Electrical Conductivity Identification of Composite Materials Using a 3-D Anisotropic Shell Element Model

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This paper presents an electrical conductivity identification technique of composite materials based on electrical resistance measurement with a frequency range from 1 kHz to 1 MHz. Resistance is measured by an impedance analyzer and also computed using the 3-D anisotropic shell element model. The inverse problem technique is then applied to identify the conductivity tensor components. This technique is validated using anisotropic materials with known electrical conductivity tensor.

***Index Terms*—Anisotropic composite material, electrical conductivity tensor, electrical resistance calculation, inverse problem, shell elements.**

I. Introduction

COMPOSITE materials are used in many manufacturing industries. Carbon fibers composites have superior specific mechanical properties compared to conventional metallic materials. In recent years, several works have been realized to study the electromagnetic and thermal behavior of carbon fiber composite [1]–[3], [5]. The main aim of these studies is the use of eddy currents in manufacturing, transforming, and health inspecting of the material.

Carbon fibers composite materials have a strong anisotropy due to the different orientation of the fibers in the composite layers. Therefore, to study and understand the behavior of these materials under the influence of magnetic field, a 3-D electromagnetic modeling is required. However, the important number of carbon fibers impregnated in each layer and their size compared to global dimensions make difficult to take them into account in the model. The microscopic electrical conductivity of the composite materials is then replaced by a homogenized anisotropic tensor.

Electrical conductivity of conductive materials can be obtained through an electrical resistance measurement. Due to the size of carbon fibers and the layers orientation, it is very difficult to measure the resistance of multilayer composite materials using the direct contact alternating current/direct current (ac/dc) methods. Hence, eddy current method will be used to overcome these difficulties.

In this paper, the measurement of electrical resistance using induced power is elaborated to identify the conductivity of the homogenized anisotropic tensor. Inverse problem is then used to seek conductivity tensor components by minimizing the difference between the computed and measured resistance.

In our case, composite material have a small thickness compared to other dimensions, therefore, a 3-D anisotropic shell element model [1]–[5] is used to compute the resistance for a working frequency range from 1 kHz up to 1 MHz.

Manuscript received October 07, 2008. Current version published February 19, 2009. Corresponding author: S. Bensaid (e-mail: samir_bensaid@hotmail.com).

Digital Object Identifier 10.1109/TMAG.2009.2012833

II. Problem Formulation

To elaborate a multilayer composite material, carbon fibers are immerged in a thin layer of resin. Several layers are then assembled together with different fibers orientations to achieve the required thickness and the mechanical strength (Fig. 1). Electrical conductivity of such materials has a tensorial form. Due to the small insulated polymer gap between the layers and within the working frequency range, electrical conductivity in z direction is neglected. Therefore, electrical conductivity tensor in the directions x and y can be expressed by

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}. \tag{1}$$

If the conductivity components of the composite layer in the direction v of the fibers and in the direction u perpendicular to the fibers are known (Fig. 1), one can obtain the conductivity tensor σ in the directions x and y using [5]

$$\begin{cases} \sigma_{xx} = \sigma_u(\cos\theta)^2 + \sigma_v(\sin\theta)^2 \\ \sigma_{xy} = (\sigma_v - \sigma_u)\cdot\cos\theta\sin\theta \\ \sigma_{yx} = (\sigma_v - \sigma_u)\cdot\cos\theta\sin\theta \\ \sigma_{yy} = \sigma_u(\sin\theta)^2 + \sigma_v(\cos\theta)^2. \end{cases} \tag{2}$$

Thus, in the case of composite materials with all layers oriented in the same direction, the tensor conductivity σ is obtained easily by measuring the conductivity in u and v directions. But generally, composite materials are composed of several layers with different fibers orientations. In this case, the fiber directions u and v are unknown. Therefore, measurement should be done in some suitable directions x and y and the four components of the tensor conductivity are determined by experimentation.

Electrical conductivity of conductive materials depends directly on electrical resistance. Due to the difficulties to establish direct contact with the fibers of the composite material, eddy current method is used to measure resistance of such materials.

