°C to 100°C ± 0.75% from 100°C to 350°C
Iron / Constantan Type J (3.25 mm diameter)	-210 to 800°C	-8.096 to 45.498	± 3°C from 0°C to 400°C ± 0.75% from 400°C to 800°C
Chromel / Alumel Type K (3.25 mm diameter)	-270 to 1250°C	-5.354 to 50.633	± 3°C from 0°C to 400°C ± 0.75% from 400°C to 1250°C
Chromel / Constantan Type E (3.25 mm diameter)	-270 to 870°C	-9.835 to 66.473	± 3°C from 0°C to 400°C ± 0.75% from 400°C to 1250°C
Platinum-Rhodium (10%) / Platinum Type S (0.51 mm diameter)	-50 to 1500°C	-0.236 to 15.576	± 2.5°C from 0°C to 600°C ± 0.4% from 600°C to 1600°C
Platinum-Rhodium (13%) / Platinum Type R (0.51 mm diameter)	-50 to 1500°C	-0.226 to 17.445	± 1.4°C from 0°C to 538°C ± 0.25% from 538°C to 1500°C
Platinum-Rhodium (30%) / Platinum-Rhodium (6%) Type B (0.51 mm diameter)	0 to 2760°C	0 to 12.426	± 0.5% from 870°C to 1700°C

VI.7 Comparison between Thermocouples and Electrical Thermometers

The following table summarizes the main advantages and disadvantages of thermocouples, resistance temperature detectors (RTDs), and thermistors.

Table VI. 3:Comparison between Thermocouples and Electrical Thermometers

	THERMOCOUPLE	RTD	THERMISTOR
Advantages	✓ Simple ✓ Robust ✓ Inexpensive ✓ Suitable for spot measurements ✓ Fast response ✓ Wide temperature range	✓ Very stable ✓ Very accurate ✓ Better linearity than thermocouple	✓ Very sensitive ✓ Small size
Disadvantages	✓ Nonlinear ✓ Low stability ✓ Low sensitivity ✓ Requires a reference temperature	✓ Expensive ✓ Slow response ✓ Low sensitivity ✓ Errors due to self-heating (Joule effect)	✓ Nonlinear ✓ Limited temperature range ✓ Low stability ✓ Fragile ✓ Self-heating

Chapter VII. Dynamic Response of a Temperature Sensor

Chapter VII: Dynamic Response of a Temperature Sensor

VII.1 Introduction

Temperature sensors are generally modeled as first-order dynamic systems. Their transient thermal behavior is described by a first-order differential equation derived from energy conservation.

Key Parameters:

- m: sensor mass (kg)
- A: effective surface area for heat transfer (m^2)
- C_p: specific heat capacity of the sensor (J/kg·K)
- h: convective heat transfer coefficient (W/m^2·K)

Governing Equation:

The rate of change of sensor temperature is proportional to the temperature difference between the medium and the sensor:

$$mC_p \frac{dT_c}{dt} = hA(T_x - T_c)$$

Standard Form:

Rewriting the equation in standard first-order form:

$$\frac{dT_c}{dt} + \frac{1}{\tau} T_c = \frac{1}{\tau} T_x$$

Time Constant:

The system time constant $\tau = mC_p/hA$ represents the time required for the sensor to reach approximately 63.2% of a step change in medium temperature. This parameter is critical for determining the sensor's response speed.

Homogeneous Solution:

When T_x is constant, the transient solution is an exponential decay:

$$Tc(t) = Ke^{-t/\tau}$$

The complete dynamic response to step and ramp input signals will be analyzed in subsequent sections, providing insight into real-world sensor performance during temperature variations.

VII.2 Response to a Step Input

The step response of a temperature sensor corresponds to its behavior when suddenly immersed in a fluid at temperature T_x. For this scenario, a particular solution to the differential equation is:

$$T_c=T_x$$

The general solution therefore becomes:

$$Tc = Ke^{-t/\tau} + T_x$$

Considering the initial conditions (at t=0, Tc(0)=0°C), the dynamic response of a temperature sensor to a step input is expressed as:

$$T_c = T_x(1 - e^{-t/\tau})$$

The following figure (Fig.VII.1) illustrates the evolution of the sensor temperature T_c when the environmental temperature T_x undergoes an abrupt increase.

Key Insights for Sensor Design

To enhance the response speed of the sensor, the time constant $\tau\tau$ must be reduced. This requires:

- **Minimizing** the sensor's mass and heat capacity
- **Maximizing** its heat exchange surface area

These design considerations directly impact the sensor's ability to track rapid temperature changes accurately.

Technical Summary

Parameter	Effect on Response Time	Design Implication
Mass (m)	Directly proportional to τ	Use lightweight materials
Heat Capacity (Cp)	Directly proportional to τ	Select low-specific-heat materials
Surface Area (A)	Inversely proportional to τ	Maximize exposed surface area
Heat Transfer Coefficient (h)	Inversely proportional to τ	Optimize fluid flow conditions

The exponential response characteristic demonstrates that the sensor approaches the final temperature asymptotically, with 63.2% of the change occurring within one time constant.

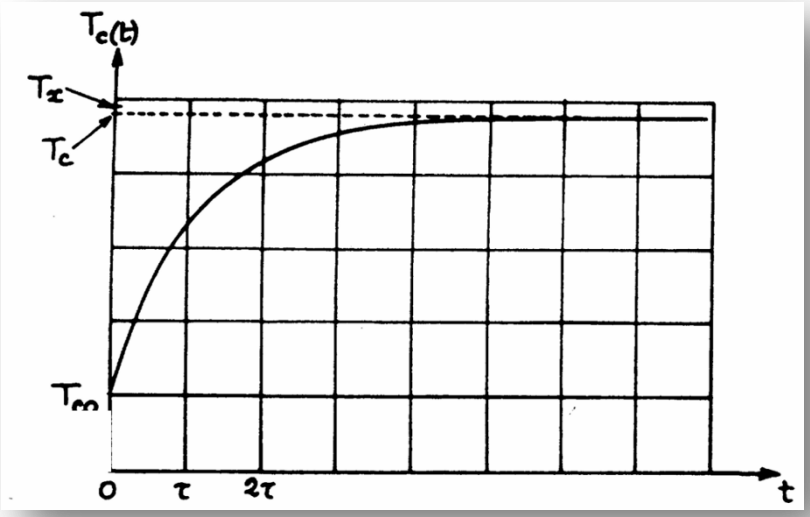


Fig.VII. 1:Evolution of the sensor temperature Tc

VII.3 Response to a Ramp Input

A temperature sensor subjected to a ramp input corresponds to a scenario where the sensor is placed in an environment whose temperature increases linearly with time. This temperature evolution is described by:

$$T_x = b \cdot t$$

where b represents the slope of the linear temperature increase.

