Computation of two-dimensional unsteady supercritical flows in open channel contraction of spillway chutes

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Abstract

Supercritical flow through open channel contractions often generates disturbances and instabilities at the surface, which appear as transverse waves. Contraction of channel, often called transitions, can be found in several hydraulic structures but mainly in free surface spillway chutes. Saint Venant's equations governing two-dimensional unsteady free surface flows can be successfully applied to this kind of problem by making some simplified assumptions, which in turn lead to a non-linear hyperbolic system of equations for which an analytical solution is not yet available. Thereafter, the equations of motion obtained are generalized with cases of unspecified bottom slopes to take into account the effect of a variable bottom slope.

In this paper, a second order two step MacCormack explicit finite differences scheme is applied, in conjunction with a geometric transformation of the irregular physical domain, to a simpler computational one of rectangular shape. The integration time steps are adjusted at each incrementation time according to Courant-Friedrichs-Lewy's stability condition. An existing example of horizontal channel contraction is reproduced herein, and the obtained results of flow pattern and water surface profiles are compared to previous experimental and numerical studies. A rectangular channel with steep slopes will be studied as a second application in order to see if the elaborate model can simulate the flow in a channel with a high bottom slope. The numerical results obtained will be compared with experimental measurements. After this a third application, in which a supercritical flow in a recti-linear contraction of channel with variable bottom slopes, is presented. The results obtained here will be compared with the results corresponding to a horizontal contraction.

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1. Introduction

Water flow study represents a vast and complex research field, particularly when dealing with open channels. The presence of discontinuities or obstacles in the channel section, even if not significant, can generate large fluctuations of the free surface and variations in the liquid flow cross-sections along the current. This complexity is more critical if the flow is unsteady and supercritical.

Channel section discontinuities, which can be contractions or expansions, are often called transitions, and are largely used in hydraulic constructions to control the water flow, especially in spillway chutes.

Unlike expansions, the contractions cause a jump in the water depth; they are the siege of transverse waves also known as oblique hydraulic jump.

In order to simulate this kind of flow, the mathematical model represented by the two-dimensional unsteady free surface equations given by Saint Venant are used. These equations are obtained by applying the principles of conservation and mass momentum, while making some simplified assumptions.

The system, such as it was given by Saint Venant, does not make it possible to take into account the effect of a considerable bottom slope. Thereafter, the equations of motion obtained were generalized for cases with unspecified bottom slopes, in order to take into account the effect of a variable bottom slope.

The equation system obtained is of non-linear hyperbolic type for which an analytical solution is impossible and, consequently, a numerical solution is indispensable. For this purpose the system has been resolved using the MacCormack finite differences explicit two-step 'Predictor-Corrector' scheme, which is a second order accurate in space and time.

Knowing that the studied transition has a relatively irregular form a geometric transformation is used, in which the physical domain is replaced by a rectangular computational one, which is simpler to analyze. Hence, the equations of motion are written in transformed coordinates.

Generally, all the explicit finite differences schemes are numerically unstable. For that, the integration time steps are adjusted at each incrementation time according to Courant-Friedrichs-Lewy's stability condition.

The numerical model has been used in a first application to analyze supercritical flow in a horizontal symmetrical channel contraction. The water surface profiles along the symmetrical axis, and along the wall of the transition, are computed. The flow pattern is also given in this contraction. The results are compared to experimental and numerical ones carried out by researchers.

The second application aims to establish whether or not the numerical model presented is able to simulate the two-dimensional supercritical flow in channels with a variable bottom slope. A rectangular channel with a steep slope will be studied and the numerical results obtained will be compared with experimental measurements.

The third application is devoted to the analysis of the supercritical flow in a recti-linear contraction of a channel with a variable bottom slope. The results obtained will be compared with the results corresponding to a horizontal contraction.

2. Open channel contraction in supercritical flow

Any change of section or direction in a channel involves an irregular and undulated surface of flow. The irregularities thus generated are negligible in the subcritical flows but become significant for torrential or supercritical flows [1, 2]. These irregularities are characterized by cross waves which are frequently present in non-prismatic channels for flows in a supercritical regime [3].

Figure 1 shows the straight-lined contraction, including stream lines and transverse waves where b_1 and b_3 are the widths in the approaching and tailwater channels, and L is the length of the transition. In this figure, the flow pattern for any arbitrary contraction angle, θ, is represented. The flow in the tailwater channel is far from being without perturbations.