The industrial conductivity meters for isotropic materials use generally a circular probe. This kind of probe is not suitable for oriented fibers and anisotropic materials. We have then designed a rectangular probe with a U ferrite core [Fig. 2(a)]. The advantage of this probe is that it can be positioned in different orientations especially in u and v direction. This additional property

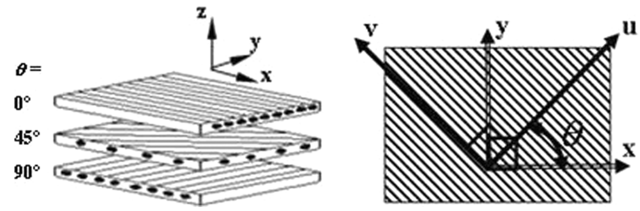
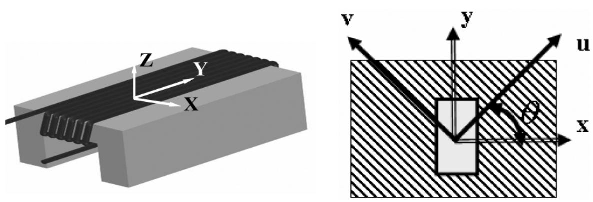


Fig. 1. Multilayer composite materials.



a) b)

Fig. 2. (a) Designed elongated probe. (b) Probe position.

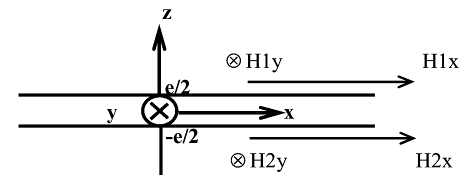


Fig. 3. Homogenized anisotropic composite plate.

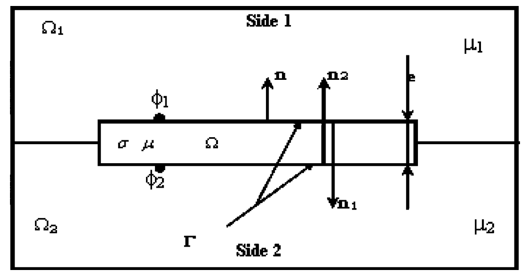


Fig. 4. Shell element problem representation.

increases substantially the sensitivity of the resistance measurement of the composite materials.

A. Computing Resistance Using a 3-D Shell Element Model

Due to the small thickness of composite sheet compared to their other dimensions, a 3-D anisotropic shell element model [1] is used to compute the resistance.

One considers in the following calculus the homogenized anisotropic composite plate (Fig. 3). To compute the resistance of the composite plate, one must solve the electromagnetic problem illustrated in Fig. 4.

The 3-D anisotropic shell element formulation in side 1 of the shell element problem is written as [1]–[5]

$$\begin{aligned}\int_{\Omega_1} & \mu_1 \cdot \mathbf{grad}\, w \cdot \mathbf{grad}\phi_1 \cdot d\Omega_1 \\ & + \frac{1}{j\omega}\int_{\Gamma} \mathbf{grad_s} w \cdot \begin{pmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{pmatrix} \cdot \mathbf{grad_s}\phi_1 \cdot d\Gamma \\ & - \frac{1}{j\omega}\int_{\Gamma} \mathbf{grad_s} w \cdot \begin{pmatrix} \beta_{xx} & \beta_{xy} \\ \beta_{yx} & \beta_{yy} \end{pmatrix} \cdot \mathbf{grad_s}\phi_2 \cdot d\Gamma_2 \\ & = \int_{\Gamma} \mu_1 \cdot w \cdot \mathbf{H_j} \cdot \mathbf{n_1} \cdot d\Gamma + \frac{1}{j\omega}\int_{\Gamma} \mathbf{grad_s} w \\ & \quad \cdot \begin{bmatrix} (\alpha_{xx}-\beta_{xx}) & (\alpha_{xy}-\beta_{xy}) \\ (\alpha_{yx}-\beta_{yx}) & (\alpha_{yy}-\beta_{yy}) \end{bmatrix} \cdot \begin{pmatrix} H_{jx} \\ H_{jy} \end{pmatrix} \cdot d\Gamma \end{aligned} \tag{3}$$

where $\mathbf{n_1}$ is the normal vector, $\mathbf{H_{js}}$ is the source field calculated by Biot and Savart's law, and α and β are the tensor component coefficients depending on the tensor conductivity σ, the magnetic permeability of the layer μ, the angular frequency of the magnetic field ω, and the thickness of the composite plate $\mathbf{e}$ [1]. The expressions of α and β tensor component coefficients are given in the Appendix.

By permuting the index 1 and 2 of the normal vector $\mathbf{n}$, the reduced scalar potential ϕ, the magnetic permeability μ, and the side name Ω in (3), one obtains the other equation corresponding to side 2 of the shell [1].

