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Kalman filter technique for multisite modelling and streamflow prediction in Algeria

Khadidja Boukharouba and Ahmed Kettab*

*Research Laboratory of Water Sciences, LRS-EAU, National Polytechnic School (E.N.P.), Algiers, 10 Av. Hacene-Badi, PO Box 182, El-Harrach 16200, Algiers, Algeria. *e-mail: kettab@yahoo.fr, b_malikadz@yahoo.com*

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Abstract

Multisite modelling and streamflow prediction are mostly required in water resources design and management. This paper aims at investigating the extent of applicability of Kalman filter (KF) for modelling and predicting streamflow records in northern Algeria, which has never been done before. KF method is based upon the recursive least squares concept and has the important property of sequential optimization, which means that the model is adaptively updated as soon as the output of the system is obtained. One of the advantages of the KF technique is that the property of stationarity is not a prerequisite. This allows for changes in model parameters and variances, which is a manner that accommodates the non-linear response of hydrologic systems. Another advantage of KF is that its application is elaborated in the time-domain. This characteristic plays an important role in real time forecasting of hydrologic time series. Besides, the KF algorithm may be initiated with minimum objective information and adjusts itself subsequently whenever more data become available. The main purpose of this paper is to apply the KF approach to the modelling and prediction of multisite streamflow in northern part of Algeria. The data used are the annual streamflow records at 10 hydrometric stations located in the above region. The obtained result is an online prediction operation where the streamflow predictor is not bound to time or space, but rather adapts itself recursively to evolving conditions related to meteorology or other physical systems in the study area. It is observed that the obtained results are satisfactory and the associated errors are quite acceptable.

Key words: Kalman filter, modelling, stochastic, prediction, multisite, estimation error covariance, transition matrix, streamflow, hydrometric station, northern part of Algeria.

Introduction

Hydrologic variables are products of complex time-varying processes which can be measured by a finite number of observations. These observations include a noise component and indicate that hydrologic variables are stochastic in nature [1] and nonlinear [2].

One of the most well-known and often-used mathematical tools that can be used for stochastic estimation from noisy measurements is the Kalman filter (KF). It describes a recursive solution to the linear filtering problem of discrete data [3]. This constitutes a currently popular recursive technique for estimating the state of a given system in the presence of noise [4]. Some years later, the research works [5, 6] have been followed by an explosive growth of their application to various problems in several areas of interest. Gelb [7] presented the importance of optimal estimation using the KF technique in practical applications.

Regarding hydrology and water-related issues, which are subject to random and unknown influences [8], the KF technique has been the subject of extensive research and application. Of particular interest here is the book presented by Chiu [9], which presents the KF as a significant amount of disclosure to water scientists by giving an introduction to the fundamentals of the KF theory and technique to identification and illustration of cases using KF in hydrology, hydraulics and water resources and determination of directions of future study and finally investigation of potential areas where applications of KF are most effective. One of these interesting areas is data assimilation [10-13]. Another particular area where KF has also been the subject of extensive research and application is the hydrological estimation and prediction [14-16]. In regard to modelling and prediction of streamflow processes, Bergman and Delleur [17] used Kalman filtering to predict daily flows, Ngan and Russel [18] studied Kalman filtering in combination with ARMAX models for daily flows, Jimenez *et al.* [19] applied Kalman filtering to identify periodic autogressive-moving average (PARMA) models for monthly sreamflow, Awwad and Valdés [20] applied Kalman filtering to identify ARMAX models for multisite daily flows and Awwad *et al.* [21] used Kalman filtering and ARMAX models in forecasting multisite flows with multiple-step-ahead intervals and a six-hour time step. Among the most recent publications referring to the KF applications one can find Moradkhani *et al.* [23], Weerts and El Serafy [24] and Ouachani *et al.* [25].