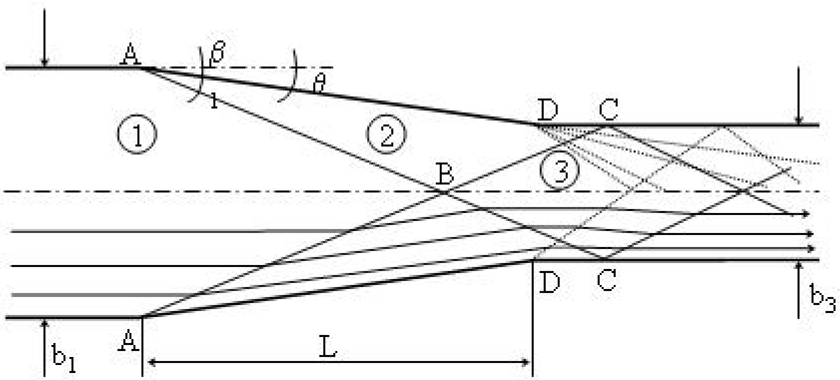

Figure 1. *Schematic flow pattern in channel contraction according to Ippen & Dawson [4]*

The design of contractions in a supercritical flow generates several complications contrary to a subcritical case. Indeed the oblique cross waves occur and can be propagated far downstream requiring considerable heights from the walls of the channel; unless the transition is designed so as to minimize this phenomenon which can entrain much air, and a stilling basin located at the chute extremity might suffer from asymmetric approach flow [4, 5].

The improvement in conditions of the flow in a symmetrical straight-lined contraction, in order to avoid the appearance of the disturbances in the tailwater channel, depends mainly on a correct choice of the angle of contraction θ. A well designed contraction is characterized by an almost uniform flow in the contracted channel [1, 6].

3. Literature review

Generally, supercritical flows through open channel contractions have received attention since the 1950s. The majority of the studies, whether experimental, analytical, or numerical, are about horizontal transitions. A basic approach to the design of high-velocity flow in horizontal contractions was reported by Ippen & Dawson [4] in a symposium that probably represented the most comprehensive treatment of the topic up to that time. This made available to the hydraulics community a unified treatment of the mechanics of supercritical flow, and a general design method for supercritical flow in hydraulic structures.

Later, Ippen & Harleman [7] conducted experiments to verify the hydrodynamic theory of oblique hydraulic jumps. Other contributions to the chute contractions were provided by Harrison [8], Jayaraman & Sethuraman [9], Täubert [10], Sturm [3], Hager [1], and Reinauer [2], among others. With the availability of fast computers, a more rigorous analysis became possible. Among others, Pandolfi [11], Villegas [12], Ellis [13], Jimenez & Chaudhry [14], and Glaister [15], carried out the computer simulation of supercritical flow in an open channel. They used shock capturing, or method of characteristics, to solve the shallow water flow equations. Thereafter, Bhallamudi & Chaudhry [16], and Rahman & Chaudhry [17], used finite difference methods to solve unsteady, depth-averaged, two-dimensional shallow water equations to simulate flow in open channel transitions. In addition, Causon *et al* [18] used finite volume approaches for the study of supercritical flow in a spillway channel. It should be noted that the majority of the studies quoted above, among others, were made for horizontal transitions only. In this work, a numerical simulation of a two-dimensional supercritical flow through a contraction of a weak and steeply sloping channel will be undertaken, while announcing the lack of experimental studies for the transitions with high bottom slopes.

4. Governing equations

Supercritical flows through the contraction of a channel are governed by two-dimensional unsteady gradually varied flow equations. These equations, established by Saint Venant, are obtained by applying the principles of conservation of the mass and momentum, in addition to making some simplified assumptions [19]. The system such as it was given by Saint Venant does not make it possible to take into account the effect of a considerable bottom slope. Thereafter, the equations of motion obtained were generalized with the cases of the unspecified bottom slopes to take into account the effect of a variable bottom slope. The principal assumptions are now:

a) The fluid is incompressible.
b) The velocities distribution is uniform on a vertical. In other words, each calculated velocity represents a mean velocity on a water column (on the vertical).
c) The vertical velocity of the flow is low.
d) The curve of the streamline of current is low.
e) The vertical acceleration of a fluid particle is very weak compared with gravitational acceleration, g, and can thus be neglected (consequence of the assumption '*d*').
f) The vertical pressures distribution is hydrostatic (consequence of the assumptions '*d*' and '*e*').
g) Bottom shear stress is large compared to other shear stress.

