A Tool for Statistical Modelling by Means of Copulas of Analog and Mixed-Signal Circuits

Article · November 2012

5 authors, including:

Ahcène Bounceur
Université de Bretagne Occidentale
164 PUBLICATIONS **1,466** CITATIONS
SEE PROFILE

Reinhardt Euler
Université de Bretagne Occidentale
142 PUBLICATIONS **637** CITATIONS
SEE PROFILE

Bilal Saoud
Université de Bouira
9 PUBLICATIONS **63** CITATIONS
SEE PROFILE

Salvador Mir
French National Centre for Scientific Research
276 PUBLICATIONS **2,215** CITATIONS
SEE PROFILE

A Tool for Statistical Modelling by Means of Copulas of Analog and Mixed-Signal Circuits

Ahcène Bounceur[1], Reinhardt Euler[1], Saoud Bilal[2], Kamel Beznia[1], Salvador Mir[3]
[1]Lab-STICC Laboratory - European University of Britanny - University of Brest
20, Avenue Victor Le Gorgeu, 29238, Brest Cedex 3, France
Email: {Ahcene.Bounceur, Reinhardt.Euler, Kamel.Beznia}@univ-brest.fr
[2] Université de Béjaia, Route de Targa Ouzemmour, 06000, Béjaia
Email: Saoud_Bilal@yahoo.fr
[3]TIMA Laboratory
46, Avenue Félix Viallet, 38031, Grenoble Cedex, France
Email: Salvador.Mir@imag.fr

Abstract— **Testing analog integrated circuits is expensive in terms of both test equipment and time. To reduce the cost, Design-For-Test techniques (DFT) such as Built-In Self-Test (BIST) have been developed. The quality of a test technique is measured in relation to its test metrics. To obtain an estimation of these metrics at the ppm (parts-per-million) level, it is first necessary to obtain a sample of the circuit output parameters via a circuit-level Monte Carlo simulation. The joint Cumulative Distribution Function (CDF) of these parameters can next be estimated. Finally, an arbitrarily large sample of circuits can be sampled from this function to calculate the test metrics. In this work, we have developed a tool which automates the task of fitting a multivariate Gaussian or a Copula-based CDF to the initial sample of output parameters. The tool generates a large sample of circuits for the estimation of test metrics and for setting optimal test limits.**

I. INTRODUCTION

An integrated test technique for analog circuits needs in general to be evaluated at the design stage, taking into account multiple variations of the circuit parameters. This evaluation requires the calculation of several test metrics or probabilities of error such as the defect level (the probability of a faulty circuit to pass the test) and the yield loss (the probability of a functional circuit to fail the test). A direct calculation of these test metrics from the Monte Carlo simulation is almost impossible, since the data volume that can be generated in a reasonable amount of time is not sufficient to correctly represent the space of faulty circuits. To obtain a precision at the ppm level, the generation of a population of several millions of circuits is necessary.

In previous work, we have considered a statistical model of the circuit output space that can be extracted from a small sample of circuits obtained via Monte Carlo simulation. Next, an arbitrarily large sample of circuits can be generated. The test metrics can then be calculated with high precision using relative frequencies, and optimal test limits can be set. Figure 1 illustrates this approach. The new set is generated by estimating the statistical model of the initial set. This model may be multinormal [1], non-parametric [2], or based on the theory of copulas [3][4]. In this work, we describe a Computer-Aided-Test (CAT) tool for the evaluation of an analog test technique at the design stage. Our tool is intended to complete the CAT platform developed in [5][6].

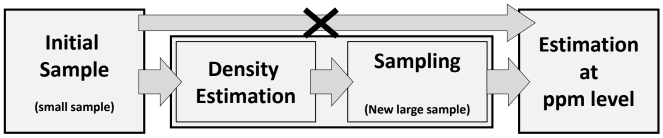


Fig. 1. Statistical methods for the generation of a large circuit population using density estimation.