The main purpose of this paper was to apply the KF approach to the modelling and prediction of multisite streamflow in northern Algeria. The obtained result is an online prediction operation where the streamflow predictor is not bound to time or space, but rather adapts itself to evolving conditions related to meteorology or other physical systems. The data used in the present study are the annual streamflow records at 10 hydrometric stations located in northern part of Algeria.

Theoretical Background

The KF is a mathematical procedure that provides an efficient computational (recursive) method for the least-squares estimation of a linear system. It does so in a predictor-corrector fashion, predicting short-term (following the last estimate) changes in the state using a dynamic model, and then correcting them with a measurement and a corresponding measurement model. The KF predictor-corrector type estimator is optimal in the sense that it minimizes the estimated error covariance, when some presumed conditions are met. The main characteristic of KF technique is that it determines accurately the forecast error covariance, which is a measure of the accuracy. Kalman filter is available in the book of Maybeck [26]. A more complete introductory discussion [6] also contains some interesting historical narrative. More extensive references include Lewis [27], Jacobs [28], Brown and Hwang[29] and Grewal and Andrews [30, 31].

The fundamental multidimensional KF recursive equations can be expressed as follows:

$$X_k = \Phi_{k/k-1}\, X_{k-1} + W_{k-1} \tag{1}$$

$$Z_k = H_k\, X_k + V_k \tag{2}$$

The state model is given by Equation 1 where W_{k-1} is the system error vector, which is assumed to be a white noise sequence with a known variance Q_k. The dimensions of both state and system error vectors are $(n\times 1)$, and the dimensions of the transition matrix $\Phi_{k/k-1}$ as well as the error variance are $(n\times n)$. Meanwhile, Equation 2 gives the measurement or observation model, which transforms the state vector X_k to a measurement vector Z_k via the H_k matrix, such transformation is assumed to be linear. The measurement error vector V_k is assumed to be a white noise sequence with known variance R_k and zero cross-correlation with the W_{k-1} sequence. This demonstrates the noiseless connection between state and measurement vectors. The dimensions of both measurement and measurement error vectors are $(m\times 1)$ and the dimensions of H_k as well as the measurement error variance R_k are $(m\times n)$; m being generally equal to or smaller than n.

Suppose that the measurements are available until time t_{k-1} (initial time), the a-priori estimate of the process based on knowledge of the process prior to t_{k-1} might be available and can be denoted by $\hat{X}_{k/k-1}$. The updated estimate $\hat{X}_{k/k}$ at time t_k incorporates the new measure Z_k in order to improve the a-priori estimate $\hat{X}_{k/k-1}$. This is done by adding a correction term, which comprises the difference $(Z_k - H_k \hat{X}_{k/k-1})$, multiplied by a blending factor K_k assumed to yield an optimum estimate $\hat{X}_{k/k}$ as far as the least squares are concerned. Hence one can obtain:

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k\,(Z_k - H_k \hat{X}_{k/k-1}) \tag{3}$$

The associated error covariance matrix $P_{k/k}$ is given by the expression:

$$P_{k/k} = (\mathrm{I} - K_k H_k)\, P_{k/k-1}\, (\mathrm{I} - K_k H_k)^T + K_k R_k K_k^T \tag{4}$$

The problem now is to find the particular vector K_k that minimizes the individual terms in the major diagonal of $P_{k/k}$ as these terms represent the estimation error covariance of the elements of the state vector, which has been estimated. After some mathematical manipulations, one can obtain the following relationship.

$$K_k = P_{k/k-1}\, H_k^T\, (H_k\, P_{k/k-1}\, H_k^T + R_k)^{-1} \tag{5}$$

The above expressed particular K_k, which minimizes the mean square estimation error, is known as Kalman gain. This gain can be used to calculate the updated estimate $\hat{X}_{k/k}$ and the updated associated error covariance matrix $P_{k/k}$. After some mathematical manipulations one can arrive at the following expression.