The equations of motion in Cartesian coordinates [20-23] are:

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(u\,h) + \frac{\partial}{\partial y}(v\,h) = 0 \tag{1}$$

Momentum equation in x direction

$$\frac{\partial}{\partial t}(u\,h) + \frac{\partial}{\partial x}\left(u^2h + g\,\frac{h^2}{2}\cos\alpha_x\right) + \frac{\partial}{\partial y}(u\,v\,h) = g\,h\,(S_{ox} - S_{fx}) \tag{2}$$

Momentum equation in y direction

$$\frac{\partial}{\partial t}(v\,h) + \frac{\partial}{\partial x}(u\,v\,h) + \frac{\partial}{\partial y}\left(v^2h + g\,\frac{h^2}{2}\cos\alpha_x\right) = g\,h\,(S_{oy} - S_{fy}) \tag{3}$$

where h is the flow depth, u is the depth averaged flow velocity in the x direction, v is the depth averaged flow velocity in the y direction, t is time, g is acceleration due to gravity, α_x and α_y are the incline angle of the channel bottom in x and y directions, respectively, S_{ox} and S_{oy} are the channel bottom slopes in x and y directions, respectively, and S_{fx} and S_{fy} are the friction slopes in x and y directions, respectively.

The bottom slopes are calculated as follows:

$$S_{ox} = \sin\alpha_x \tag{4}$$

$$S_{oy} = \sin\alpha_y \tag{5}$$

The friction slopes are calculated as follows:

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h} \left(\frac{b + h}{b\, h}\right)^{1/3} \tag{6}$$

$$S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h} \left(\frac{b + h}{b\, h}\right)^{1/3} \tag{7}$$

Where *n* is the Manning roughness coefficient, and *b* is the channel width.

5. Resolution of governing equations

The unsteady free surface flows are governed by a system of non-linear hyperbolic partial differential equations. Such equations can be solved theoretically only in particular cases not easily found in reality. Thus several problems in hydraulics require, for lack of an analytical solution, a numerical solution of the partial differential equations. One of the classic methods in approaching this solution is with the use of the finite differences method.

In the current case, the MacCormack explicit two steps 'Predictor-Corrector' finite differences scheme, which is a second order equation accurate in space and time, has been chosen [24-27].

5.1. MacCormack scheme

Several methods investigated by Lax & Wendroff [28] have become popular for solving hyperbolic systems. These methods, known as two-step schemes, are based on second-order Taylor series expansions in time. An interesting and simpler variation of the Lax-Wendroff scheme was introduced by MacCormack [24], and has been widely used in computational fluid dynamics.

5.2. General formulation

This scheme consists of a two-step 'Predictor-Corrector' sequence. Flow variables are known at *k* time level, and their values are to be determined at $k + 1$ time level.

In order to illustrate the principal steps of the MacCormack scheme, the governing equations in terms of the flow variables $U = (h, uh, vh)^t$ are written in the Cartesian coordinates, and in the following conservative form:

$$U_t + E_x + F_y + S = 0 \tag{8a}$$

in which

$$E_x = \begin{bmatrix} u\,h \\ u^2 h + g\,\dfrac{h^2}{2}\cos\alpha_x \\ u\,v\,h \end{bmatrix}; \quad F_y = \begin{bmatrix} v\,h \\ u\,v\,h \\ v^2 h + g\,\dfrac{h^2}{2}\cos\alpha_x \end{bmatrix}; \tag{8b}$$

$$S = \begin{bmatrix} 0 \\ -\,g\,h\,(S_{ox} - S_{fx}) \\ -\,g\,h\,(S_{oy} - S_{fy}) \end{bmatrix}$$

Then, for grid points i and j, the following finite difference equation may be written for Equation 8a and Equation 8b:

Predictor step

$$\widetilde{U}_{i,j} = U_{i,j}^{k} - \tau_x\,(E_{i,j}^{k} - E_{i,j}^{k}) - \tau_y\,(F_{i,j+1}^{k} - F_{i,j}^{k}) - \Delta t S_{i,j}^{k} \tag{9}$$

Corrector step

$$\hat{U}_{i,j} = \widetilde{U}_{i,j}^{k} - \tau_x\,(\widetilde{E}_{i,j} - \widetilde{E}_{i-1,j}) - \tau_y\,(\widetilde{F}_{i,j} - \widetilde{F}_{i,j-1}) - \Delta t \widetilde{S}_{i,j} \tag{10}$$

where $\tau_x = \Delta t / \Delta x$, and $\tau_y = \Delta t / \Delta y$.