The 3-D anisotropic shell element formulation expressed by the magnetic scalar potential "ϕ" is solved in a Matlab® workspace. Once the magnetic scalar potential is determined, one calculates the magnetic and electrical fields on both sides of composite plate (Fig. 3) using (4) and (5). The power induced inside the composite plate is then computed using the Poynting vector integral (6)

$$\begin{pmatrix} \mathbf{H}_{1s} \\ \mathbf{H}_{2s} \end{pmatrix} = \begin{pmatrix} \mathbf{H_{js}} - \mathbf{grad_s}(\phi_1) \\ \mathbf{H_{js}} - \mathbf{grad_s}(\phi_2) \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{2x} \\ E_{2y} \end{pmatrix} = \mathbf{n_1} \begin{pmatrix} -\alpha_{xx} & -\alpha_{xy} & \beta_{xx} & \beta_{xy} \\ -\alpha_{yx} & -\alpha_{yy} & \beta_{yx} & \beta_{yy} \\ -\beta_{xx} & -\beta_{xy} & \alpha_{xx} & \alpha_{xy} \\ -\beta_{yx} & -\beta_{yy} & \alpha_{yx} & \alpha_{yy} \end{pmatrix} \begin{pmatrix} H_{1x} \\ H_{1y} \\ H_{2x} \\ H_{2y} \end{pmatrix} \tag{5}$$

$$P = \frac{1}{2}\left(\int\int_{S_2} (\mathbf{E_{2s}}\mathbf{H^*_{2s}}) \cdot dS - \int\int_{S_1} (\mathbf{E_{1s}}\mathbf{H^*_{1s}}) \cdot dS \right). \tag{6}$$

The real part of P represents the injected active power in the composite load. This power can be related to the resistance variation of the probe with and without the presence of the composite sheet [6]. Therefore, the resistance variation $\mathrm{dR_{cal}}$ of probe can be expressed as follows:

$$\mathrm{dR_{cal}} = \frac{\mathrm{real}(P)}{I^2} \tag{7}$$

where I is the electrical current source of the probe.

B. Identification of Conductivity Tensor Using Inverse Problem

The measured resistance is greatly dependent on the probe orientation and on the frequency value. The best situation for the one layer is the measurement in u and v directions. For a material with multiorientation layers, one searches the probe orientation, which gives the greatest resistance. The measurement will be done in this direction and the direction perpendicular to it.

To identify the conductivity tensor components, the computed values are compared to the measured ones using an inverse problem algorithm for two orientations of the probe.

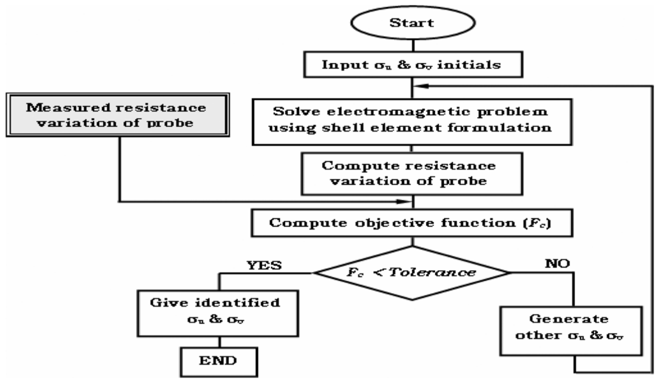


Fig. 5. Inverse problem algorithm.

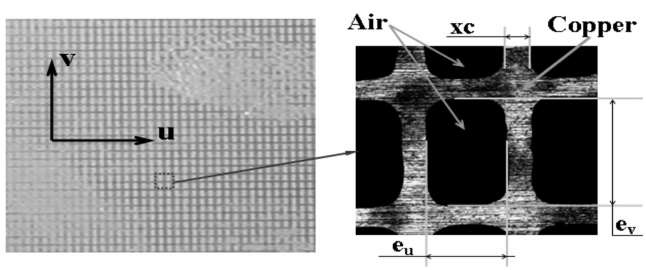


Fig. 6. Printed circuit board achieved with copper pathways thickness of 35 μm.

TABLE I
CHARACTERISTIC DIMENSIONS OF REALIZED SPECIMENS OF ANISOTROPIC PRINTED CIRCUIT BOARD MATERIALS

	Specimen A	Specimen B
e_u [μm]	470	360
e_v [μm]	840	950
xc [μm]	150	260

The objective function is expressed as a function of the probe orientation and the frequency values

$$F_c = \frac{1}{2}\left(\frac{\mathrm{dR_{meas\_1}} - \mathrm{dR_{cal\_1}}}{\mathrm{dR_{meas\_1}}}\right)^2 + \frac{1}{2}\left(\frac{\mathrm{dR_{meas\_2}} - \mathrm{dR_{cal\_2}}}{\mathrm{dR_{meas\_2}}}\right)^2 \quad (8)$$

where $\mathrm{dR_{meas}}$ and $\mathrm{dR_{cal}}$ are, respectively, the measured and the calculated resistance variation of the probe. The indices 1 and 2 correspond successively to the measurement in $\theta = \pi/2$ and $\theta = 0$ probe position.

