3-D Simulation of Induction Heating of Anisotropic Composite Materials

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This paper presents a three-dimensional modeling of induction heating of conductive composite materials using the shell elements. These elements are generalized by taking into account the anisotropic behavior of the load. Magnetic fields and temperatures on the composite load are then calculated and compared with experimental measurements.

***Index Terms*—Composite material, finite elements, induction heating, shell elements.**

I. Introduction

COMPOSITE materials are used in high technological industrial applications such as space and military purposes. Their structure is a combination of fibers reinforcement and a polymer matrix. In this paper, the case of the conductive composites with carbon fibers is considered where the induction heating process can be used. This process presents several advantages compared to classical techniques as quoted in the literature [1], [7]. In fact, with induction heating, the power is induced in the heart of the material and reduces in this way the cycle time of manufacturing.

To form a composite sheet, parallel carbon fibers are immerged in a thin layer of resin. Several layers are then assembled in different orientation to achieve the required thickness and the mechanical strength. In this case, the electric conductivity is a tensor in the $\mathbf{xy}$ plane, and null in the $\mathbf{z}$ direction

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}. \tag{1}$$

The thermal conductivity is also a tensor

$$\lambda = \begin{pmatrix} \lambda_{xx} & \lambda_{xy} & 0 \\ \lambda_{yx} & \lambda_{yy} & 0 \\ 0 & 0 & \lambda_z \end{pmatrix}. \tag{2}$$

In a composite material, there are several million fibers, so one can not take into account the real geometry in the simulation. Homogenization techniques [1], [3] are then used to evaluate the equivalent terms of σ and λ.

The thickness of the composite sheet is very low compared to its other dimensions. The shell elements then can be used for three-dimensional (3-D) modeling to reduce the finite element mesh size.

In this paper, an anisotropic shell element formulation is introduced to solve the electromagnetic problem. The thermal problem is then solved to calculate the temperature distribution inside the composite material. Finally the simulation results are compared with the experimental ones.

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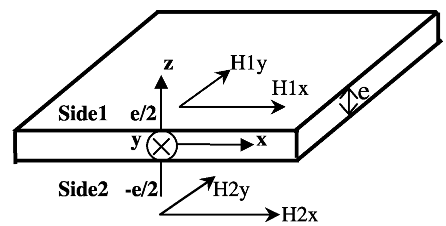

Fig. 1. Thin conducting plate.

II. Electromagnetic Problem

Shell elements are introduced in [2], [4] to solve 3-D electromagnetic problem on thin plates for isotropic materials.

To generalize the same problem for an anisotropic conducting plate, the following differential equations are solved:

$$\begin{cases} \frac{d^2H_x(z)}{dz^2} - j\omega\mu \cdot \sigma_{xx}H_x(z) = j\omega\mu \cdot \sigma_{xy}H_y(z) \\ \frac{d^2H_y(z)}{dz^2} - j\omega\mu \cdot \sigma_{yy}H_y(z) = j\omega\mu \cdot \sigma_{yx}H_x(z) \end{cases}. \tag{3}$$

The magnetic fields $(H_{1x}\ H_{1y}\ H_{2x}\ H_{2y})$ on the side "1" and the side "2" (Fig. 1) are used for initial conditions.

The solution of (3) is

$$\begin{cases} H_x(z) = A + B \\ H_y(z) = K_1 \cdot A + K_2 \cdot B \end{cases} \tag{4}$$

with

$$\begin{aligned} A = &\frac{1}{(K_2 - K_1) \cdot \sinh(eP_1)} \\ &\times \left[(K_2H_{1x} - H_{1y}) \cdot \sinh\left(\frac{e}{2}P_1 + P_1z\right) \right. \\ &\left. + (K_2H_{2x} - H_{2y}) \cdot \sinh\left(\frac{e}{2}P_1 - P_1z\right) \right] \\ B = &\frac{1}{(K_2 - K_1) \cdot \sinh(eP_2)} \\ &\times \left[(H_{1y} - K_1H_{1x}) \cdot \sinh\left(\frac{e}{2}P_2 + P_2z\right) \right. \\ &\left. + (H_{2y} - K_1H_{2x}) \cdot \sinh\left(\frac{e}{2}P_2 - P_2z\right) \right] \end{aligned}$$