$\widetilde{U}$ and $\hat{U}$ are the intermediate values for U. The new values of U are then obtained from:

$$U_{i,j}^{k+1} = \frac{1}{2}\,(U_{i,j}^{k} + \hat{U}_{i,j}) \tag{11}$$

The grid points are defined by subscripts i, j and k. The scheme first uses forward space differences to predict an intermediate solution from known information at the k time level. Backward space differences are then used in the second step to correct the predicted values. The solution at the unknown time level is calculated by using the results obtained in the predictor and the corrector steps. Hence it is possible to use backward finite differences in the predictor part, and forward finite differences in the corrector part.

6. Coordinate transformation

In order to solve the equations of motion (1-3) by finite differences techniques, it is compulsory to simplify the boundaries of the transition. For that, the physical domain is transformed to a rectangular computational one by the following coordinate's transformation:

$$\xi = x \tag{12}$$

$$\eta = \frac{y}{b(x)} \tag{13}$$

where, $b(x)$ is the distance between the symmetry line and the upper boundary at distance x.

To apply the finite differences method on the computational plane, the governing equations are transformed into the following conservation form in terms of ξ and η [16, 22]:

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial \xi} - \frac{\eta}{b(\xi)} b'(\xi) \frac{\partial (uh)}{\partial \eta} + \frac{1}{b(\xi)} \frac{\partial (vh)}{\partial \eta} = 0 \tag{14}$$

Momentum equation in ξ direction

$$\frac{\partial (uh)}{\partial t} + \frac{\partial}{\partial \xi}\left(u^2 h + g\,\frac{h^2}{2}\cos \alpha_\xi\right) - \frac{\eta}{b(\xi)} b'(\xi) \frac{\partial}{\partial \eta}\left(u^2 h + g\,\frac{h^2}{2}\cos \alpha_\xi\right) + \frac{1}{b(\xi)} \frac{\partial (uvh)}{\partial \eta} = gh\,(S_{o\xi} - S_{f\xi}) \tag{15}$$

Momentum equation in η direction

$$\frac{\partial (vh)}{\partial t} + \frac{\partial (uvh)}{\partial \xi} - \frac{\eta}{b(\xi)}\, b'(\xi) \frac{\partial (uvh)}{\partial \eta} + \frac{1}{b(\xi)} \frac{\partial}{\partial \eta}\left(v^2 h + g\,\frac{h^2}{2}\cos \alpha_\xi\right) = g\,h\,(S_{o\eta} - S_{f\eta}) \tag{16}$$

where

$$b'(\xi) = \frac{\partial b(\xi)}{\partial \xi} \tag{17}$$

7. Stability condition

Finite difference techniques are commonly employed in solving numerically partial differential equations. The solutions obtained from explicit finite difference schemes are conditionally stable where the stability condition is given by the Courant-Friedrichs-Lewy (CFL) restriction. For two-dimensional flows, this condition is given by the following equation [16, 25, 27]:

$$C_n = \frac{\left(V + \sqrt{g\, h \cos \alpha_\xi}\right)\Delta t}{b\,(\xi)\,\Delta\xi\,\Delta\eta} \sqrt{\Delta\xi^2 + [\mathrm{b}\,(\xi)\,\Delta\eta]^2} \tag{18}$$

where V is the resultant velocity at the grid point, C_n is the Courant number, $\Delta\xi$ and $\Delta\eta$ are the distance increment in ξ axis and η axis, respectively, and Δt is the time interval. For the MacCormack schemes, the value of the Courant number must be inferior or equal to one ($C_n \leq 1$).

A characteristic feature of the explicit schemes is the choice of time level, which is governed by the stability criteria. The magnitude of time step is given by the CFL stability condition.

8. Application

8.1. Symmetrical channel contraction with small bottom slope

In this application a supercritical free surface flow through a symmetrical channel is analyzed (Figure 2). This case has been studied in experiments by Coles & Shintaku [16], and numerically by Bhallamudi & Chaudhry [16].

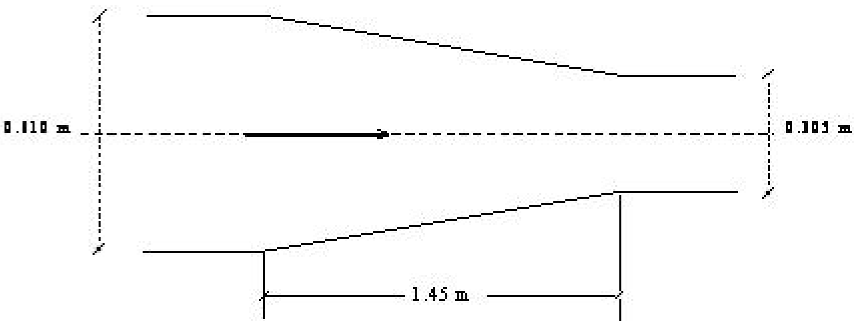

Figure 2. *Symmetrical recti-linear contraction*

The principal data of this application are:

(a) The upstream width of the contraction is $b_1 = 0.61$m
(b) The downstream width of the contraction is $b_3 = 0.305$m
(c) The length of the transition is $L = 1.45$m
(d) The upstream boundary conditions of the transition are:
- The upstream Froude number is $F_o = 4$
- The upstream depth is $h_o = 0.0305$m
- The longitudinal upstream velocity is $u_o = 2.188$m/sec
- The transversal upstream velocity is $v_o = 0$m/sec

(e) The bottom and the friction slopes are supposed to be negligible (equal to zero)
(f) The Courant number is $C_n = 0.8$
(g) The grid used is $\Delta\xi = 0.0483$m, and $\Delta\eta = 0.0476$m

The water surface profiles along the centerline, and along the wall, are respectively given in Figure 3 and Figure 4.

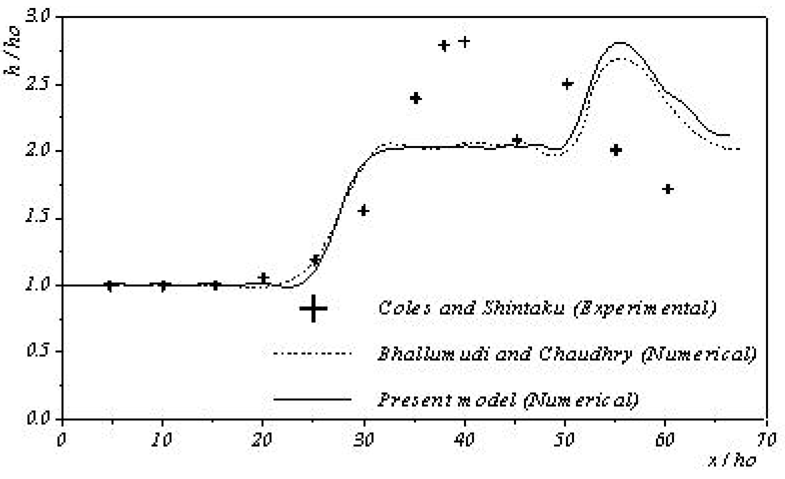

Figure 3. *Water surface profile along the centerline in the channel contraction*

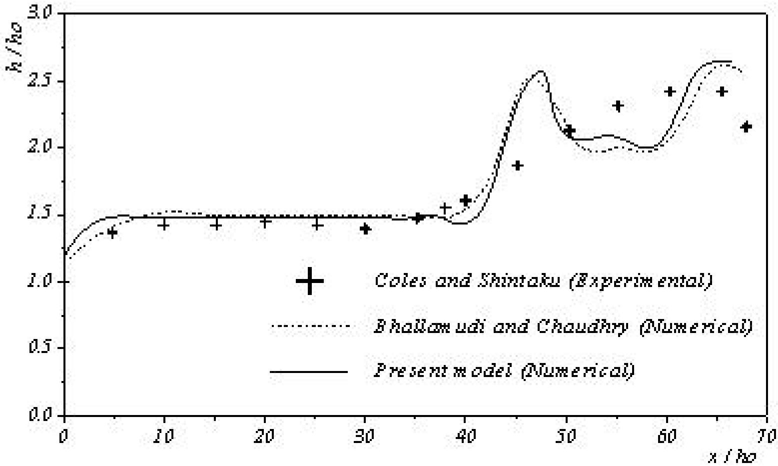

Figure 4. *Water surface profile along the wall in the channel contraction*

From these figures it is noticed that the results obtained by using a discretization with the MacCormack explicit finite differences scheme agree well with the numerical results obtained by Bhallamudi & Chaudhry [16].

By comparing these results with the experimental measurements, it can be noted that they are rather good along the solid side wall of the transition and relatively less good along the symmetrical axis where there is a shift between the peak of the calculated water surface profile and the peak observed in experiments. This difference is primarily due to the violation of the assumption of hydrostatic pressure distribution near step gradients. Hydrostatic pressure distribution is valid at all points except in the vicinity of a shock, such as a hydraulic jump and oblique hydraulic jump. In order to obtain good results it is necessary to include the Boussinesq terms to account for non-hydrostatic pressure distribution due to the acceleration effects.

On the other hand, crossing of the waves near the centerline results in air entrainment. The effect of air entrainment is not included in the present model.

The model gives results which can confidently be used for selecting the wall height of recti-linear channel contraction.