The resistance of probe is measured using a precision impedance analyzer. The resistance variation of the probe is computed using a 3-D anisotropic shell element model and then compared to the measured one until the verification of convergence criterion (tolerance) of the objective function F_c. The conductivity tensor components are then identified. These steps are illustrated by the inverse problem algorithm (Fig. 5).

III. Results and Discussion

The proposed identification technique of conductivity tensor components is first validated on a set of anisotropic materials realized by ourselves using the printed circuit board (Fig. 6). Table I shows the characteristic dimensions of the boards.

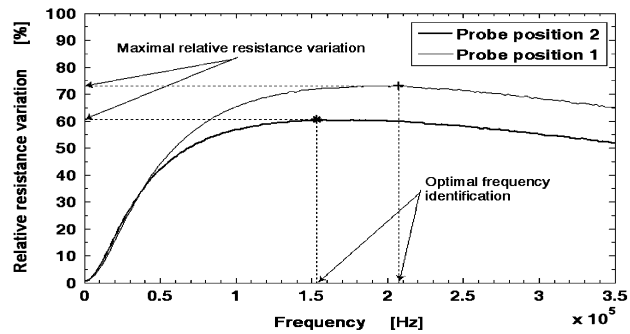


Fig. 7. Relative variation of resistance of the probe depending on the frequency (in case of specimen A).

TABLE II
IDENTIFIED CONDUCTIVITY TENSOR COMPONENTS COMPARED TO THE KNOWN CONDUCTIVITY

		Specimen A	Specimen B
σ_u [MS·m^{-1}]	Known	8.64	12.25
	Measured	8.25	12.7
	Difference [%]	**4.5**	**3.7**
σ_v [MS·m^{-1}]	Known	13.79	23.9
	Measured	13.18	24.6
	Difference [%]	**4.4**	**2.93**

The working frequency has a great influence on the sensitivity of the probe. Fig. 7 illustrates the relative resistance of the anisotropic material (specimen A) as a function of frequency. The relative resistance has a maximum at the frequencies 150 and 200 kHz for the two positions of the probe.

The identified conductivity tensor components of A and B specimens are then compared to the ones given in Table II. The difference between the identified conductivity tensor and the given one is less than 5%.

The computed resistance depending on frequency of the two specimens using the identified conductivity tensor is then compared to the measured one. Fig. 8 shows the results for the specimen A. A good concordance of results with an error less than 4% is observed.

In the second step, the identification technique of conductivity tensor components is applied to identify the conductivity of carbon fiber composite materials.

The conductivity tensor of a one layer woven carbon fiber composite material is measured. The obtained conductivity tensor components in the directions u and v are, respectively, 14 587 and 15 470 S/m. This conductivity tensor is then introduced in a 3-D anisotropic shell element model to compute resistance according to the frequency (Fig. 9). One has obtained a difference less than 2% between the computed resistances and the measured ones.

Identified conductivity tensor of the one layer carbon fibers composite material can be introduced in a 3-D multilayer anisotropic shell element model [5] to study the electromagnetic behavior of the multilayer multioriented carbon fiber composite materials.

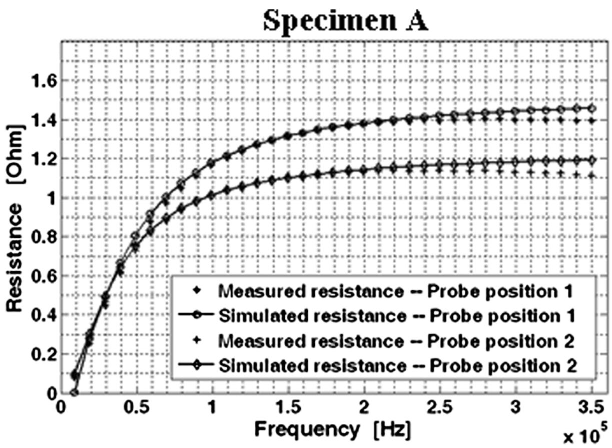


Fig. 8. Measured and simulated resistance variation of probe depending on the frequency in case of specimen A.