$$P_{k/k} = (1 - K_k H_k)\, P_{k/k-1} \tag{6}$$

With the above-described procedure, it becomes possible to project ahead the updated estimation $\hat{X}_{k+1/k}$ using the transition matrix as follows:

$$\hat{X}_{k+1/k} = \Phi_{k+1/k}\, \hat{X}_{k/k} \tag{7}$$

where the contribution of W_k in Equation 1 is not considered, because it has zero mean and no correlation with the previous W's. The associated error covariance matrix is then given as:

$$P_{k+1/k} = \Phi_{k+1/k} P_{k/k}\, \Phi^T_{k+1/k} + Q_k \tag{8}$$

These are the necessary expressions for prediction at time t_{k+1}. For the next step estimations $\hat{X}_{k+1/k}$ and $P_{k+1/k}$ are considered as initial conditions. For more details on the theoretical development of the KF algorithm and relating equations see Sen and Latif [32].

In the present paper, it is assumed that the randomness of the system is white Gaussian noise with a covariance matrix, Q. It is also assumed that the measurement noise is random with a covariance matrix, R, that is not correlated with the system noise, and finally it is assumed that on average, the estimate of the state will equal the true state.

Application and Discussion of Results

In the present paper, the KF technique has been applied using data of annual streamflow series observed at 10 gauging stations, 5 of them located at sites of major dams in northern Algeria. Fig. 1 shows the location of the hydrometric stations and their rivers. The recorded time series were provided by the National Agency of Hydraulic Resources (ANRH) of Algiers, and have a common observation period of 25 years (1968-1992). As such, for n = 10 the state and measurement vectors have (10×1) dimensions, while the covariance matrices as well as the system transition and measurement matrices have (10×10) dimensions.

Initial conditions: One of the most significant steps in the application of the KF technique is the problem formulation into state and measurement equations as given by Equations 1 and 2, respectively. Then, the KF processing requires specification of initial state vector and associated initial error covariance matrix, system and measurement noise covariances, as well as state transition and measurement matrices.

Initial state vector and associated error covariance matrix: In the presence of prior information, the initial state vector $\hat{X}_{k/k-1}$, for k=1 is constituted by the mean annual streamflows at the corresponding 10 gauging stations $(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_{10})$ as follows.

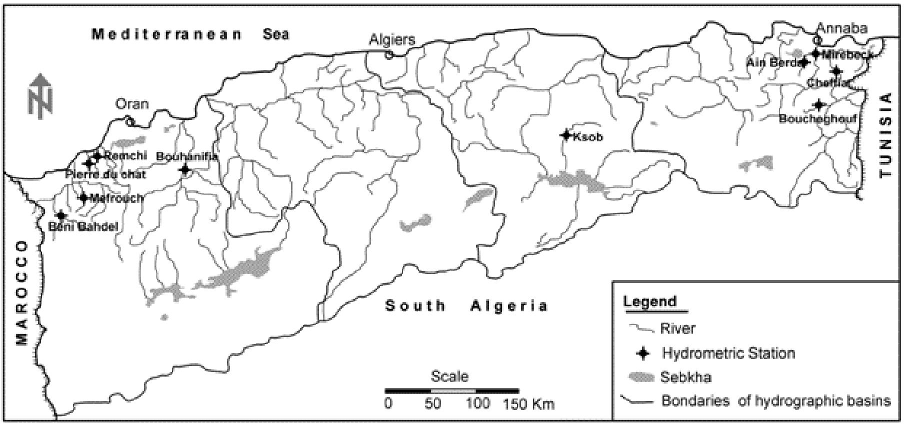

Figure 1. Northern Algeria hydrographic basins and location of the considered hydrometric stations.

$$\widehat{X}_{1/0} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{X}_{10} \end{bmatrix}$$

Meanwhile, to specify the associated initial covariance matrix $P_{k/k-1}$, which is not exactly known, one can start with large elements in the corresponding major diagonals. In this manner, the algorithm will have flexibility to adjust itself to sensible values in a relatively short time. In the present paper one can choose such (10×10) matrix as:

$$\boldsymbol{P}_{1/0} = \begin{bmatrix} 1000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1000 \end{bmatrix}$$