Figure 5 shows a plane representation of the flow in the symmetrical recti-linear contraction, where the formed pattern of cross waves can be observed in a clearer way, which leads to agitations and perturbations in the downstream rectangular channel. Therefore, this type of configuration must be avoided. It can be said that the results obtained here (Figure 5) conform to the observation made by Ippen & Dawson [4] (Figure 1).

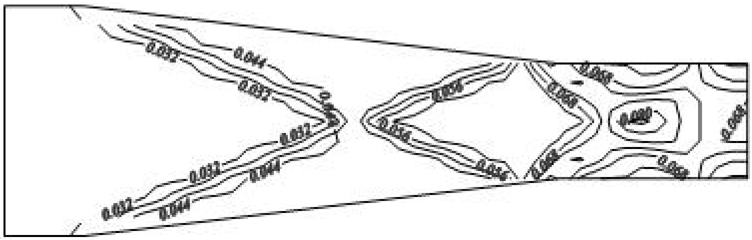

Figure 5. *Curves of equal heights in symmetrical channel contraction*

8.2. Rectangular channel with high bottom slope

The objective of the second application is to test the validity of the elaborate numerical model to simulate the unsteady torrential flows in steeply sloping channels. For that a rectangular channel of spillway chutes in a reduced model has been chosen [22, 29] , which has a bottom slope of approximately 28°. The flow conditions in this channel are summarized as follows:

Flow conditions in the rectangular channel		
Discharge (m^3/sec)	0.0689	0.093
The upstream flow depth (m)	0.1000	0.1300
The longitudinal upstream velocity (m/sec)	1.6805	1.7450
The transversal upstream velocity (m/sec)	0	0

The following data are also used:

(a) The channel width is 0.41m
(b) The length of channel is 0.74m
(c) The grid used is $\Delta\xi$ = 0.0211m, and $\Delta\lambda\eta$ = 0.01m
(d) The Courant number is C_n = 0.5
(e) The Manning number is n = 0.01
(f) The bottom slope is α = 28.1245°

Figure 6 and Figure 7 provide the water surface profiles along the side wall of the prismatic channel for a discharge of 0.0689m^3/sec (real discharge = 2000m^3/sec), and an exceptional discharge of 0.093m^3/sec (real discharge = 2700m^3/sec).

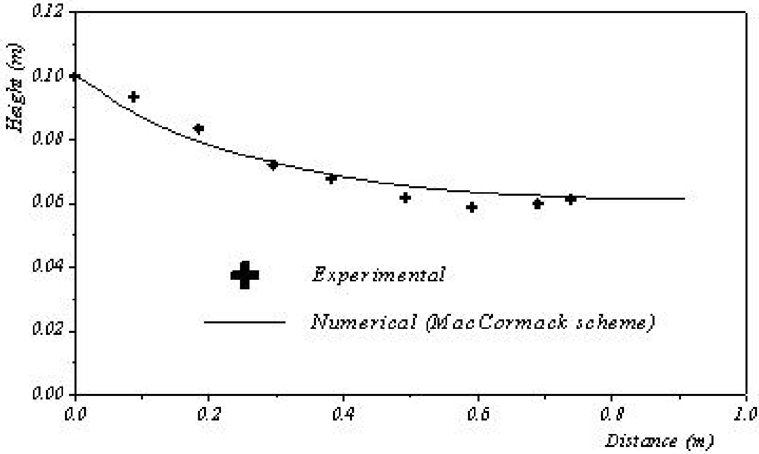

Figure 6. *Water surface profile along the wall in the rectangular channel for a discharge of 0.0689m^3/sec*

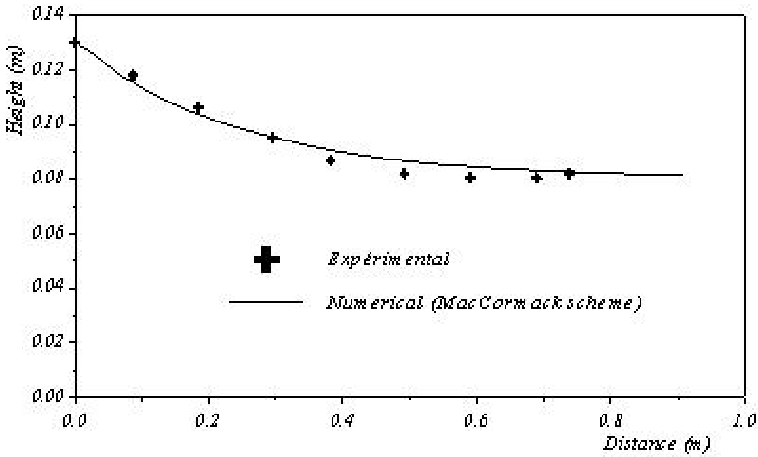

Figure 7. *Water surface profile along the wall in the rectangular channel for a discharge of 0.093m^3/sec*

By comparing the results obtained numerically with the experimental results, a rather good agreement is noticed. So it can be concluded that the elaborate numerical model is ready to simulate the torrential flows in channels with a high bottom slope.