For this case, a particular solution to the differential equation is:

$$T_c = b \cdot (t - \tau)$$

The general solution to the problem become

$$Tc = K' e^{-t/\tau} + b \cdot (t - \tau)$$

Considering the initial conditions (at t=0, $T_c(0)=T_x(0)=0°C$), we find $K'=b\tau$.

The sensor's response to a ramp input is therefore expressed as:

$$T_c = b \cdot t - b \cdot \tau(1 - e^{-t/\tau})$$

The Fig.VII.2 illustrates the evolution of the sensor temperature T_c when the environmental temperature T_x varies linearly with time. The exponential term dominates initially but decays rapidly, becoming negligible for $t>3\tau$. The sensor response then becomes linear with a constant time lag of τ relative to the environmental temperature. Accurate measurements are achieved starting from $t>3\tau$.

Key Observations and Practical Implications

Transient Phase Characteristics:

- The exponential term $-b\tau(1-e^{-t/\tau})$ creates an initial measurement error
- This transient error diminishes as the system approaches steady state
- After $t>3\tau$, the response becomes effectively linear

Steady-State Behavior:

- The sensor tracks the environmental temperature with a constant lag: $T_c=b(t-\tau)$
- The steady-state error is exactly $b\tau$
- Measurement precision improves significantly once the transient phase concludes

Design Considerations:

- To minimize tracking error in ramp response scenarios, reduce the time constant τ
- This can be achieved through:
    - Lower sensor mass and heat capacity
    - Enhanced heat transfer surface area
    - Improved convective heat transfer coefficients

Application Notes:

This analysis is particularly relevant for applications involving:

- Industrial processes with controlled temperature ramps
- Thermal cycling tests
- Environmental monitoring with diurnal temperature variations
- Any scenario where temperatures change at a consistent rate

The ramp response demonstrates how first-order systems inevitably lag behind linearly changing inputs, with the time constant τ directly determining both the transient duration and the steady-state tracking error.

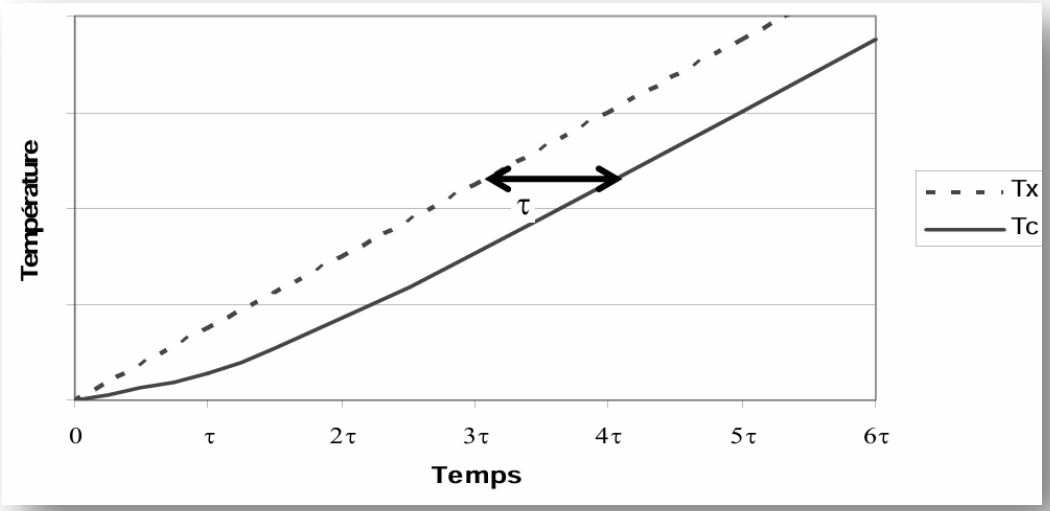


Fig.VII. 2:Evolution of the sensor temperature Tc with time

General Conclusion

This Energy Audit Course Handbook provides a comprehensive and structured framework for understanding and implementing energy audit processes, aligning with both international standards and specific national regulations in Algeria. It systematically guides the reader from fundamental energy concepts through the entire audit lifecycle—from preparation and on-site assessment to detailed analysis, reporting, and economic evaluation.

The handbook emphasizes that energy auditing is not merely a technical exercise but a crucial management tool for enhancing profitability, competitiveness, and environmental stewardship. By adopting the methodologies outlined—such as establishing energy balances, utilizing Sankey diagrams, calculating key performance indicators (KPIs), and applying both analytical and empirical evaluation methods—organizations can transition from simple energy monitoring to active energy management.

Furthermore, the document underscores the importance of economic feasibility and strategic prioritization, providing clear guidelines for assessing project viability based on return on investment (ROI), payback period, and risk. It also introduces modern approaches like outsourcing and Energy Performance Contracts (EPCs), highlighting the evolving role of Energy Service Companies (ESCOs) in the energy efficiency landscape.

In the context of rising energy costs and regulatory pressures, this handbook serves as an essential resource for future engineers and practitioners. It equips them with the necessary skills to identify significant energy-saving opportunities, propose cost-effective measures, and contribute to a sustainable energy future, thereby fostering a lasting culture of energy efficiency within industrial and tertiary sectors.