This choice will lead to an increase in the covariance matrix $P_{k/k-1}$and the Kalman gain matrix K_k, thus allowing the adaptive filter to weight more heavily the new information (the measureZ_k in Equation 3). This figure gives a value of 10*4 to the initial covariance matrix trace (major diagonal elements sum). Such a value is expected to decrease continuously and asymptotically to a stable positive value near zero. This reduction is accomplished through the KF algorithm, which adapts itself automatically as a consequence to having new objective information. If this asymptotic, supposedly stable, value of the covariance matrix trace does not change significantly during the calculation procedure, it means that the filter has converged. This characteristic attributes to the covariance matrix trace the merit of being a confident measure of the KF performance.

System and measurement noise covariances: As measurements are expected to be less noisy to the system dynamic, the system covariance matrix Q and the measurement noise matrix R are given the values expressed by Equations 9 and 10, respectively. They are both (10×10) in dimension.

$$\boldsymbol{Q} = \begin{bmatrix} 100 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 100 \end{bmatrix} \tag{9}$$

$$\boldsymbol{R} = \begin{bmatrix} 50 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 50 \end{bmatrix} \tag{10}$$

However, the choice is rather subjective in the sense that high value choice leads to long-time calculations but reach to the same stable prediction value at the end. The procedure is repeated with different values of the noise covariance matrix and it is observed that the same stable values are reached at the end.

State transition and measurement matrices: One of the first difficulties in the application of KF arises from the estimation of the state transition matrix $\boldsymbol{\Phi}_{k/k-1}$. In this paper, the transition matrix has been considered as the cross-correlations between all the considered records. Concerning the measurement matrix H_k, it has also been chosen as equivalent to (10×10) unity matrix, since all gauging stations are providing their observations.

$$\boldsymbol{H}_k = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{11}$$

Recursive equations of KF: Kalman gain matrix K_k is the first step calculation by the software developed here. For the one-step prediction, and considering all the previous information and assumptions, the KF gain matrix, which is (10×10), can be computed for k = 1, using Equation 5 as follows:

$$\boldsymbol{K}_1 = \begin{bmatrix} 1000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1000 \end{bmatrix} \left[\begin{bmatrix} 1000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1000 \end{bmatrix} + \begin{bmatrix} 50 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 50 \end{bmatrix} \right]^{-1} \tag{12}$$

Using measurement to update estimate vector: This is the second step calculation for the same time index at k = 1. Here, the estimate is updated using the new measurement Z_k to improve the initial estimate $\widehat{X}_{1/0}$. This is done by adding a correction term, which comprises the weighted difference $(Z_k - H_k \widehat{X}_{k/k-1})$ multiplied by the blending factor K_1 according to Equation 3 so as to obtain,

$$\widehat{X}_{1/1} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{X}_{10} \end{bmatrix}_{1/0} + \begin{bmatrix} k_{1,1} & k_{1,2} & \cdot & \cdot & k_{1,10} \\ k_{2,1} & k_{2,2} & \cdot & \cdot & k_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ k_{10,1} & k_{10,2} & \cdot & \cdot & k_{10,10} \end{bmatrix}_1 \left(\begin{bmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ \cdot \\ Z_{10} \end{bmatrix}_1 - \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{X}_{10} \end{bmatrix}_{1/0} \right) \tag{13}$$

Error covariance matrix associated with the updated estimate vector: By substituting the error covariance matrix $P_{1/0}$ associated with initial state vector $\widehat{X}_{1/0}$, KF gain matrix K_1 and measurement matrix H_k into Equation 6 one obtains the error covariance matrix associated with the updated estimate vector as follows,

$$\boldsymbol{P}_{1/1} = \begin{bmatrix} 1000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1000 \end{bmatrix}_{1/0} - \begin{bmatrix} k_{1,1} & k_{1,2} & \cdot & \cdot & k_{1,10} \\ k_{2,1} & k_{2,2} & \cdot & \cdot & k_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ k_{10,1} & k_{10,2} & \cdot & \cdot & k_{10,10} \end{bmatrix}_{1} \begin{bmatrix} 1000 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1000 \end{bmatrix}_{1/0} \tag{14}$$