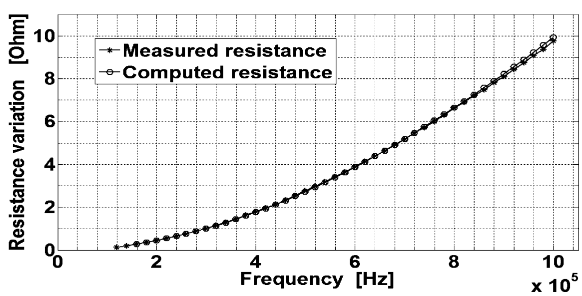


Fig. 9. Measured and simulated resistance variation of probe depending on the frequency in case of a one layer woven carbon fiber composite material.

NB: The probe positions 1 and 2 correspond successively to the measurement in $\theta = \pi/2$ and $\theta = 0$ directions [Fig. 2(b)].

IV. Conclusion

In this work, an eddy currents identification technique of electrical conductivity tensor components of anisotropic materials is presented. A specific probe, with a high sensitivity of resistance according to the orientations, is designed. The 3-D anisotropic shell element is used to compute resistance. Inverse problem is applied to identify conductivity tensor components. The method is validated on the anisotropic printed circuit board materials.

Conductivity tensor of a one layer woven carbon fiber composite material is measured and then introduced in a 3-D anisotropic shell element model to compute the resistance. The confrontation between the computed resistance and the measured one shows a very good concordance.

Appendix

The α and β tensors components coefficients are expressed as follows:

$$\begin{cases} \alpha_{xx} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{K_2 P_1}{\tanh(eP_1)}(\sigma_{yx}K_1 + \sigma_{xx}) - \frac{K_1 P_2}{\tanh(eP_2)}(\sigma_{yx}K_2 + \sigma_{xx})\right] \\ \alpha_{xy} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{P_2}{\tanh(eP_2)}(\sigma_{yx}K_2 + \sigma_{xx}) - \frac{P_1}{\tanh(eP_1)}(\sigma_{yx}K_1 + \sigma_{xx})\right] \\ \alpha_{yx} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{K_2 P_1}{\tanh(eP_1)}(\sigma_{yy}K_1 + \sigma_{xy}) - \frac{K_1 P_2}{\tanh(eP_2)}(\sigma_{yy}K_2 + \sigma_{xy})\right] \\ \alpha_{yy} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{P_2}{\tanh(eP_2)}(\sigma_{yy}K_2 + \sigma_{xy}) - \frac{P_1}{\tanh(eP_1)}(\sigma_{yy}K_1 + \sigma_{xy})\right] \end{cases}$$

$$\begin{cases} \beta_{xx} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{K_2 P_1}{\sinh(eP_1)}(\sigma_{yx}K_1 + \sigma_{xx}) - \frac{K_1 P_2}{\sinh(eP_2)}(\sigma_{yx}K_2 + \sigma_{xx})\right] \\ \beta_{xy} = \frac{1}{(K_2 - K_1 \cdot det)} \\ \quad \times \left[\frac{P_2}{\sinh(eP_2)}(\sigma_{yx}K_2 + \sigma_{xx}) - \frac{P_1}{\sinh(eP_1)}(\sigma_{yx}K_1 + \sigma_{xx})\right] \\ \beta_{yx} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{K_2 P_1}{\sinh(eP_1)}(\sigma_{yy}K_1 + \sigma_{xy}) - \frac{K_1 P_2}{\sinh(eP_2)}(\sigma_{yy}K_2 + \sigma_{xy})\right] \\ \beta_{yy} = \frac{1}{(K_2 - K_1 \cdot \det)} \\ \quad \times \left[\frac{P_2}{\sinh(eP_2)}(\sigma_{yy}K_2 + \sigma_{xy}) - \frac{P_1}{\sinh(eP_1)}(\sigma_{yy}K_1 + \sigma_{xy})\right] \end{cases}$$

where

$$K_1 = \frac{\sigma_{yy}}{\sigma_{xy}} - \frac{P_1^2}{j\omega\mu\sigma_{xy}} \quad K_2 = \frac{\sigma_{yy}}{\sigma_{xy}} - \frac{P_2^2}{j\omega\mu\sigma_{xy}}$$

$$P_1 = \sqrt{\frac{j\omega\mu}{2}\left[(\sigma_{xx} + \sigma_{yy}) + \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \cdot \sigma_{xy}^2}\right]}$$

$$P_2 = \sqrt{\frac{j\omega\mu}{2}\left[(\sigma_{xx} + \sigma_{yy}) - \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \cdot \sigma_{xy}^2}\right]}.$$

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