K1 and K_2 are given by

$$K_1 = \frac{P_1^2}{j\omega\mu\sigma_{xy}} - \frac{\sigma_{xx}}{\sigma_{xy}} \quad \text{and} \quad K_2 = \frac{P_2^2}{j\omega\mu\sigma_{xy}} - \frac{\sigma_{xx}}{\sigma_{xy}}.$$

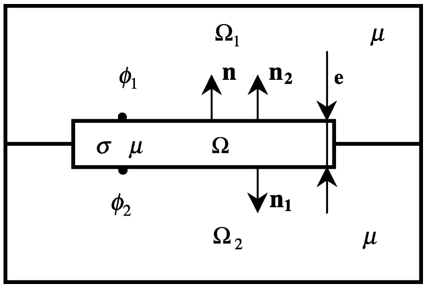

Fig. 2. Typical problem to solve.

P_1, P_2 are roots of characteristic equation of the differential equation system (3), and are given by

$$P_1 = \sqrt{\frac{j\omega\mu}{2}\left[(\sigma_{xx}+\sigma_{yy})+\sqrt{(\sigma_{xx}-\sigma_{yy})^2+4\cdot(\sigma_{xy})^2}\right]} \quad (5)$$

$$P_2 = \sqrt{\frac{j\omega\mu}{2}\left[(\sigma_{xx}+\sigma_{yy})-\sqrt{(\sigma_{xx}-\sigma_{yy})^2+4\cdot(\sigma_{xy})^2}\right]} \quad (6)$$

where ω is the angular frequency of the magnetic field, and μ is the permeability of the load.

Introducing (4) in Faraday's law combined with the local form of Ohm's law, one obtains the surface impedance description [2]

$$\begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{2x} \\ E_{2y} \end{pmatrix} = \mathbf{n} \times \begin{pmatrix} \alpha_1 & \alpha_2 & -\beta_1 & -\beta_2 \\ \alpha_3 & \alpha_4 & -\beta_3 & -\beta_4 \\ \beta_1 & \beta_2 & -\alpha_1 & -\alpha_2 \\ \beta_3 & \beta_4 & -\alpha_3 & -\alpha_4 \end{pmatrix} \begin{pmatrix} H_{1x} \\ H_{1y} \\ H_{2x} \\ H_{2y} \end{pmatrix} \quad (7)$$

where $\alpha_{1,2,3,4}$ and $\beta_{1,2,3,4}$ are scalar coefficients depending on K_1, K_2, P_1, and P_2, $\mathbf{n}$ is the normal vector, and E_{1x}, E_{1y}, E_{2x}, E_{2y} are the electrical fields on both sides of composite plate (Fig. 2).

The tangential magnetic fields on both sides of the plate according to reduced scalar potential are written

$$\begin{pmatrix} \boldsymbol{H_{1s}} \\ \boldsymbol{H_{2s}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{H_{js}} - \mathbf{grad}_s(\phi_1) \\ \boldsymbol{H_{js}} - \mathbf{grad}_s(\phi_2) \end{pmatrix} \quad (8)$$

where H_{js} is the source field calculated by Biot and Savart's law.

The shell integral formulation with reduced scalar potential (ϕ) on side "1" is written as

$$\int_{\Omega_1} \mu.\mathbf{grad}\, w \cdot \mathbf{grad}(\phi_1) d\Omega_1 + \frac{1}{jw}\int_\Gamma \mathbf{grad}_s w \cdot \begin{pmatrix} -\alpha_1 & -\alpha_2 & \beta_1 & \beta_2 \\ -\alpha_3 & -\alpha_4 & \beta_3 & \beta_4 \end{pmatrix} \begin{pmatrix} grad_x(\phi_1) \\ grad_y(\phi_1) \\ grad_x(\phi_2) \\ grad_y(\phi_2) \end{pmatrix} d\Gamma$$
$$= \int_\Gamma \mu w \boldsymbol{H}_j \cdot \boldsymbol{n_1} d\Gamma - \frac{1}{jw}\int_\Gamma \mathbf{grad}_s w \begin{pmatrix} (\beta_1-\alpha_1) & (\beta_2-\alpha_2) \\ (\beta_3-\alpha_3) & (\beta_4-\alpha_4) \end{pmatrix} \begin{pmatrix} H_{jx} \\ H_{jy} \end{pmatrix} d\Gamma. \quad (9)$$

The other equation corresponding to the side "2" of the shell is obtained by permuting of the index, 1 and 2 of normal vector $(\mathbf{n})$ and reduced scalar potential (ϕ).