One step prediction of the updated estimate: The updated estimate $\widehat{X}_{1/1}$ is projected ahead via the state transition matrix according to Equation 1, where the term W_{k-1} is ignored simply because it has zero mean and is not correlated with the previous W's. This leads to the one-step prediction of the updated estimate $\widehat{X}_{2/1}$ so as,

$$\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_{10} \end{bmatrix}_{2/1} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdot & \cdot & \cdot & \phi_{1,10} \\ \phi_{2,1} & \phi_{2,2} & \cdot & \cdot & \cdot & \phi_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \phi_{10,1} & \phi_{2,10} & \cdot & \cdot & \cdot & \phi_{10,10} \end{bmatrix}_{2/1} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_{10} \end{bmatrix}_{1/1} \tag{15}$$

One-step prediction error covariance matrix: It is possible to compute the one-step prediction error covariance matrix $P_{2/1}$ associated with $\widehat{X}_{2/1}$ according to Equation 8 as follows,

$$\boldsymbol{P}_{2/1} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdot & \cdot & \phi_{1,10} \\ \phi_{2,1} & \phi_{2,2} & \cdot & \cdot & \phi_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \phi_{10,1} & \phi_{2,10} & \cdot & \cdot & \phi_{10,10} \end{bmatrix}_{2/1} \begin{bmatrix} p_{1,1} & p_{1,2} & \cdot & \cdot & p_{1,10} \\ p_{2,1} & p_{2,2} & \cdot & \cdot & p_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{10,1} & p_{10,2} & \cdot & \cdot & p_{10,10} \end{bmatrix}_{1/1} \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdot & \cdot & \phi_{1,10} \\ \phi_{2,1} & \phi_{2,2} & \cdot & \cdot & \phi_{2,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \phi_{10,1} & \phi_{2,10} & \cdot & \cdot & \phi_{10,10} \end{bmatrix}_{2/1}^{T} + \begin{bmatrix} 100 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 100 \end{bmatrix} \tag{16}$$

For the next iteration $\widehat{X}_{2/1}$ and $P_{2/1}$ are considered as initial conditions available at hand, the multisite KF recursive Equations 12-16 and their looping have been computed over the whole 25 years of record as two portions. The first 15 year observations of the measurement sequence have been used for estimating the model parameter, the rest of the period is used for the model verification. Their consecutive execution during the period of observation represents the application of KF approach to modelling and prediction of the streamflow records mentioned earlier. At the end of calculations, some of the predicted values are underestimated and some of them are overestimated. The difference between the measurements and their predictions leads to a sequence of residuals. For the purpose of accuracy of the calculation procedure, the serial correlation of those residuals has been checked by Anderson's test [33] and their relative error percentage has been calculated according to Şen *et al.* [22] procedure.

Due to space limitation it is not possible to present all of the prediction results, neither in space nor in time dimensions. Only some of the results, in graphical form, are presented here as examples for illustration. Fig. 2 provides the multisite observed and predicted streamflows for 1992, which corresponds to the final iteration together with the corresponding relative error percentage. Meanwhile, for single sites, Fig. 3 provides the annual observed and predicted streamflow values at Bouchegouf gauging station (1968-1992). It is obvious, from both figures that the observed and predicted values follow each other closely. The agreement between the predicted and measured streamflows shows clearly the effectiveness of the multisite KF for modelling annual streamflows. Positive (+) and negative (-) signs are respectively assigned to the over-estimation and under-estimation differences. It should be noted here that the differences in Fig. 2 and Fig. 3 for all the stations are the best ones that can be obtained upon using the linear filtering theory, simply due to the optimality of the KF estimator.