This paper is organized as follows. In Section II we present the theory allowing to generate a large sample of circuits by means of Archimedean copulas. Section III is devoted to the presentation of the developed tool. The case study representing a BIST technique for CMOS imager pixels together with the results of its evaluation will be the subject of Section IV. The last Section contains our conclusions.

II. THE THEORY OF ARCHIMEDEAN COPULAS

In this Section we will briefly describe the fundamentals of the theory of copulas. We will mainly focus on the copula of Clayton for illustration, although our tool allows the generation of other non-Gaussian copulas such as the Gumbel one. Since the latter is not used in our example case-study, it will be omitted here.

A. Sample generation using copulas

The theory of copulas allows the estimation of a multidimensional distribution by separating the dependence structure between the variables (of each dimension) and the marginal distributions. The theory is applied to generate from an initial Monte Carlo simulated sample, a much larger sample for the circuit under study. For basic definitions and properties of copulas we refer to [7]. A short introduction to copulas for test metrics estimation is given in [3]. To illustrate the application of the theory of copulas to the generation of a larger sample, we consider an example of a bivariate distribution as represented in Figure 2.

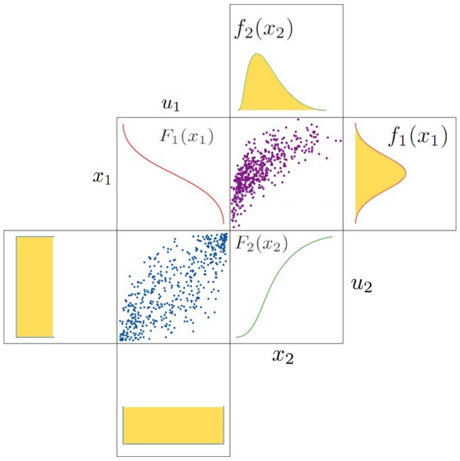


Fig. 2. Calculating the copula from an initial circuit sample.

The scatter plot of a bivariate random vector $X = (X_1, X_2)$ is shown in the upper right corner (purple colour) of Figure 2. In order to separate the dependencies between these two random variables from their marginal distributions, we apply the transformation $u_i = F_i(x_i)$, where each initial sample point (x_1, x_2) is transformed into a new point (u_1, u_2), using the marginal CDF of each variable (F_1 for x_1 and F_2 for x_2). The result of this transformation is the bivariate random vector $U = (U_1, U_2)$ shown in the lower left corner (blue colour). This new sample distribution corresponds to an empirical copula for which the marginal distributions are uniform. This complete and scale-free description of dependence is more suitable to be fitted to well known multivariate parametric laws called copulas.

Once a parametric form of the copula has been fitted, we can use it for the generation of a much larger sample. We can sample an arbitrary large number of points from the estimated copula, and each point can be transformed back to the initial distribution using the inverse CDF of each marginal variable $x_i = F_i^{-1}(u_i)$. The sample data generated will then have the same joint CDF as the initial set of data. This is illustrated in Figure 3. As in [3], we have considered a Gaussian copula for this example that can easily be recognized by his eye-like elliptical form (blue colour). To sample the Gaussian copula, we first transform the initial empirical copula into a multivariate Gaussian distribution using standardised marginal distributions. We next fit a multivariate Gaussian distribution to this data which is easy to sample using readily available techniques (black colour). Each generated point in the multivariate Gaussian distribution is transformed back to the copula using the standardized marginals (blue colour). Finally, each copula point is transformed to the initial distribution using the inverse CDF of each marginal variable (purple colour).

B. Archimedean copulas

In the sequel we will illustrate the use for test metrics estimation of another kind of copula which belongs to the family of Archimedean copulas. Archimedean copulas include a large variety of copula families that can be easily constructed to model non linear dependencies and non elliptical distributions. For example, Archimedean copulas can describe asymmetric dependencies, where the dependence coefficients in the upper and the lower tails are different. In this work, we will use as dependence coefficient Kendall's τ instead of the classical linear correlation factor ρ. An estimator $\hat{\tau}$ of this coefficient is calculated as follows:

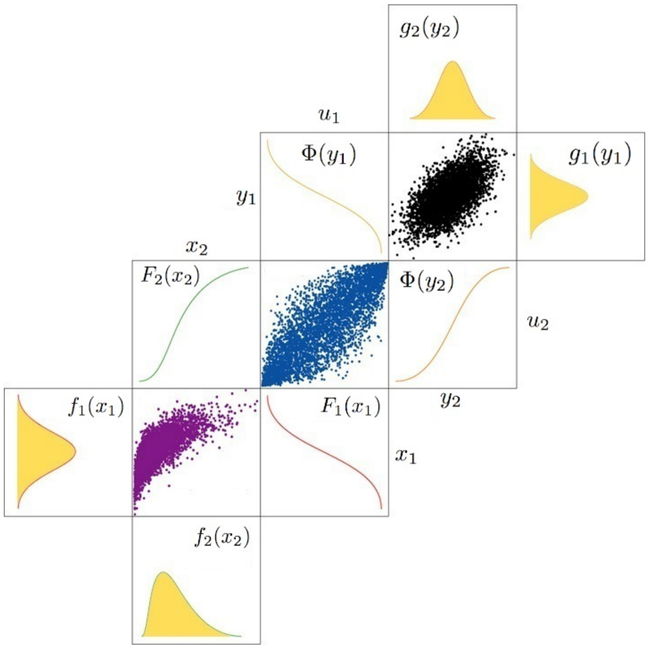


Fig. 3. Sample generation using a Gaussian copula.

$$\hat{\tau} = \frac{2}{n(n-1)} \sum_{i<j} sgn[(x_i - x_j)(y_i - y_j)], \quad i, j = 1, ..., n \tag{1}$$

where

$$sgn(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{if } z < 0 \end{cases}$$

and where $(x_1, y_1), \ldots, (x_n, y_n)$ are n observations from a vector (X, Y) of continuous random variables.

Archimedean copulas have the following form:

$$C(u_1, u_2) = \begin{cases} \varphi^{-1}(\varphi(u_1) + \varphi(u_2)) & if \\ & \varphi(u_1) + \varphi(u_2) \leq \varphi(0) \\ 0 & otherwise \end{cases} \tag{2}$$

where φ is called the generator function of the Archimedean copula. Notice that the generator function allows to write the copula as a sum of functions of the marginals. For $0 \leq u \leq 1$, φ is defined as $\varphi(1) = 0$, $\varphi'(u) < 0$ and, $\varphi''(u) > 0$. This equation can be generalized to d dimensions.

Using the generator function φ, the Kendall's τ of an archimedean copula can be written as follows :

$$\tau = 1 + 4 \int_0^1 \frac{\varphi(u)}{\varphi'(u)} du \tag{3}$$

It is estimated using Equation (1).

C. Clayton Copula

The Clayton copula [8][9] is an archimedean copula whose generator function is defined as:

$$\varphi(u)=\frac{1}{\theta}(u^{-\theta}-1) \tag{4}$$

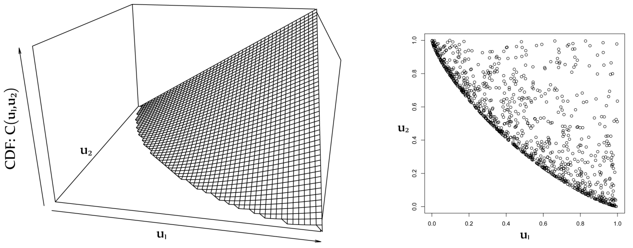
with, $\theta \in]-1,0[\cup]0,\infty[$. For the two dimensional case $C(u_1,u_2)$ can be written as follows:

$$C(u_1,u_2)=\max\left\{\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-\frac{1}{\theta}},0\right\} \tag{5}$$

The generalized form of this equation is given in [7]. The parameter θ depends on Kendall's τ and is calculated as follows:

$$\theta=-\frac{2\tau}{\tau-1} \tag{6}$$

Figure 4(a) shows the CDF of the Clayton copula with a parameter $\theta=-0.63$ and Figure 4(b) shows a set of 1000 samples generated from this copula.