Once the magnetic fields on both sides are calculated with (8), one can compute the current density induced $(\mathbf{J})$ and then the injected power by volume unity (P, heat source) in composite plate

$$\boldsymbol{J} = \mathbf{rot}\boldsymbol{H} \quad (10)$$
$$\boldsymbol{E} = \sigma^{-1} \cdot \boldsymbol{J} \quad (11)$$
$$P = \boldsymbol{J}^T \cdot \boldsymbol{E} = \boldsymbol{J}^T \cdot \sigma^{-1} \cdot \boldsymbol{J}. \quad (12)$$

III. Thermal Problem

The thermal problem is solved in the composite plate. The equation to be solved is written as [6]

$$\rho C_p \frac{\partial T}{\partial t} + \mathrm{div}(-\lambda\, \mathbf{grad}\, T) = P \quad (13)$$

with the Fourier boundary condition

$$-\lambda \cdot \frac{\partial T}{\partial \boldsymbol{n}} = h \cdot (T - T_a) \quad (14)$$

where P is the induced power density, ρ is the volumic masse, C_p is the specific heat, h is the convective coefficient, T is the temperature in composite plate, and T_a is the ambient temperature.

To solve the thermal equation, a 3-D finite element method is used.

The 3-D electromagnetic and thermal formulations are solved using our package developed under a Matlab workspace.

IV. Results and Discussions

A. *Influence of Anisotropic Behavior*

Due to the direction of carbon-fibers, the composite material presents a high degree of anisotropy which is reflected in many physical properties including thermal and electrical conductivity [8]. To determinate the terms of electrical and thermal tensor conductivity, the homogenization techniques are used to obtain σ_u and σ_v in the main directions of the fibers (Fig. 3). Once σ_u and σ_v are determined, the equivalent terms of the tensor σ in x and y directions can be calculated in the static case with the following expressions [3]:

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} (\sigma_u \cos^2\theta + \sigma_v \sin^2\theta) & ((\sigma_u-\sigma_v)\cdot\cos\theta\cdot\sin\theta) \\ ((\sigma_u-\sigma_v)\cdot\cos\theta\cdot\sin\theta) & (\sigma_v\cos^2\theta+\sigma_u\sin^2\theta) \end{bmatrix}. \quad (15)$$

To see the influence of the anisotropic behavior of composite material in the case of induction heating one use a rectilinear inductor oriented in (y) direction. Figs. 3 and 4 show the inductor position for the plates with different orientation.

One takes: $\sigma_u = 1.1 \cdot 10^5$ S/m, and $\sigma_v = 0.8 \cdot 10^5$ S/m.

The inductor used in these simulations has a square form with one turn. The thickness of the composite is 4.5 mm, the intensity of the current is 150 A, and the electromagnetic frequency is 400 kHz. The airgap between the inductor and the load is 0.5 mm.

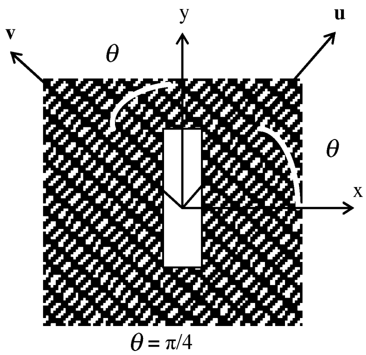

Fig. 3. Calculation of the tensor of conductivity.