The relatively big differences between the predicted and measured streamflows observed during the first time instances are due to the adaptation of the filter. During the first iterations of the KF algorithm, the gain matrix K_k takes initially erroneous values and hence the estimation is automatically bad. At this step, the measure Z_k (as objective information) is more efficient than the model estimate. After three iterations, the confidence in the model as a prediction mechanism, however, steadily improves and the KF gain value K_k as shown in Fig. 4 becomes smaller and consequently the model prediction becomes very close to the measured value.

The optimality of the results thus obtained [34] has been proven by the convergence of the prediction error covariance matrix, and more exactly by its major diagonal elements as shown in Fig. 5. This convergence to a stable value becomes evident just after the first iterations of the algorithm and remains always positive. This result conforms with the expected performance of KF algorithm and affirms the adequacy of the adjusted model to the real process. The differences between the predicted and observed streamflows for each year in record (multisite predictions) for all stations of interest have been checked for independence. This is done by testing the first serial correlation coefficient of the residuals at the 5% significance level. The relative percentage error for the obtained predictions is also calculated for the whole stations during the 25 years. Except for the first years, where relatively big errors are due to multisite KF adaptation, relative percentage error as provided by Fig. 2 for 1992 vary from -12 at Mirebeck and Beni-bahdel, which is an under-estimation, to +12 at Bouhanifia, which is an over-estimation, with an overall average relative percentage error of 9.8%.

The Anderson test has been applied to the first serial correlation coefficient of the residuals shows insignificance at the 5% significance level for all individual sites. The percentages of relative error of the predicted annual streamflows for Bouchegouf as provided by Fig. 3 vary from -75 corresponding to 1968 where the prediction was automatically bad, to +7 in 1983 and 1985 where KF has practically converged, to reach the value of -4 in 1992. The average value calculated over the 25 years is about 10.32%, while it is about 5.52% without the first 3 years.

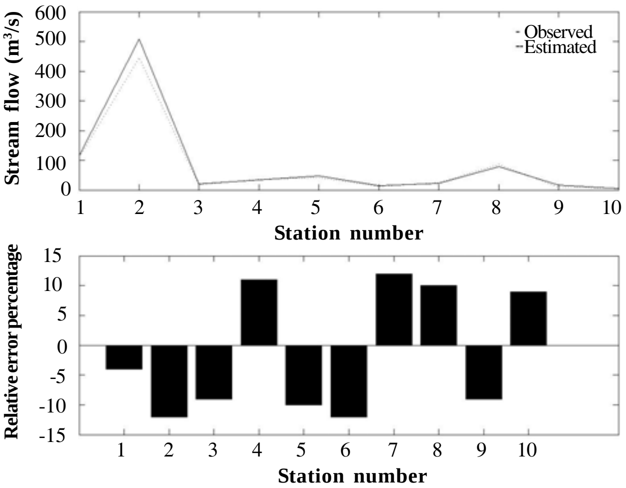

Figure 2. Observed and predicted streamflow values with the corresponding relative percentage error at selected gauging stations in northern Algeria for 1992.

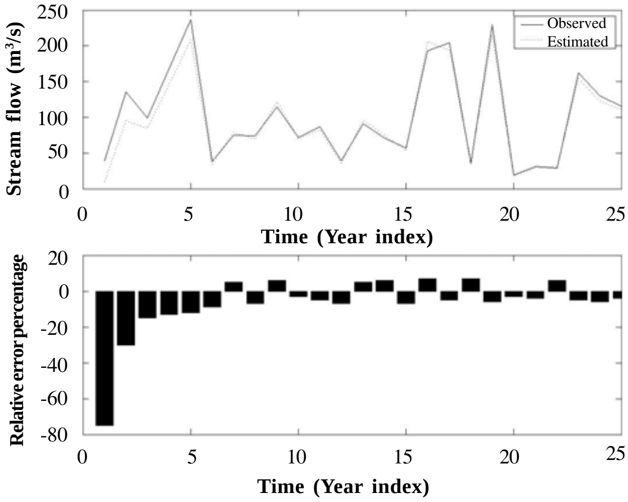

Figure 3. Observed and predicted annual streamflow values with the corresponding relative percentage error at Bouchegouf gauging station in northern Algeria (1968-1992).