Chapter VIII: Measuring Instruments

VIII.1. Introduction to Dimensional Metrology

Dimensional metrology uses two main families of instruments:

Variable dimension instruments (indirect measurement)

Fixed dimension instruments (direct measurement by comparison)

VIII.2. Variable Dimension Instruments

a) Linear Measurement Instruments

- Caliper: internal, external, and depth measurement
- Micrometer: high precision (0.01 mm)
- Comparator: measurement of deviations from a reference dimension
- Depth gauge: specific depth measurement
- Protractor: angular measurement

b) Coordinate Measuring Machines (CMM)

- Principle: 3D measurement through displacement along X, Y, Z axes
- History: Developed in the 1960s, popularized in the 1980s
- Applications: Complete geometric control of complex parts
- Precision: Very high (up to a few microns)

VIII.3. Fixed Dimension Instruments

a) For Bore Measurement

- Plug gauges: verification of internal diameters
- Cylindrical ring gauges: bore control by go/no-go testing

b) For Shaft Measurement

- Male gauges: verification of external diameters
- Snap gauges: diameter measurement by comparison

c) For Thread Measurement

- Thread gauges: pitch and profile control
- Thread rings: rapid male/female thread verification

d) For Angle Measurement

- Fixed angle gauges: verification of specific angles

Sine bars: precision angular measurement

d) For Length Measurement

Length gauges: reference standards

Thickness gauges: clearance and center distance control

VIII.4. Instrument Selection Criteria

The choice of measuring instrument depends on:

a) Technical Characteristics

Measurement capacity

Accuracy class

Fidelity and trueness

Measurement range

Measurement uncertainty

b) Operational Factors

Measurement procedure

Work environment

Operator competence

Frequency of use

c) Part Characteristics

Material (steel, plastic, etc.)

Geometric shape

Dimensional tolerance

Surface roughness

VIII.5. Industrial Applications

a) Incoming Inspection

Verification of purchased parts

Use of fixed dimension instruments for rapid control

c) In-Process Control

Continuous monitoring of dimensional characteristics

Use of variable dimension instruments

d) Final Inspection

Complete verification before shipment

Use of CMM for complete 3D control

VIII.6. Good Metrological Practices

Regular calibration according to calibration plan

Preventive maintenance of instruments

Operator training

Metrological traceability to national standards

Measurement uncertainty management

VIII.7. Technological Evolution

Digitization of manual instruments

Automation of measurement processes

Integration with MES (Manufacturing Execution System)

Development of non-contact sensors

Real-time statistical analysis

VIII.8. Caliper Gauge or Vernier Caliper

This is an inspection instrument used by mechanics for quick measurements. The caliper provides a measurement whose accuracy varies depending on the type of vernier scale used. The vernier can be 1/10, 1/20, or 1/50, allowing for measurement evaluation to 1/10 of a mm (0.1 mm), 1/20 of a mm (0.05 mm), and 1/50 of a mm (0.02 mm), respectively.

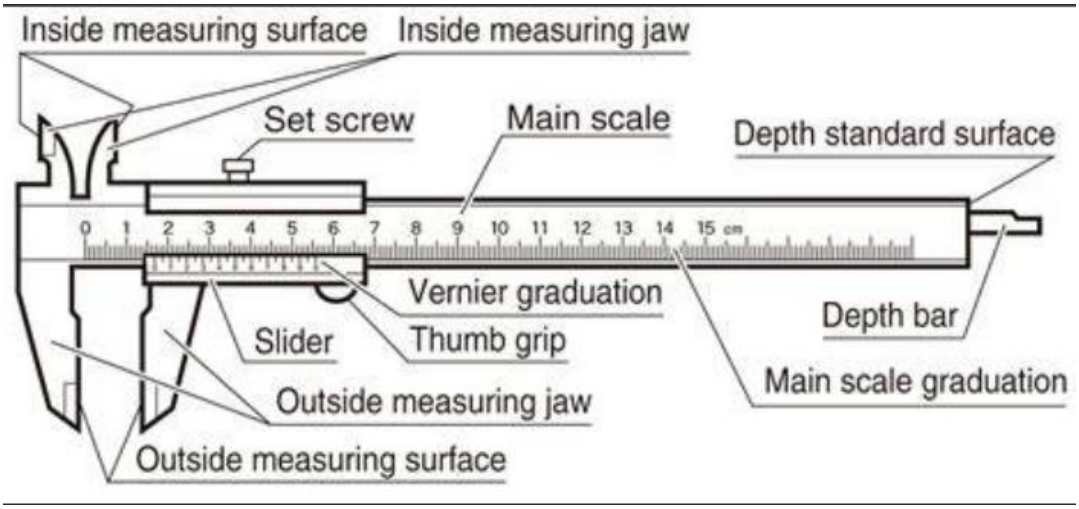


Fig.VIII. 1:Vernier Caliper

VIII.8.1 Types of Vernier Calipers

a) Universal Vernier Caliper (Standard Type):

- The most common and versatile type.
- Equipped with:
  - Internal jaws (for inside measurements),
  - External jaws (for outside measurements),
  - A depth rod (for depth measurements),
  - A main scale and vernier scale (for precise readings).
- Capable of taking **inside, outside, depth, and step** measurements.

b) Depth Vernier Caliper:

- Designed specifically for measuring the **depth of holes, recesses, and steps**.
- Features a **specialized base** that spans the hole.
- Has a **narrow, protruding beam** with a **sliding jaw** that extends vertically into the recess for accurate depth readings.

c) Vernier Caliper for Stock:

- Intended for industrial applications, such as measuring the thickness or diameter of stock materials.
- May include special jaw shapes or extensions to adapt to specific workpieces and measurement needs.

d) Digital Vernier Caliper:

- A modern version of the traditional tool.
- Replaces the vernier scale with an electronic digital display.
- Offers features such as:
  - Direct numerical readout,
  - Metric–inch conversion,
  - Data output,
  - Zero setting at any point.
- Enhances precision by reducing reading errors and increasing measurement speed.
-



Fig.VIII. 2: Types of Vernier

VIII.8.2. Operating Procedure for Vernier Calipers

a) Measurement Process:

1. **Positioning:** Gently bring the caliper jaws into contact with the workpiece
2. **Clamping:** Apply moderate pressure while ensuring full jaw contact
3. **Securing:** Lock the movable jaw using the locking screw (moderate tightening)
4. **Reading:** Perform measurement reading following the sequence below

b) Measurement Reading Sequence:

1. **Main Scale Reading:** Read the number of WHOLE millimeters located to the LEFT of the vernier zero mark (nominal dimension)
2. **Vernier Alignment:** Identify the vernier graduation that aligns perfectly with any main scale graduation
3. **Fraction Calculation:** Count the number of vernier divisions between the vernier zero and the aligned graduation

c) Measurement Capabilities and Precision:

- **External measurements:** Outside dimensions and diameters
- **Internal measurements:** Inside diameters and openings
- **Step measurements:** Shoulder and recess dimensions

- **Depth measurements:** Hole depths and recess depths
- **Groove measurements:** Groove widths and undercuts

d) Instrument Precision Levels:

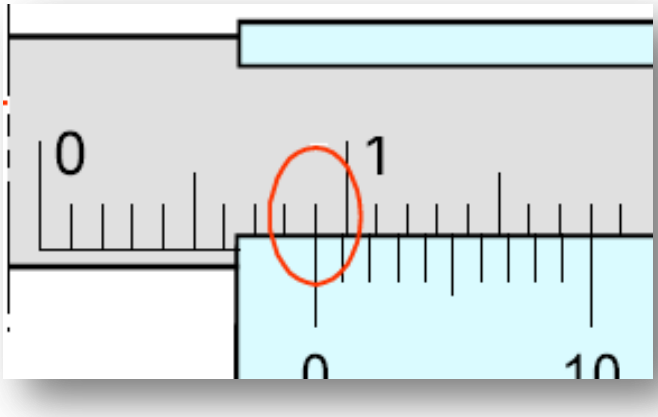
- 1/10 vernier: ±0.1 mm accuracy
- 1/20 vernier: ±0.05 mm accuracy
- 1/50 vernier: ±0.02 mm accuracy

VIII.8.3 Vernier Reading Principle

VIII.1.4.1 .Vernier with 1/10 mm resolution

a) When the Vernier zero aligns with a main scale graduation

If the zero line of the Vernier scale is directly opposite a graduation mark on the main scale, then the reading is simply the value of that main scale graduation.

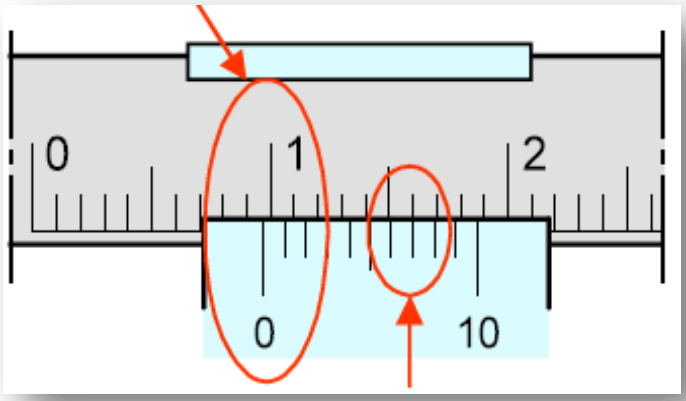


In the example: The measurement is 9 mm.

Explanation: This is the simplest case, where no fractional millimeter measurement is needed because the vernier zero aligns perfectly with a main scale mark.

b) When the Vernier Zero is NOT Aligned with a Main Scale Graduation

1. Read on the main scale the number of **whole millimeters** to the left of the vernier zero.
2. Identify the line on the **vernier scale** that is **perfectly aligned** with any line on the main scale.
3. This vernier graduation indicates the **tenths of a millimeter**.



In the example:
9 mm + 0.7 mm = **9.7 mm**

a) Vernier with 1/20 mm resolution

On a 1/20 vernier scale, a 19 mm segment on the main scale is divided into 20 equal parts on the vernier. Each of these vernier divisions is therefore 0.95 mm long.

On the main scale, each graduation represents 1 mm.

The difference between one main scale division (1 mm) and one vernier division (0.95 mm) is:

1.00 mm - 0.95 mm = **0.05 mm (1/20 mm)**

This 0.05 mm difference is the resolution of the instrument.

Important note: When taking **internal measurements**, remember to **add the thickness of the caliper's jaws** to the reading. This is typically **10 mm** (2 jaws × 5 mm each). Always confirm the specific jaw thickness value stamped on your caliper.

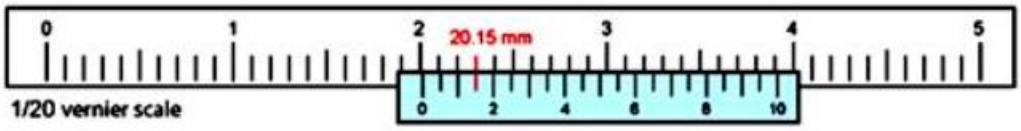
Reading

In the example below, the zero (0) of the vernier scale is located after the 20 mm graduation on the main scale and before the 21 mm graduation.

If we look for the line on the vernier scale that is exactly aligned with a line on the main scale, we find the line marked **3**.

In this case, the reading is as follows:

20 mm + 3 * 0.05 mm = 20 mm + 0.15 mm = 20.15 mm (here 1/20=0.05)



b)Vernier with 1/50 mm resolution

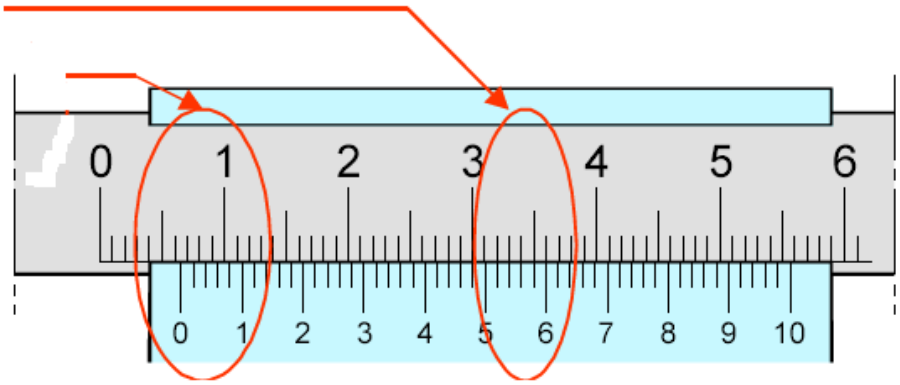
1. Read the number of whole millimeters on the main scale to the left of the vernier's zero line.
2. Identify the line on the vernier scale that is perfectly aligned with any line on the main scale.
3. Read the number on the vernier scale that appears **before** the aligned graduation. This number indicates the number of **tenths of a millimeter (0.1 mm)**.