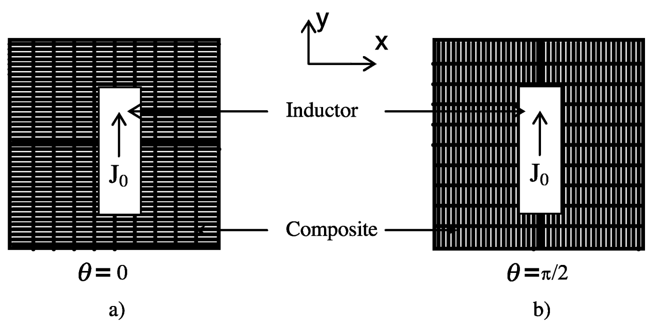

Fig. 4. Placement of inductor. a) Inductor in low conductivity direction; b) inductor in high conductivity direction.

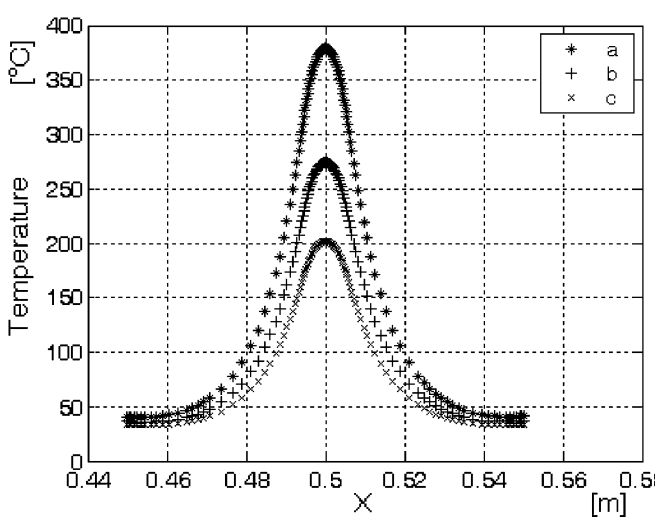

Fig. 5. Temperature distribution for different orientations of the fibers.

The figures (Fig. 5) shows the variation of the along the line $y = 0$ temperature for a heating time of 20 s, at the surface of the load.

When the inductor is in the low conductivity direction (case a: $\theta = 0$), induction heating has the best performances. The power induced in the plate is 274 W, and the maximum temperature is 380 °C. Whereas in the case b ($\theta = \pi/2$), the power is 159 W and the maximum temperature is 202 °C, in the case c ($\theta = \pi/4$) the power is 204 W and the maximum temperature is 275 °C.

This can be explained by the fact that, with the parameters used in this simulation (thickness, conductivity range of the plate and the frequency of the inducting current), the power induced in the plate decreases with increasing conductivity. This is shown in Fig. 6 for isotropic plate with the same thickness.

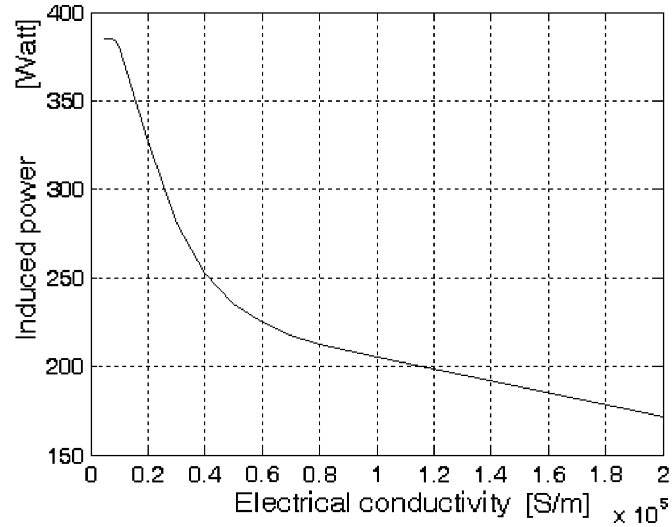

Fig. 6. Induced power in load as a function of electrical conductivity.

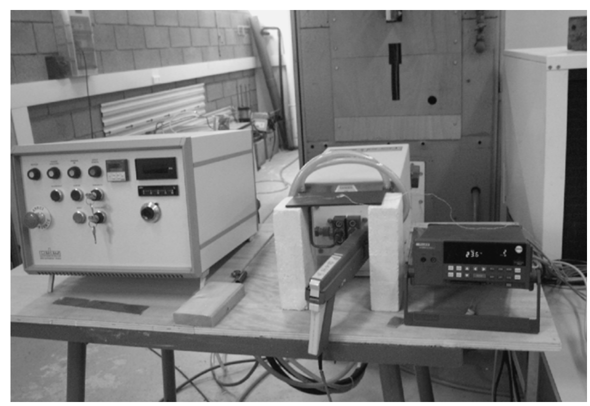

Fig. 7. Induction heating setup.