(a) (b)

Fig. 4. (a) CDF of a Clayton copula with $\theta=-0.63$, (b) 1000 samples generated from the Clayton copula.

D. Simulating a Clayton copula

To generate a set of points by means of the bivariate copula of Clayton, we will apply the algorithm of conditional distributions [9]. A point (u_1,u_2) is generated in two steps as follows:

- Generate two independent random variables, u_1 and v uniformly distributed over $[0,1]$.
- Calculate $u_2=\left[u_1^{-\theta}\left(v^{-\theta/(1+\theta)}\right)+1\right]^{-1/\theta}$.

Executing these two steps n times, it is possible to generate a set of n samples $(u_1^1,u_2^1),(u_1^2,u_2^2),\ldots,(u_1^n,u_2^n)$.

III. THE NEW TOOL

The developed tool allows the generation of multidimensional samples following either a multivariate Gaussian distribution or a copula-based distribution with arbitrary marginals (Gaussian, Clayton or Gumbel copulas).

Figure 5 shows the architecture of the tool where a data file originating from a Monte Carlo simulation of the circuit under test is first loaded. The file contains the set of output parameter values (performances and test measures) of the circuit. From this sample, the joint CDF of all output parameters must be estimated. The tool considers two cases. If the data follow a gaussian distribution, the parameters of a multivariate normal distribution are fitted and a large sample is generated from this model. If the multinormal model cannot be fitted, the tool proceeds with the estimation of the copula. First, the CDFs of each marginal distribution are calculated. The transformation of the points in the initial sample using the marginal CDFs results in an empirical copula to which a Gaussian, Clayton or Gumbel copula will be fitted. If this fitting is successful, an arbitrarily large sample of circuits can be generated from the copula-based model. New points are sampled from the estimated copula, and these points are transformed using the inverse of the estimated marginal CDFs. Finally, the test metrics will be estimated on the basis of this new large sample.

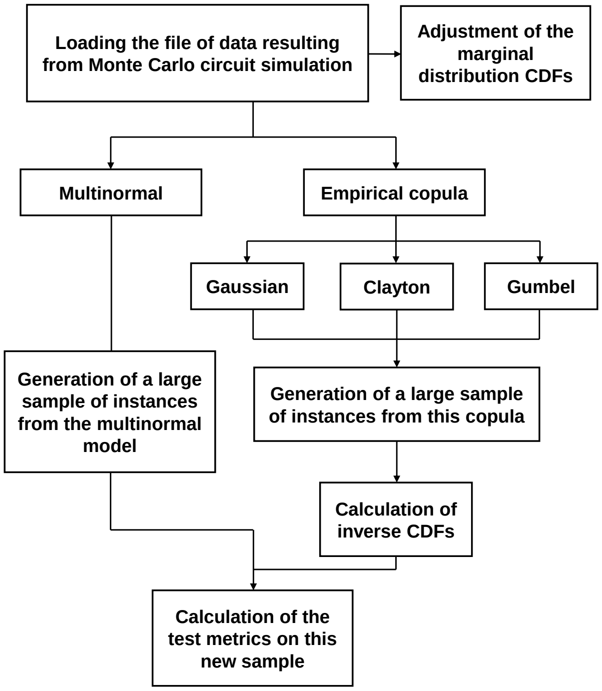


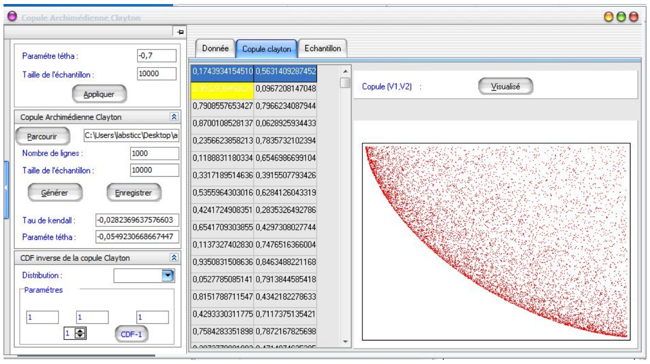
Fig. 5. Architecture of the developed tool.