8.3. Contraction of a channel with a variable bottom slope

In the third application the symmetrical recti-linear contraction given by Figure 2, which was studied in the first application with a null slope, will be taken again here by keeping the same dimensions and same conditions of flow. In the present case a torrential flow through this contraction will be studied, and the bottom slope is accentuated considerably in order to see how the water surface compares with the profile obtained for a horizontal transition. The study is made for slopes of 15°, 25° and 40°. It was selected to compare water surface profiles for the various slopes chosen, with the case of a horizontal transition studied in the first application, considering the lack of application containing from experimental measurements.

The profiles of the water surface along the symmetrical axis calculated for the three slopes is shown in Figure 8.

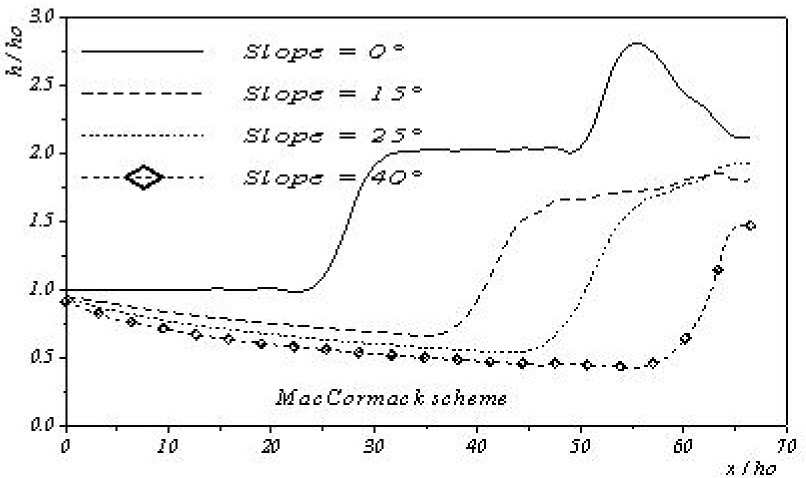

Figure 8. *Effect of the increase in the bottom slope on the water surface in a symmetrical recti-linear contraction (symmetrical axis)*

The following observations can be made:

• The increase in the bottom slope will cause a lowering of the free surface compared to

the case of the horizontal channel.
• The profiles of the water surface obtained for the three slopes present only one heightening contrary to the case of the horizontal transition.
• The heightening of the free surface for the slopes of 15°, 25° and 40° does not exceed the first heightening observed in the horizontal channel contraction.
• The more the bottom slope is increased, the more the system of shock waves moves far downstream.

Therefore the increase in the bottom slope in a symmetrical recti-linear contraction, crossed by a supercritical flow, mainly generates an increased rate of flow in the direction of the flow. This considerable speed involves the system of transverse waves more and more downstream, and prevents the formation of the great tops of waves which is observed in the horizontal or slightly sloping contractions.

In addition, Figure 9 gives the water surface profiles in the solid side wall for the various studied slopes.

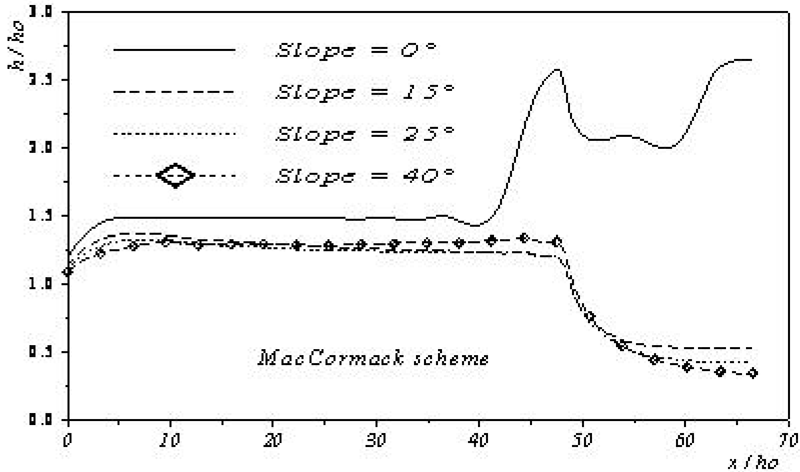

Figure 9. *Effect of the increase in the bottom slope on the water surface in a symmetrical recti-linear contraction (solid side wall)*

From this figure the following points can be observed:

• No significant heightening of the water surface is observed for the three studied slopes, compared to the results slope equal to zero.