4. Count the number of small divisions **after** this number until the aligned line. Multiply this count by 2 to get the number of **hundredths of a millimeter (0.01 mm)**.

Example:

1. Number of whole mm before the vernier's zero: **6 → 6.00 mm**
2. Number on the vernier before the aligned line: **5 → 0.50 mm**
3. 4 graduations after the number 5: 4 × 0.02 = **0.08 mm, (here 1/50=0.02)**

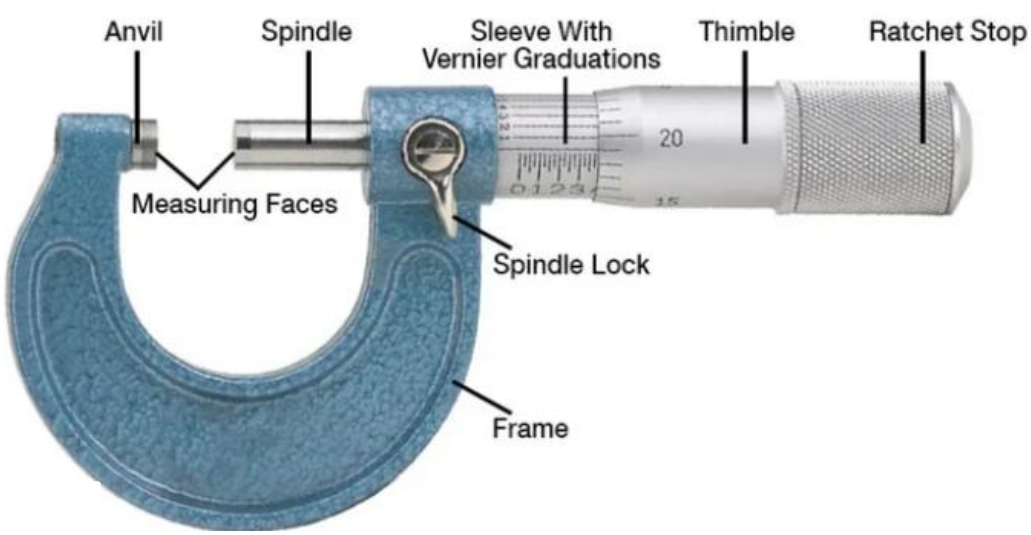
Total Measurement: 6.00 + 0.50 + 0.08 = 6.58 mm



VIII.9. Micrometer or Micrometer Caliper

The micrometer is capable of measuring very small distances, such as the diameter of a pipe, a rod, a nut, a bolt, as well as washers and nails.

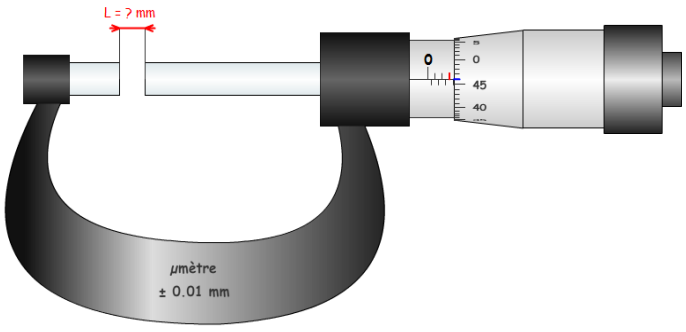
The following (Fig.VIII.3) represents the main components of the micrometer.



VIII.9. 1.Types of Micrometers and Their Uses

1. Outside Micrometer

- **Function**: Measures external diameters and thicknesses
- **Principle**: Uses a calibrated screw to move a mobile jaw toward a fixed jaw (anvil)
- **Precision**: Up to 0.01 mm (1/100 mm) or 0.001" (1/1000 inch)
- **Key Components**:
  - Anvil: Fixed part
  - Spindle: Mobile jaw
  - Sleeve: Main scale
  - Thimble: Secondary scale
  - Frame: U-shaped structure



2. Inside Micrometer

- **Function**: Measures internal diameters and bores
- **Feature**: Extensible rods to adapt to different dimensions
- **Precision**: Similar to outside micrometer

3. Depth Micrometer

- **Function**: Measures depth of holes and grooves
- **Components**: Reference base and interchangeable rods
- **Use**: Base rests on surface, rod descends to bottom of hole

4. Tube Micrometer

- **Function**: Measures wall thickness of tubes
- **Design**: Special tips to fit curved surfaces

How to Read a Manual Micrometer

Reading Steps:

1. **Sleeve Reading**
    - Read value exposed on main scale (generally in mm or inches)
    - Note completely visible graduation to left of thimble
2. **Thimble Reading**
    - Read thimble graduation aligned with sleeve reference line
    - Each graduation represents 0.01 mm (1/100 mm)
3. **Total Measurement Calculation**
    - **Measurement = Sleeve value + (thimble value × precision)**

Practical Example:

- Sleeve: 5.5 mm
- Thimble: graduation 37
- Precision: 0.01 mm
- **Total measurement = 5.5 + (37 × 0.01) = 5.87 mm**