Figure 6 shows the interface of the tool: a set of 10000 circuits (in black) is generated by means of a copula of Clayton, which itself has been estimated from a set of 1000 circuits (in red).

IV. CASE STUDY AND RESULTS

In our case study we consider a BIST technique for the pixels of a CMOS imager as presented in [10]. For illustration purposes, we will consider one typical functional performance that will be replaced by a simple test measure.

The circuit performance is the pixel Dark Signal Non Uniformity (DSNU). The DSNU is a measurement of a pixel output when the imager receives no illumination (dark signal). This output parameter measures the variability on the pixel performance, typically depending on circuit parameters such as transistor thresholds and leakages. This test is costly since it requires a carefully controlled dark condition and a large



Paramétre tétha : -0,7

Taille de l'échantillon : 10000

C:\Users\labsticc\Desktop\a

Nombre de lignes : 1000

Taille de l'échantillon : 10000

Tau de kendall : -0,0282369637576603

Paraméte tétha : -0,0549230668667447

1 1 1

1

0,1743934154510	0,5631409287452
0,9502839850021	0,0967208147048
0,7908557653427	0,7966234087944
0,8700108528137	0,0628925934433
0,2356623858213	0,7835732102394
0,1188831180334	0,6546986699104
0,3317189514636	0,3915507793426
0,5355964303016	0,6284126043319
0,4241724908351	0,2835326492786
0,6541709303855	0,4297308027744
0,1137327402830	0,7476516366004
0,9350831508636	0,8463488221168
0,0527785085141	0,7913844585418
0,8151788711547	0,4342182278633
0,4293330311775	0,7117375135421
0,7584283351898	0,7872167825698

Fig. 6. Interface of the tool.

integration time to measure the effects of dark currents. The specification for this imager is $DSNU \in [-0.032, 0.032]$ V, fixed at 3σ which leads to a yield of $\simeq 3000dppm$ (defect parts-per-million, i.e. 3000 pixels out of specifications in 1 million).

The low-cost test measurement that we consider is a purely electrical test signal that is measured very fast and is thus independent of the optical condition of the imager. The pixel is electrically stimulated as indicated in [10] and the pixel output is measured in a very short time response. We call this test measure VA. By simulation, we have observed a non-linear relationship between the functional DSNU performance and the purely electrical BIST measurement VA.

A. *The statistical model of the pixel behaviour*

Figure 7 shows the statistical model (joint distribution) of 1000 pixels. Clearly, this model (in black) does not follow a multivariate Gaussian distribution (in gray).

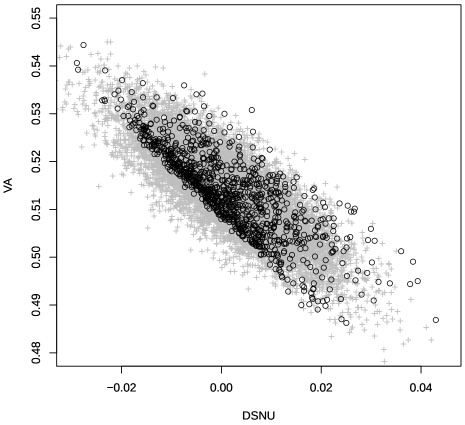


Fig. 7. Initial set of 1000 pixels obtained from circuit-level Monte Carlo simulation (black) vs. set of 1000 samples obtained from a multivariate Gaussian distribution (gray).

Since the statistical model of this joint distribution is unknown, we are going to calculate the empirical copula of this set to check whether it follows a known copula. Figure 8 shows the estimated copula of the pixels. It is obtained by transforming each point of the initial sample by the CDF of each marginal. It does not have the typical elliptic shape of a Gaussian copula. However, by comparing it to the sample of Figure 4, we conclude that its form is identical to that of the bivariate copula of Clayton. For a formal verification of this, we have applied the fitting test for Archimedean copulas as presented in [11].