B. Experimentation

The experimentation was performed with an induction generator with a voltage hold constant at 220 V. The temperatures of the load were measured by means of thermocouples, using a data acquisition device. Fig. 7 shows the general sight of the induction heating setup for our application. The investigations for temperature measurements were carried of carbon-fiber reinforcement polyphenylene sulfide (CF/PPS).

The composite plate has the thickness of 3.2 mm, length of 200 mm and width of 100 mm. The injected current is 91 A, the electromagnetic frequency is 220 kHz and the load is placed at a distance of 0.9 mm on top of the inductor. The load is heated during 50 s. The temperatures are measured each 2 s.

Fig. 8 shows the measured and simulated temperature evolution in the center point of the surface near to the inductor.

The agreement between the measured and the simulated temperatures are very good. The difference between the temperatures does not exceed 4%. Fig. 9 shows the simulated and mea-

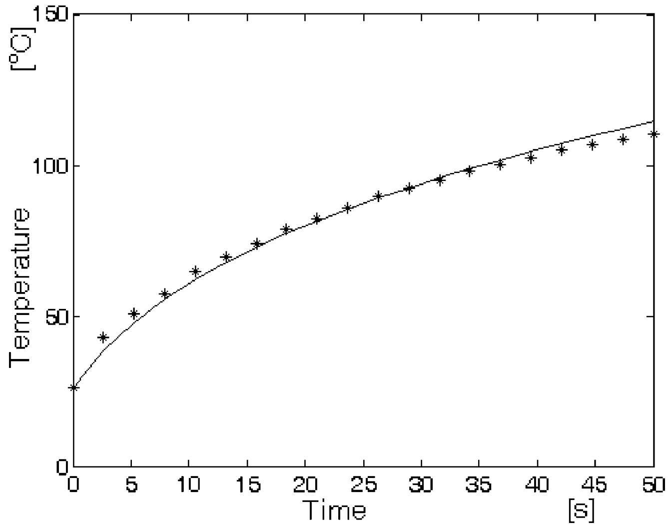

Fig. 8. Temperature evolution on surface point.

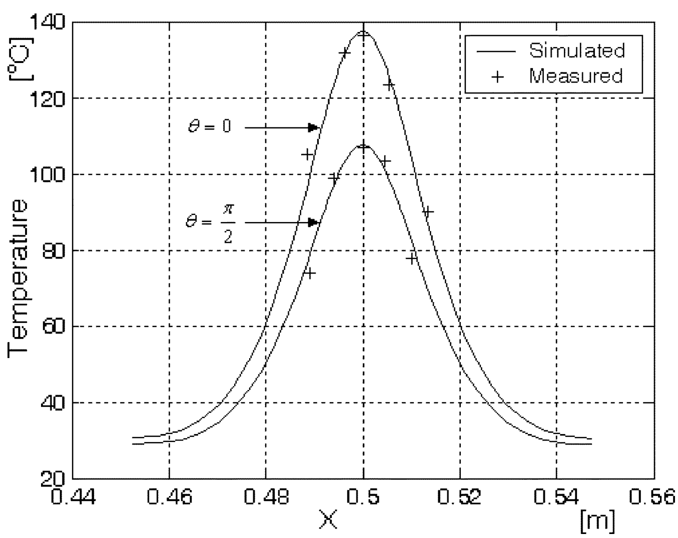

Fig. 9. Temperature distribution for 0 and $\pi/2$ orientations.

sured temperature distributions of the two cases $(\theta = 0)$ and $(\theta = \pi/2)$.

V. Conclusion

A formulation with the shell elements has been presented to solve the electromagnetic problem in thin plate of conductive anisotropic material. The model is verified with an experimental application.

The simulated and experimental results show that the anisotropic characteristic of composite material has a great impact in their induction heating. This is very important in the case of composite materials with layer of different orientations.

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