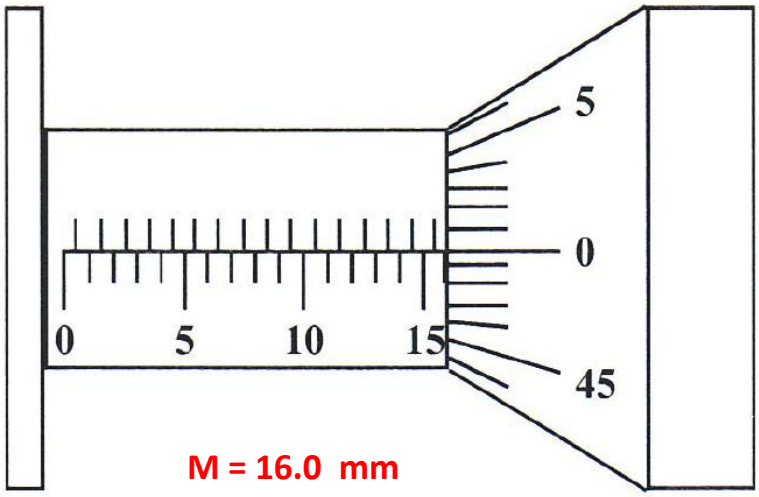
Advantages of Digital Micrometers

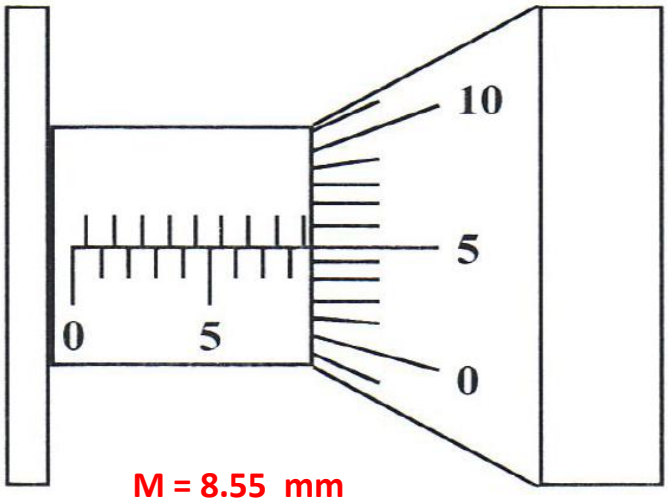
- Direct reading on digital display
- Automatic mm/inch conversion
- Measurement hold function
- Data export to software
- Improved precision up to 0.001 mm

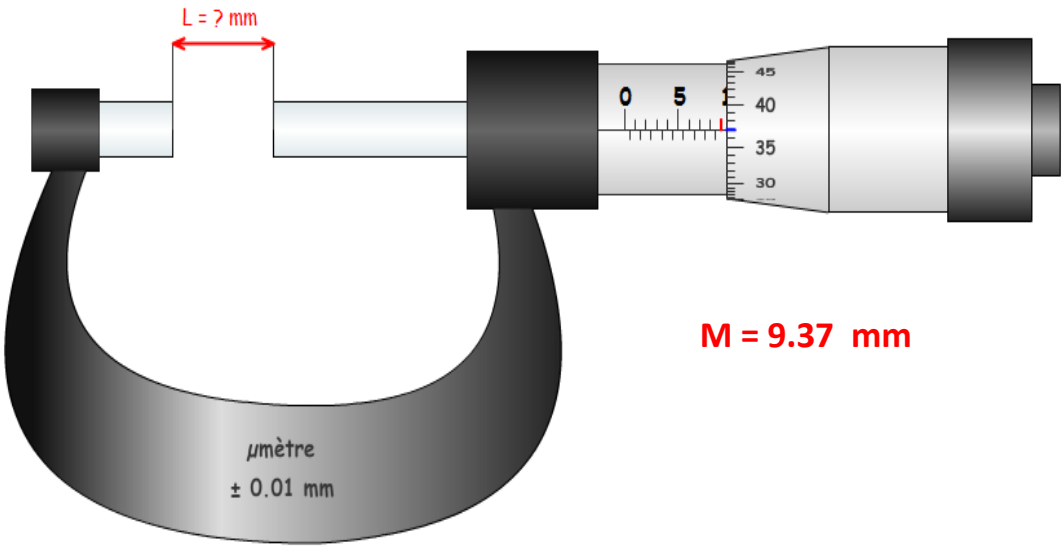
Good Measurement Practices

- Clean measuring surfaces
- Apply constant pressure with torque limiter
- Regularly check zero
- Proper storage to avoid shocks
- Regular calibration to maintain precision

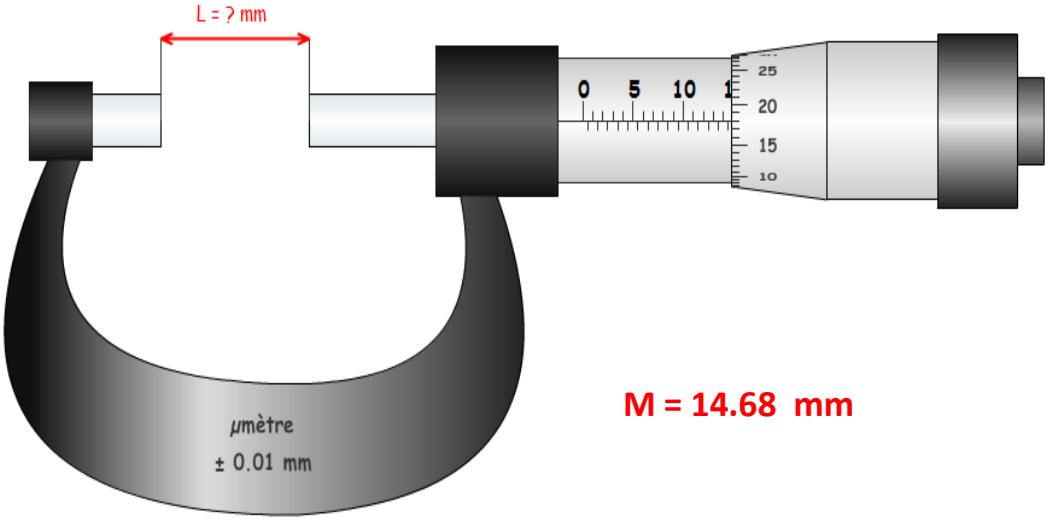
Micrometers remain essential in fields requiring high precision like aerospace, where tolerances can reach 1/10,000 of an inch.







M = 9.37 mm



M = 14.68 mm

Multiple Choice Questions (MCQs)

Each question offers several options, among which one or more may be correct. Check the correct answer(s).

1) What is metrology?

a) The science of measurement.

b) The study of materials used in instrument manufacturing.

c) The analysis of tolerances in mechanical systems.

d) A branch of chemistry.

2) What is the difference between precision and accuracy?

a) Precision measures the closeness of a value to the true value, while accuracy measures the repeatability of results.

b) Precision measures the repeatability of results, while accuracy measures the closeness of results to the true value.

c) Both terms mean the same thing.

d) None of the above.

3) What is a measurable quantity?

a) A physical, chemical, or biological characteristic that can be quantified.

b) A type of measuring instrument.

c) A base unit defined in the International System of Units (SI).

d) A reference standard used in laboratories.