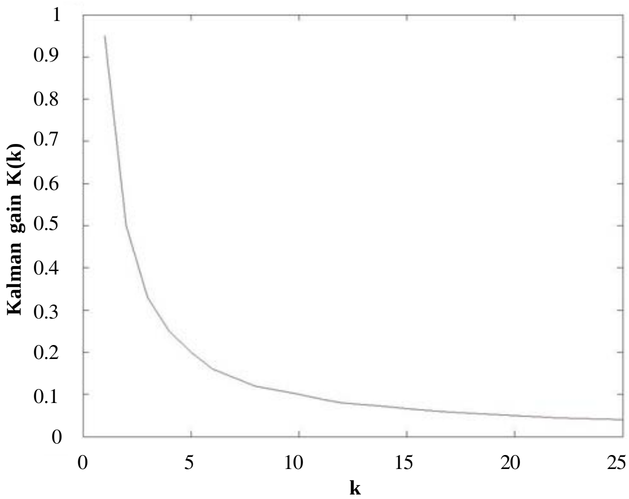

Figure 4. Kalman gain.

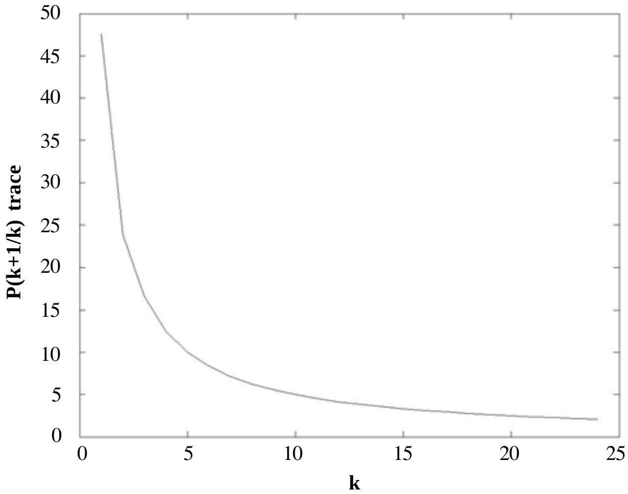

Figure 5. Prediction error variance.

The means and standard deviations of the observed and predicted streamflows for all the selected stations are calculated in time and space and presented in Tables 1 and 2, respectively. It is clear from those tables that the standard deviations for the observed values are greater than those of the predicted ones. This might lead to a slight tendency of multisite KF to underestimation.

From all aforementioned results it is clear that KF technique is a

Table 1. Mean, standard deviation and relative error of observed and predicted annual streamflow values (1968-1992).

No.	Station code	Station name	Observed value		Predicted value		Relative error (%)	
			Mean (m^3/s)	St.Dev. (m^3/s)	Mean (m^3/s)	St.Dev. (m^3/s)	Mean	St.Dev.
1	140501	Bouchegouf	101.73	65.39	97.42	60.52	4.24	7.45
2	140601	Mirebeck	331.49	238.21	323.45	220.54	2.43	7.42
3	140602	Ain Berda	12.32	11.04	10.65	9.17	13.56	16.94
4	160703	Remchi	73.60	64.03	74.65	58.65	1.43	8.40
5	160801	Pierredu chat	167.85	153.13	172.32	143.54	2.66	6.26
6	160403	Beni-Bahdel	52.54	42.83	53.11	43.42	1.08	1.38
7	111504	Bouhnifia	90.83	60.01	91.85	53.65	1.12	10.60
8	031501	Cheffia	139.86	95.21	136.4	87.10	2.47	8.52
9	050902	Ksob	31.86	22.42	29.78	18.28	6.53	18.47
10	160701	Mefrouch	11.34	7.37	11.46	5.45	1.06	26.05
		Average	101.34	75.96	100.11	70.03	3.66	11.15

Table 2. Mean and standard deviation of observed and predicted annual streamflow values at 10 gauging stations (1968-1992).