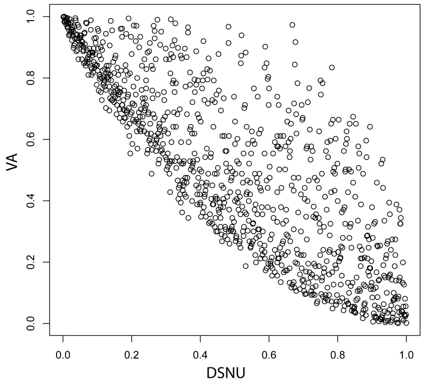
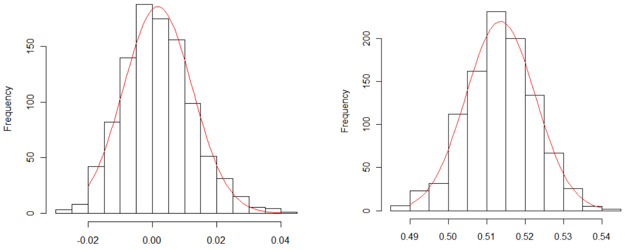


Fig. 8. Empirical copula of the initial set.

B. *Generation of a large sample*

To generate a sample of large size from the Clayton copula, we make use of the conditional distribution algorithm presented in Section II.D. Next, by calculating the inverse CDFs of the marginals we will obtain a large sample with the same statistical model than that of the pixels. To be able to calculate the inverse CDFs, we have observed the marginal distributions of the output parameters (DSNU and VA). Figure 9 indicates that these distributions are Gaussian. They have been validated by the classical univariate adjustment test of Kolmogorov-Smirnov. The parameters of these Gaussians are given as follows:

- DSNU : μ=1.7mV, σ=10mV
- VA : μ=513mV, σ=9mV



(a) DSNU

(b) VA

Fig. 9. The marginal distributions of the output parameters of the pixels.

Thereafter, we have estimated the parameter $\hat{\theta}$ of the copula of Clayton from the initial sample of pixels by means of Equation (6). This parameter is calculated from Kendall's τ, in our case: $\hat{\theta}$ = - 0,77 and τ = - 0,63. Using this parameter we have generated a set of 16000 pixels based on the copula of Clayton. Figure 10 shows this set (in black) in comparison to the initial one of 1000 pixels (in red).

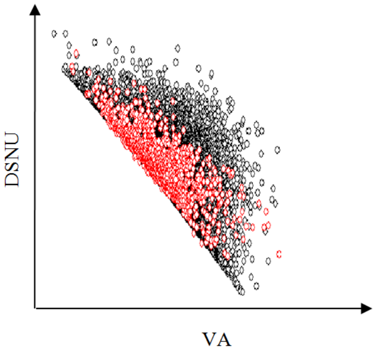


Fig. 10. 1000 pixels generated from circuit-level Monte Carlo simulation (red) vs. 16000 generated from the Clayton copula (black).

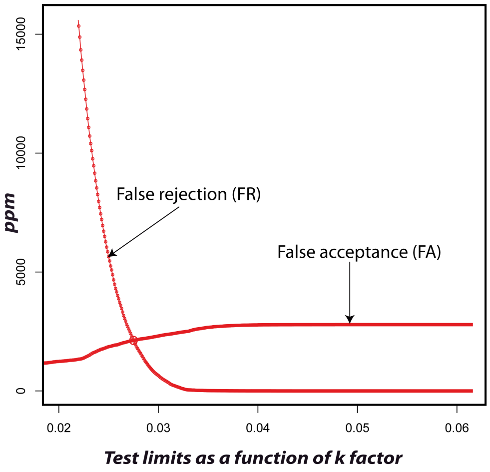


Fig. 11. Test metrics vs. test limits estimated form the set of 10^6 circuits generated from the Clayton copula.