• The surface profiles for the slopes of 15°, 25° and 40° keep an almost constant height in the transition.
• On the other hand, just at the end of the contraction (at the entry of the channel downstream), the depth of the flow falls quickly.
• The increase in the bottom slope thus involves a reduction in the water surface profiles along the solid side wall.

It is noted here that the section "x/ho = 47.75" is with a length "X = 1.45m", corresponding to the length of the contraction (i.e. just with the entry of the downstream rectangular channel), and there is an abrupt lowering of the water surface for the three slopes taken into account in this study. This is primarily due to the fact that the velocity of flow increased with the increase of the bottom slope, which leads to the concentration of water flow along the axis of symmetry, not along the wall of the downstream rectangular channel and, consequently, it does not have here the birth of waves along the wall.

9. Conclusion

The interest granted to the supercritical flows in open channels, through various research works raised in the literature specialized in this field, indicates the importance of this subject. An analysis of supercritical flows crossing non-prismatic open channels is presented. This type of flow is complex since it always generates irregular and ondular free surfaces. A mathematical model which allows the simulation of this phenomenon has been given. Resolving these equations can be made by using the MacCormack explicit two-step "Predictor-Corrector" finite differences scheme. The calculation of the water surface profiles in symmetrical channel contraction gives satisfactory results with the MacCormack numerical scheme along the side wall, and relatively less good results along the symmetrical axis of the channel where the assumption of a hydrostatic pressure distribution is not valid. Hydrostatic pressure distribution is valid at all points except in the vicinity of a shock. Although some details are lost in the vicinity of the shock, if the shallow water equations are used, the overall results are adequate for engineering purposes.

Based on experimental results, Ippen & Harleman [7] have shown that the error introduced by the assumption of uniform velocity distribution is negligible. In addition, it is shown that the steady state of the flow in this study is reached at approximately 1.51 seconds, which is a great advantage compared to the results obtained by Bhallamudi & Chaudhry [16], where the steady state is reached at approximately three seconds by using the same scheme. Thus it can be concluded that the MacCormack scheme is apt to simulate the supercritical flows in symmetrical recti-linear channel contractions with the presence of shocks, and the obtained results can be used with confidence in the dimensioning of such hydraulic structures, especially for selecting the wall height.

The aptitude of the numerical model presented to simulate the supercritical flows in steeply sloping channels was checked by treating the flow through a channel with a rectangular cross-section (a reduced model of rectangular spillway chutes) with a bottom slope of approximately 28°. The profiles of the water surface obtained are very close to the experimental measurements taken at the laboratory.

Thereafter the study was extended to the case of steeply sloping channel contractions, where symmetrical contractions with variable bottom slopes were analyzed. A reduction in depth and an increased speed are the principal consequences generated by the passage of a supercritical flow through the studied transition. The considerable increase in speed further involves the system of transverse waves with the downstream, and prevents the formation of great tops of waves, which always exist in the horizontal transitions.

Finally, one can say that the numerical model presented makes it possible to simulate the supercritical flows through prismatic or non-prismatic channels with small or high bottom slopes.

Notations

The following symbols are used in this paper:

b_1 = Upstream width
b_3 = Downstream width
C_n = Courant number
g = Acceleration due to gravity
F_o = Upstream Froude number
h = Flow depth
h_o = Upstream height
L = Length of contraction
n = Manning roughness coefficient
S_{fx} = Friction slopes in x direction
S_{fy} = Friction slopes in y direction
S_{ox} = Bottom slope in x direction
S_{oy} = Bottom slope in y direction
t = Time
U = Depth averaged flow velocity in x direction
u_o = Longitudinal upstream velocity
V = Resultant velocity
v = Depth averaged flow velocity in y direction
v_o = Transversal upstream velocity
x = Longitudinal spatial coordinate
y = Transversal spatial coordinate
α_x = Incline angle of the channel bottom in x direction
α_y = Incline angle of the channel bottom in y direction
β = Shock angle

ξ = Transformed space coordinates
η = Transformed space coordinates
θ = Contraction angle
Δt = Time interval
$\Delta\xi$ = Distance increment in ξ direction
$\Delta\eta$ = Distance increment in η direction

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