4) Which of the following instruments is used to measure external dimensions precisely?

a) Micrometer

b) Caliper

c) Depth gauge

d) Protractor

5) What is the main function of a dial indicator?

a) To measure large lengths.

b) To check deviations or variations of a dimension from a reference.

c) To calculate angles in mechanisms.

d) To evaluate the roughness of a surface.

6) In which case would one use a profile projector?

a) To measure the thickness of a material.

b) To observe internal defects in metals.

c) To compare a part with a model using its projected shadow.

d) To measure the temperature of an object.

7) What types of geometric defects can be identified with a surface plate?

a) Flatness.

b) Concentricity.

c) Alignment.

d) The thickness of a part.

8) What is the main objective of a geometric defect inspection?

a) To check the conformity of a part's dimensions.

b) To identify deviations from specified geometric tolerances.

c) To determine the physical properties of the material.

d) To ensure the thermal balance of the part.

9) Which method can be used to detect roundness defects of a part?

a) Using a cylindrical measuring bench.

b) Profile projector.

c) Dial indicator mounted on a rotary table.

d) Liquid immersion test.

10) During dimensional inspection, which tool is best suited for measuring an internal diameter precisely?

a) Inside micrometer

b) Caliper

c) Thickness gauge

d) Protractor

11) An operator measures the length of a part several times with a caliper and gets similar results that are far from the expected value. What is the probable cause?

a) A precision problem.

b) An accuracy problem.

c) A parallax error.

d) A human error.

12) To check the concentricity of a cylindrical part, which instrument is best suited?

a) Dial indicator on a V-block.

b) Micrometer.

c) Caliper.

d) Profile projector.

13. What are the three branches of metrology?

A) Industrial metrology

B) Legal metrology

C) Scientific or fundamental metrology

D) Chemical metrology

14. Match the International System (SI) units to their base quantities:

Length ( )

Time ( )

Mass ( )

Electric current ( )

Thermodynamic temperature ( )

Amount of substance ( )

Luminous intensity ( )

A) Meter (m)

B) Second (s)

C) Kilogram (kg)

D) Ampere (A)

E) Kelvin (K)

F) Mole (mol)

G) Candela (cd)

15. Which physical quantities can influence a sensor's response?

A) Temperature

B) Pressure

C) Humidity

D) Mass

16. How can random and systematic errors be reduced during a measurement?

A) Repeating the measurement to reduce random errors

B) Applying corrections to reduce systematic errors

C) Using a high-precision sensor

D) Taking a single measurement

17. Which of the following corresponds to a general operating principle of a sensor?

A) The conversion of a physical quantity into a usable signal

B) The automatic suppression of human errors

C) Direct measurement without physical transformation

D) Systematic calibration before each measurement

Exercise 1

A student conducts a simple pendulum experiment to measure the acceleration due to gravity *g*. He repeats the time measurement several times; his results are recorded in the following table:

g(m/s2)	9,91	10,07	9,89	10,1	9,87	10.06

1. Calculate the average value of *g* and the standard deviation.
2. Given the standard deviation, state the confidence interval for the reported value of *g*.
3. What is the main source of uncertainty in this experiment? Justify your answer.

Répondre ici

Exercise 2

The measurement of the temperature at the outlet of a chimney gives the following values (in degrees Celsius):
135.0, 136.1, 138.9, 142.7, 140.1, 137.2, 142.7, 140.0, 142.8, 134.5

1. Calculate the average temperature of the exhaust gases at the outlet of this chimney.
2. Calculate the standard deviation.

3. Determine the confidence interval under the hypothesis that the gas temperature at the outlet of this chimney is 139.0 °C.

Répondre ici

Exercise 3

Repeated measurements of the height of a building yielded the following results: 33.48, 33.46, 33.49, 33.50, 33.49, 33.51, 33.48, 33.50, 33.47, 33.48, 33.49, 33.50, 33.47, 33.51, 33.50, 33.48 m.

1. Calculate the average value of these measurements.
2. Calculate the standard deviation.
3. Properly state the result in the form: h = h_average ± Δh

Répondre ici

Exercise 4

Draw the histogram of the distribution (frequency of a measurement versus the measurement value).

The measurement of the same current was carried out with 22 identical multimeters. The results are shown in the following table: I (mA)

(Note: The data table for current values (I in mA) is missing from the original text and would need to be provided to complete the translation and tasks.)

1. Calculate the average and standard deviation of this measured current.
2. Draw the histogram showing the frequency of a measurement versus the measurement value.

119,5	118,6	119,9	119,5	119,2	120,3	119,9	119,2	119,2	119,4	119,9
120,0	119,0	120,1	119,8	119,4	120,5	120,1	119,4	119,4	119,5	120,1

Répondre ici

Exercise 5

A series of weighings of a sample of mass m is performed with an electronic balance. The results are as follows:

m (g)	22,85	22,87	22,81	22,79	22,84

1. Calculate the average value and the standard deviation.
2. The displayed values are certified to 5%. Calculate the reported value with its uncertainty.

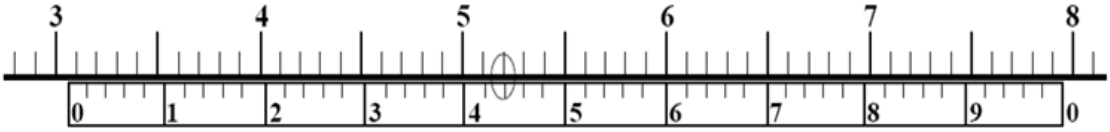
Répondre ici

Exercise 6

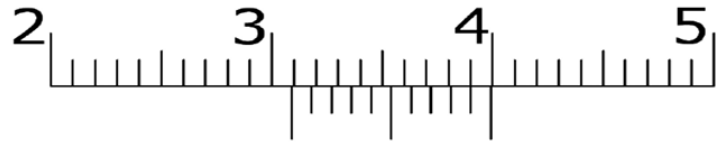
Measurement with a caliper is based on the Vernier method, illustrated by the following example:
(Note: The example mentioned is not provided in the original text.)

Donnez pour chacun des cas suivant la valeur lue et la précision :

A)……..



B)……..