	Observed value		Predicted value			Observed value		Predicted values	
Year	Mean (m^3/s)	St.Dev. (m^3/s)	Mean (m^3/s)	St.Dev. (m^3/s)	Year	Mean (m^3/s)	St.Dev. (m^3/s)	Mean (m^3/s)	St.Dev. (m^3/s)
1968	64.54	46.19	59.56	40.1	1981	55.30	54.11	50.3	49.2
1969	151.81	165.53	145.5	150.4	1982	51.59	51.78	49.4	52.6
1970	124.13	130.68	115.15	120.4	1983	102.97	153.46	97.8	149.3
1971	202.52	167.37	180.4	157.2	1984	157.75	282.92	155.7	279.4
1972	241.33	197.16	220	170	1985	45.87	35.99	43.8	33.9
1973	150.21	160.59	149.1	165.4	1986	177.67	237.73	170.7	227.8
1974	133.08	137.8	140	147.5	1987	22.82	15.78	22.1	14.9
1975	97.08	78.39	87	60.3	1988	26.06	22.42	24.4	17.3
1976	102.6	105.31	100.3	90.7	1989	35.85	36.46	36.9	30.5
1977	67.61	63.22	66.2	60.3	1990	120.57	160.02	117.6	155.4
1978	74.06	68.53	73.1	65.4	1991	76.37	88.95	73.8	78.9
1979	73.09	82.53	69.4	78.4	1992	86.13	151.87	80.4	130.4
1980	89.72	87.84	86.4	80.8	Average	101.23	111.30	99.70	104.26

suitable tool for determining the correct time-varying parameters of the process model. As such, it allows for parameter prediction and adjustment every time a new objective information is available. Figs 2 and 3 are just two simple examples of the application of the KF multisite method to modelling and prediction of annual streamflows. Judging by the smallness of the differences between the means of the observed and predicted streamflows, and so by the standard deviations, one might claim that the KF is an efficient tool for modelling multisite as it is efficient for modelling the annual streamflows at single sites. In other terms KF technique is efficient either in time or space dimensions.

Conclusions

In the present study, the Kalman filter (KF) technique is applied for modelling and predicting the annual streamflows at a number of sites simultaneously (multisites). The estimator developed for the purpose of this study has the particularity of adapting itself automatically as a new piece of information becomes available. In this way, optimal streamflow predictions can be obtained temporally and spatially.

Another optimality aspect of KF technique is that it incorporates all information that can be provided to it. It processes all available measurements regardless of their precision to estimate the current value of the variables of interest, with use of (a) knowledge of the system and measurement device dynamics, (b) the statistical description of the system noises, measurement errors and uncertainty in the dynamics models, and (c) any available information about initial conditions of the variables of interest. As such KF provides not only the state variables estimation but also by mean of the error covariance matrix the estimation of the assigned confidence, which is a great advantage of the technique.

In the present investigation, ten annual streamflow series in Algeria recorded over a common time period (1968-92) of 25 years, have been worked out. The average streamflow at each station has been adopted as initial state vector, and large diagonal elements of the error covariance matrix have been selected in order to weight heavily the new information that has become available.

Streamflow predictions obtained for each of the considered stations show to be in close agreement with the corresponding measured streamflows for the same period of record. Likewise the predictions obtained spatially for each year equally follow closely the observed streamflows at the selected stations. This close agreement indicates that KF provides an efficient tool for modelling and predicting annual streamflows. Last, but not least, the suitability of the KF model has been checked by the Anderson test for the first serial correlation coefficient of the residuals as well as the prediction percentage of relative errors. The overall average relative percentage error of the predicted values is less than 10% which is highly acceptable.

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