C. Test metrics estimation

To estimate the test metrics, a sample of one million pixels has been generated following the method presented in the previous section. This sample will be used to fix the test limits of the measure VA on the basis of a compromise between the Defect Level (D_L) and the Yield Loss (Y_L) of the test technique. These test metrics are defined as follows:

$$\begin{aligned} D_L &= \text{is the proportion of the faulty circuits that pass the test;} \\ Y_L &= \text{is the proportion of the circuits failing the test within} \\ &\quad\ \text{those that are functional.} \end{aligned}$$

where the faulty circuits are those whose DSNU does not verify the above given specification ([-0.032, 0.032]); the functional circuits are those which respect this specification. The circuits passing the test are those verifying the test limits fixed by the test engineer, and those failing the test are those not verifying these limits.

Figure 11 illustrates, for different test limits, the values of the defect level together with the corresponding yield loss. The test limits have been chosen in the interval $[0.51 - k, 0.51 + k]$ V where the factor k varies between 0 and 0.06 with a step of 0.001. The chosen test limit is the one where $D_L = Y_L = 2124ppm$.

V. CONCLUSIONS AND FUTURE WORK

In this paper we have presented a tool allowing the generation of a very large sample of circuits from an initial one of much smaller size obtained from circuit-level Monte Carlo simulation. The new sample is obtained from a statistical model obtained from the data in the initial sample. The tool can use either the multivariate Gaussian model or models based on Gaussian as well as Archimedean copulas (of Clayton and Gumbel type). The tool has been validated on a case-study that represents a BIST technique for CMOS imager pixels. The statistical model for this case-study has required a Clayton copula and we have shown how to generate a large sample of one million pixels from an initial sample of one thousand. The new sample has been used to estimate the test metrics (defect level and yield loss) with high precision and to set the test limits.

REFERENCES

[1] A. Bounceur, S. Mir, E. Simeu, and L. Rolíndez, *Estimation of test metrics for the optimisation of analogue circuit testing*, *Journal of Electronic Testing: Theory and Applications*, vol. 23, no. 6, pp. 471-484, 2007.
[2] H. Stratigopoulos, S. Mir, and A. Bounceur. *Evaluation of analog/RF test measurements at the design stage*. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 28(4), pp. 582-590, April 2009.
[3] A. Bounceur, S. Mir, H.-G. Stratigopoulos. *Estimation of Analog Parametric Test Metrics Using Copulas*. IEEE Transactions on CAD of Integrated Circuits and Systems. 30(9), pp. 1400-1410, 2011.
[4] K. Beznia, A. Bounceur, S. Mir, R. Euler. *Parametric test metrics estimation using non-Gaussian copulas*. IEEE International Mixed-Signals, Sensors, and Systems Test Workshop (IMS3TW'11), Santa Barbara, USA, 2011.
[5] A. Bounceur, S. Mir, L. Rolíndez and E. Simeu. *CAT platform for analogue and mixed-signal test evaluation and optimization*, in IFIP International Federation for Information Processing, Volume 249, VLSI-SoC : Research trends in VLSI and Systems on Chip, eds. De Micheli, G., Mir, S., Reis, R., (Boston : Springer), pp.281-300, 2007.
[6] Y. Lechuga, A. Bounceur, R. Mozuelos, M. Martínez, S. Bracho and S. Mir. Test limits evaluation for an ADC Design-for-Test approach by using a CAT platform. 23rd International Conference on Design of Circuits and Integrated Systems, Grenoble, France, November 2008.
[7] R.-B. Nelsen. *An Introduction to Copulas*. Lecture Notes in Statistics. Springer, New York, 1999.
[8] D.G. Clayton, *A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence*. Biometrika, 65, pp. 141-151, 1978.
[9] C. Genest and R.J. MacKay, *Copules archimédiennes et familles de lois bidimensionnelles dont les marges sont données*. Canad. J. Statist., 14, pp.145-159, 1986.
[10] L. Lizarraga, S. Mir, G. Sicard, *Experimental Validation of a BIST Technique for CMOS Active Pixel Sensors*, 27th IEEE VLSI Test Symposium (VTS), pp.189-194, 2009.
[11] C. Genest, B. Remillard and D. Beaudoin. *Goodness-of-fit tests for copulas: A review and a power study*. Insurance: Mathematics and Economics, 44, pp. 199-214, 2009.