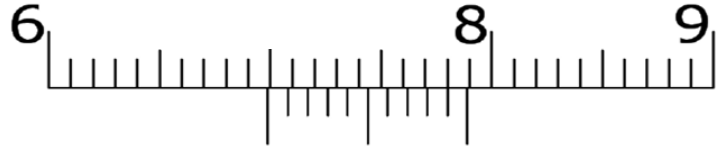
B)

C)……..



C)

D)……..



D)

Exercise 7

a) Prove that: Pascal × second is equivalent to Kilogram / (meter × second).
b) Give the dimensional equation for energy: $E = (1/2) \times m \times c^2$.

Répondre ici

Exercise 8

The measurement of a soil sample's mass gives 474.5 ± 3 g.

- What is the value of the absolute uncertainty of this measurement?
- Write the expression for the true value of the result!
- Calculate the relative uncertainty of this measurement (Ir).
- What can you conclude about the precision quality of this measurement: (normal precision, low precision, high precision)?

Répondre ici

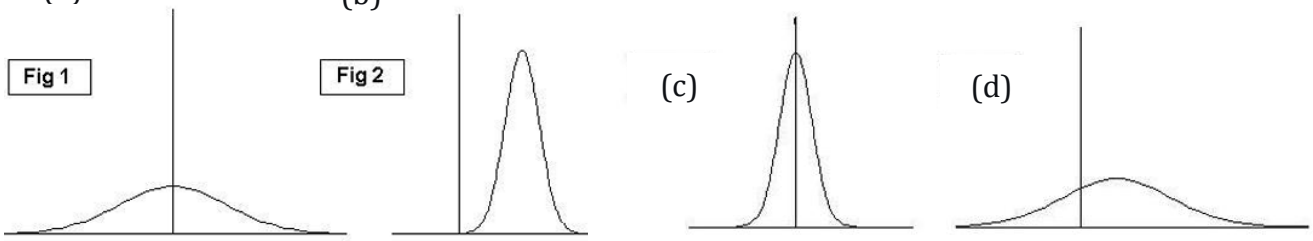
Exercise 9

Depending on the accuracy and precision characteristics of the sensor used in the measurement operation, the distribution of measurement results can be represented mathematically through different types of graphs.
Match each graph with its corresponding definition:

1. Precise but not accurate sensor: Systematic errors are significant but random errors are small.
2. Accurate but not precise sensor: Systematic errors are reduced but random errors are significant.
3. Neither accurate nor precise sensor: Both systematic and random errors are significant.
4. Accurate and precise sensor: Both systematic and random errors are small.

(a) (b)

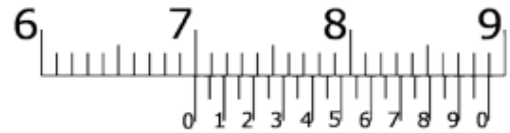
Fig 1 Fig 2



Répondre ici

Exercise 10

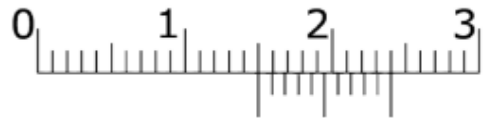
1) Give the displayed measurement values with unit precision using a caliper.



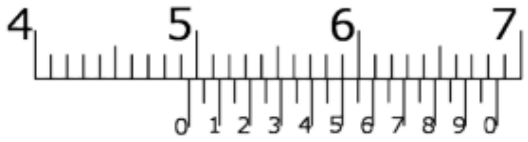
Lecture est de :
Précision est de :



Lecture est de
Précision est de



Lecture est de :
Précision est de :

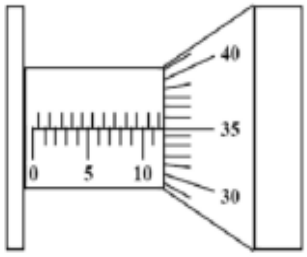


Lecture est de :
Précision est de :

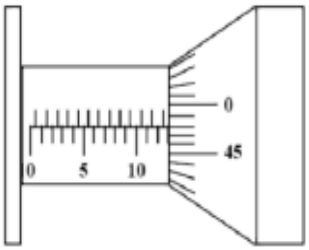


Lecture est de :
Précision est de :

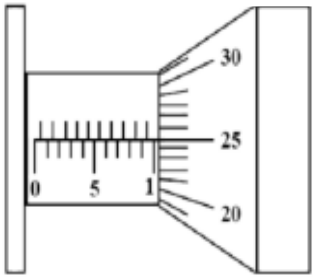
2) Complete the following readings according to the micrometer principle:
(Note: The specific readings/diagrams for the caliper and micrometer are missing from the original text and would need to be provided to complete this exercise.)



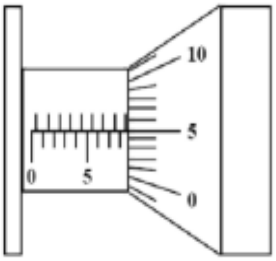
Lecture est de



Lecture est de



Lecture est de :



Lecture est de :

References

[1] Asch, G. (1998). *Les capteurs en instrumentation industrielle* (5e éd.). Dunod.

[2] Bergougnoux, L. (s.d.). *Conditionnement électronique des capteurs*. Polytechnique Marseille.

[3] Bouden, C. (1996). *Textbook for instrumentation lectures*. École Nationale d'Ingénieurs de Tunis.

[4] Gharbi Ernez, L. (2005). *Cours de mesure et instrumentation*. École Nationale d'Ingénieurs de Tunis.

[5] Bentley, J. P. (2005). *Principles of measurement systems* (4th ed.). Pearson Education.

[6] Bera, R. N. (2019). *Scientific metrology*. Springer Nature. 10.1007/978-981-10-4938-5

[7] CIPM (1980), BIPM Proc.-Verb. Com. Int. Poids et Mesures 48, C1-C30 (in French); BIPM (1980), Rapport BIPM-80/3, Report on the BIPM enquiry on error statements, Bur. Intl. Poids et Mesures (Sèvres, France) (in English)

[8] KAARLS, R. (1981), BIPM Proc.-Verb. Com. Int. Poids et Mesures 49, A1-A12 (in French); Giacomo, P. (1981), Metrologia 17, 73 -74 (in English)

[9] Kirkup, L., & Frenkel, R. B. (2006). *An introduction to uncertainty in measurement*. Cambridge University Press.